WORKED EXAMPLES

FOR THE DESIGN OF CONCRETE BUILDINGS

Based on BSI publication
DD ENV 1992-1-1: 1992.
Eurocode 2: Design of concrete structures.
Part 1. General rules and rules for buildings.

BCA ARUP TIETZ

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FOREWORD

Eurocode 2: *Design of concrete structures,* Part 1: *General rules and rules for buildings* (EC2)[1] sets out both the principles for the design of all types of concrete structure, and design rules for buildings. Rules for other types of structure and particular areas of technology, including precast concrete elements and structures, will be covered in other parts of EC2.

EC2 contains a considerable number of parameters for which only indicative values are given. The appropriate values for use in the UK are set out in the National Application Document (NAD)[1] which has been drafted by BSI. The NAD also includes a number of amendments to the rules in EC2 where, in the draft for development stage of EC2, it was decided that the EC2 rules either did not apply, or were incomplete. Two such areas are the design for fire resistance and the provision of ties, where the NAD states that the rules in BS 8110[2] should be applied.

Attention is drawn to Approved Document A (Structure) related to the Building Regulations 1991[3], which states that Eurocode 2, including the National Application Document, is considered to provide appropriate guidance for the design of concrete buildings in the United Kingdom.

Enquiries of a technical nature concerning these worked examples may be addressed to the authors directly, or through the BCA, or to the Building Research Establishment.

CONTENTS

1 INTRODUCTION AND SYMBOLS

1.1 Introduction and symbols

The main objective of this publication is to illustrate through worked examples how EC2[1] may be used in practice. It has been prepared for engineers who are generally familiar with design practice in the UK, particularly to BS 8110[2].

The worked examples relate primarily to in-situ concrete building structures. The designs are in accordance with EC2: Part 1 as modified by the UK National Application Document[1]. Where necessary, the information given in EC2 has been supplemented by guidance taken from other documents.

The core example, in Section **2**, is a re-design of the in-situ concrete office block used in the BCA publication *Designed and detailed (BS 8110: 1985)*, by Higgins & Rogers[4]. Other design aspects and forms of construction are fully explored by means of further examples in Sections **3** to **12**.

Equations and charts for the design of beam and column sections, taken from the *Concise Eurocode for the design of concrete buildings*[5], are given in Section **13**. Publications used in the preparation of this book, and from which further information may be obtained, are listed in the References. Unless otherwise stated, all references to BS 8110 refer to Part 1.

Two conventions have been adopted in the preparation of this book. Statements followed by '. . . . . . . OK' mark places where the calculated value is shown to be satisfactory. Green type is used to draw attention to key information such as the reinforcement to be provided.

The calculations are cross-referenced to the relevant clauses in EC2 and, where appropriate, to other documents; all references in the right-hand margins are to EC2 unless indicated otherwise.

The symbols used throughout the publication are listed and defined below, and are generally those used in EC2 itself.

1.2 Symbols

A	Area of cross-section
A_c	Area of concrete cross-section
A_{ct}	Area of concrete within tensile zone
$A_{ct,ext}$	Area of concrete tensile zone external to links
A_k	Area enclosed within centre-line of thin-walled section
A_p	Area of prestressing tendons
A_s	Area of tension or, in columns, total longitudinal reinforcement
A'_s	Area of compression reinforcement
$A_{s,min}$	Minimum area of tension or, in columns, total longitudinal reinforcement
$A_{s,prov}$	Area of tension reinforcement provided
$A_{s,req}$	Area of tension reinforcement required
$A_{s,surf}$	Area of surface reinforcement
A_{sf}	Area of transverse reinforcement within flange of beam
A_{sl}	Area of tension reinforcement effective at a section or, for torsion, area of additional longitudinal reinforcement
A_{sw}	Area of shear reinforcement or torsion links
$A_{sw,min}$	Minimum area of shear reinforcement
$E_{c,eff}$	Effective modulus of elasticity of concrete

E_{ci}	Secant modulus of elasticity of concrete at transfer
E_{cm}	Secant modulus of elasticity of concrete
E_s	Modulus of elasticity of reinforcement or prestressing steel
F_c	Force due to concrete in compression at ultimate limit state
F_s	Force in tension reinforcement or prestressing tendons at ultimate limit state
F_{Sd}	Design value of tie force in pilecap
$F_{Sd,sup}$	Design value of support reaction
F_t	Tie force in corbel or due to accidental action
F_v	Vertical force applied to corbel or, for sway classification of structures, sum of all vertical loads under service conditions
G_k	Characteristic value of permanent action or dead load
G_{kf}	Characteristic dead floor load
G_{kr}	Characteristic dead roof load
H	Overall depth of tank
H_c	Horizontal force applied to corbel
I	Second moment of area of cross-section
I_{I}	Second moment of area of uncracked concrete section
I_{II}	Second moment of area of cracked concrete section
I_b	Second moment of area of beam section
I_c	Second moment of area of concrete section
I_{col}	Second moment of area of column section
I_{slab}	Second moment of area of slab section
I_x	Second moment of area of section in x direction
I_y	Second moment of area of section in y direction
J	St Venant torsional stiffness of rectangular section
J_{tot}	St Venant torsional stiffness of total section
K	Deflection-curvature factor dependent upon the shape of the bending moment diagram
K_1	Reduction factor for calculation of second order eccentricity
K_2	Coefficient taking account of decrease in curvature due to increasing axial force
M	Bending moment
M_c	Moment of force, F_c, about tension reinforcement
M_{cr}	Moment causing cracking
M_{cx}	Moment of force, N_c, about x axis
M_{cy}	Moment of force, N_c, about y axis
M_o	First order moment
M_{Rd}	Design moment of resistance
$M_{Rd,c}$	Moment of force, $N_{Rd,c}$, about mid-depth of section
$M'_{Rd,c}$	Moment of force, $N'_{Rd,c}$, about mid-depth of section
$M_{Rd,s}$	Moment of force, $N_{Rd,s}$, about mid-depth of section

M_{Sd}	Design value of applied moment
M_{Sdx}	Design moment in x direction
M_{Sdy}	Design moment in y direction
M_{Sd1}	First order moment at end 1
M_{Sd2}	First order moment at end 2
$M_{Sd,cs}$	Design moment in column strip
$M_{Sd,ms}$	Design moment in middle strip
M_{span}	Moment in span
M_{sup}	Moment at support
$M_{t,max}$	Maximum moment transfer value
M_x	Moment about x axis
M_y	Moment about y axis
N	Axial force
N_c	Axial force due to concrete in compression
N_{Rd}	Design resistance to axial force
$N_{Rd,c}$	Design resistance to axial force due to concrete
$N'_{Rd,c}$	Design resistance to axial force due to concrete of hypothetical section of depth $x > h$
$N_{Rd,s}$	Design resistance to axial force due to reinforcement
N_{Sd}	Design value of applied axial force
$N_{Sd,m}$	Mean applied axial force
P	Prestressing force or point load
P_{av}	Average prestressing force along tendon profile
$P_{m,o}$	Initial prestressing force at transfer
$P_{m,t}$	Mean effective prestressing force at time t
$P_{m,\infty}$	Final prestressing force after all losses
P_o	Maximum initial prestressing force at active end of tendon
P_{req}	Required prestressing force
P_t	Final prestressing force at service
Q_k	Characteristic value of variable action or imposed load
Q_{kf}	Characteristic value of imposed floor load
Q_{kr}	Characteristic value of imposed roof load
R_A	Reaction at support A
R_B	Reaction at support B
S	First moment of area of reinforcement about centroid of section
S_I	First moment of area of reinforcement about centroid of uncracked section
S_{II}	First moment of area of reinforcement about centroid of cracked section
T_d	Design value of tensile force in longitudinal reinforcement
T_{Rd1}	Maximum torsional moment resisted by concrete struts
T_{Rd2}	Maximum torsional moment resisted by reinforcement

T_{Sd}	Design value of applied torsional moment
$T_{Sd,fl}$	Torsional moment applied to flange
$T_{Sd,tot}$	Total applied torsional moment
$T_{Sd,w}$	Torsional moment applied to web
V_A	Shear force at support A
V_B	Shear force at support B
V_{cd}	Design shear resistance provided by concrete
V_{ext}	Shear force at exterior support
V_{int}	Shear force at interior support
V_{Rd1}	Design shear resistance of member without shear reinforcement
V_{Rd2}	Maximum design shear force to avoid crushing of notional concrete struts
V_{Rd3}	Design shear resistance of member with shear reinforcement
V_{Sd}	Design value of applied shear force
V_{Sdx}	Design shear force in x direction
V_{Sdy}	Design shear force in y direction
$V_{Sd,max}$	Maximum design shear force
V_{wd}	Design shear resistance provided by shear reinforcement
W_b	Section modulus at bottom fibre
W_{cp}	Section modulus at centroid of tendons
W_k	Characteristic value of wind load
W_t	Section modulus at top fibre
a	Distance or deflection or maximum drape of tendon profile
a_{I}	Deflection based on uncracked section
a_{II}	Deflection based on cracked section
a_c	Distance of load from face of support (corbel) or from centre-line of hanger bars (nib)
a_{cs}	Deflection due to concrete shrinkage
a_i	Distance from face of support to effective centre of bearing
a_l	Horizontal displacement of the envelope line of tensile force
a_{tot}	Total deflection
a_v	Distance between positions of zero and maximum bending
a_x	Deflection at distance x along span
a_1,a_2	Values of a_i at ends of span
b	Width of section or flange width or lateral cover in plane of lap
b_{av}	Average width of trapezoidal compression zone
b_e	Width of effective moment transfer strip
b_{eff}	Effective width of flange
b_{min}	Minimum width of support beam
b_r	Width of rib
b_{sup}	Width of support

b_t	Mean width of section over the tension zone
b_w	Minimum width of section over the effective depth
c	Cover to longitudinal torsion reinforcement
c_1, c_2	Support widths at ends of beam
d	Effective depth of section
d'	Depth to compression reinforcement
d_{av}	Average effective depth for both directions
d_b	Depth to bar considered
d_{crit}	Distance of critical section for punching shear from centroid of column
d_f	Effective depth of flange
d_H	Effective depth for punching shear check in column head
d_{max}	Maximum effective depth for both directions
d_{min}	Minimum effective depth for both directions
d_x	Effective depth in x direction
d_y	Effective depth in y direction
d_1	Effective depth to bars in layer 1
d_2	Effective depth to bars in layer 2
e_a	Additional eccentricity due to geometrical imperfections
e_{ay}	Additional eccentricity in the y direction
e_{az}	Additional eccentricity in the z direction
e_e	Equivalent eccentricity at critical section
e_{oy}	First order eccentricity in y direction
e_{o1}, e_{o2}	First order eccentricities at ends of column
e_{tot}	Total eccentricity
e_y	Eccentricity in y direction
e_z	Eccentricity in z direction
e_2	Second order eccentricity
e_{2y}	Second order eccentricity in y direction
e_{2z}	Second order eccentricity in z direction
f_b	Stress in concrete at bottom fibre
f_{bd}	Design value of ultimate bond stress
f_{cd}	Design cylinder strength of concrete
f_{ci}	Cube strength of concrete at transfer
f_{ck}	Characteristic cylinder strength of concrete
$f_{ct,eff}$	Effective tensile strength of concrete at time cracking is expected to occur
f_{ctm}	Mean value of axial tensile strength of concrete
f_{cu}	Characteristic cube strength of concrete
f_{pd}	Design tensile strength of prestressing steel
f_{pk}	Characteristic tensile strength of prestressing steel

f_{Rdu}	Design value of ultimate bearing stress
f_s	Stress in reinforcement
f_t	Stress in concrete at top fibre
f_{yd}	Design yield strength of reinforcement
f_{yk}	Characteristic yield strength of reinforcement
f_{yld}	Design yield strength of longitudinal torsion reinforcement
f_{ywd}	Design yield strength of shear reinforcement or torsion links
f_{ywk}	Characteristic yield strength of shear reinforcement or torsion links
g_k	Characteristic dead load per unit area
h	Overall depth of section or liquid in tank
h'	Reduced value of h for separate check about minor axis of column section with biaxial eccentricities
h_a	Active height of deep beam
h_c	Overall depth of corbel at face of support
h_f	Overall depth of flange
h_H	Depth of column head
h_{max}	Larger dimension of rectangular section
h_{min}	Smaller dimension of rectangular section
h_{tot}	Total height of structure in metres
i	Radius of gyration of section
k	Coefficient or factor
k_A	Restraint coefficient at end A
k_B	Restraint coefficient at end B
k_{bottom}	Restraint coefficient at bottom
k_c	Minimum reinforcement coefficient associated with stress distribution
k_{top}	Restraint coefficient at top
k_1	Crack spacing coefficient associated with bond characteristics
k_2	Crack spacing coefficient associated with strain distribution
l	Length or span
l'	Length of tendon over which anchorage slip is taken up
l_b	Basic anchorage length
$l_{b,min}$	Minimum anchorage length
$l_{b,net}$	Required anchorage length
l_c	Diameter of circular column
l_{col}	Height of column between centres of restraints
l_{eff}	Effective span
$l_{eff,slab}$	Effective span of slab
l_H	Distance from column face to edge of column head
l_n	Clear distance between faces of support

l_o	Distance between positions of zero bending or effective height of column or, for deep beams, clear distance between faces of support
l_{ot}	Length of compression flange between lateral supports
l_s	Required lap length or floor to ceiling height in metres
$l_{s,min}$	Minimum lap length
l_t	Greater of distances in metres between centres of columns, frames or walls supporting any two adjacent floor spans in direction of tie under consideration
l_x	Effective span in x direction
l_y	Effective span in y direction
l_1, l_2	Lengths between centres of supports or overall dimensions of rectangular column head
m_{Sd}	Minimum design moment per unit width
n	Ultimate design load per unit area or number of tendons or number of sub-divisions
p'	Average loss of prestressing force per unit length due to friction
q	Equivalent load per unit length due to prestressing force profile
q_k	Characteristic imposed load per unit area
r	Radius of bend or radius of curvature
r_I	Radius of curvature based on uncracked section
r_{II}	Radius of curvature based on cracked section
r_{cs}	Radius of curvature due to concrete shrinkage
r_{csI}	Radius of curvature due to concrete shrinkage based on uncracked section
r_{csII}	Radius of curvature due to concrete shrinkage based on cracked section
r_{tot}	Total radius of curvature
s	Spacing of shear reinforcement or torsion links or horizontal length of tendon profile
s_f	Spacing of transverse reinforcement within flange of beam
s_{max}	Maximum spacing of shear reinforcement or torsion links
s_{rm}	Average final crack spacing
t	Thickness of supporting element or wall of thin-walled section
t_{min}	Minimum thickness of wall
u	Circumference of concrete section or critical section for punching shear
u_k	Circumference of area A_k
v_{Rd1}	Design shear resistance per unit length of critical perimeter, for slab without shear reinforcement
v_{Rd2}	Maximum design shear resistance per unit length of critical perimeter, for slab with shear reinforcement
v_{Rd3}	Design shear resistance per unit length of critical perimeter, for slab with shear reinforcement
v_{Sd}	Design value of shear force per unit length of critical perimeter
w	Support width or quasi-permanent load per unit length
w_k	Design crack width

w_{min}	Minimum width of support
x	Neutral axis depth or distance along span from face of support or distance along tendon or column dimension in x direction
x'	Maximum depth of concrete in compression in direction of minor axis for column section with biaxial eccentricities
x_c	Depth of concrete in compression at position of minor axis for column section with biaxial eccentricities
y	Drape of tendon at distance x along profile or column dimension in y direction
y_t	Distance from centroid of uncracked section to extreme tension fibre
z	Lever arm of internal forces
z_{cp}	Distance from centroid of section to centroid of tendons
α	Reduction factor for concrete compressive stress or modular ratio or deformation parameter
α_I	Value of parameter based on uncracked section
α_{II}	Value of parameter based on cracked section
α_a	Effectiveness coefficient for anchorage
α_e	Effective modular ratio
α_n	Reduction coefficient for assumed inclination of structure due to imperfections
α_{sx}, α_{sy}	Moment coefficients in x and y directions
α_1	Effectiveness coefficient for lap
β	Coefficient with several applications including shear resistance enhancement, effective height of column, St Venant torsional stiffness, punching shear magnification, design crack width
β_{red}	Reduced value of shear resistance enhancement coefficient
β_1	Coefficient associated with bond characteristics
β_2	Coefficient associated with duration of load
γ_c	Partial safety factor for concrete material properties
γ_F	Partial safety factor for actions
γ_G	Partial safety factor for permanent action or dead load
$\gamma_{G,inf}$	Partial safety factor for permanent action, in calculating lower design value
$\gamma_{G,sup}$	Partial safety factor for permanent action, in calculating upper design value
γ_P	Partial safety factor for actions associated with prestressing force
γ_Q	Partial safety factor for variable action or imposed load
γ_S	Partial safety factor for steel material properties of reinforcement or prestressing tendons
δ	Ratio of redistributed moment to moment before redistribution
ϵ_b	Strain in concrete at bottom of section
ϵ_{cs}	Basic concrete shrinkage strain
$\epsilon_{cs\infty}$	Final concrete shrinkage strain
ϵ_p	Minimum strain in tendons to achieve design tensile strength
ϵ_{pm}	Strain in tendons corresponding to prestressing force $P_{m,t}$

ϵ_s	Strain in reinforcement
$\epsilon_s(t,t_o)$	Estimated concrete shrinkage strain
ϵ_{sm}	Mean strain in reinforcement allowing for tension stiffening effect of concrete
ϵ_u	Ultimate compressive strain in concrete
ϵ_{yd}	Initial yield strain in reinforcement
ζ	Distribution coefficient
η	Moment coefficient
Θ	Angle of rotation or angle between concrete struts and longitudinal axis
λ	Slenderness ratio
λ_{crit}	Critical slenderness ratio
λ_m	Mean slenderness ratio of all columns in storey considered
λ_{min}	Slenderness ratio beyond which column is considered slender
μ	Coefficient of friction between tendon and duct or applied moment ratio
μ_{lim}	Limiting value of applied moment ratio for singly reinforced section
ν	Efficiency factor or assumed inclination of structure due to imperfections
ν_{red}	Reduced value of assumed inclination of structure
ν_u	Longitudinal force coefficient
ρ	Tension reinforcement ratio or density of liquid
ρ'	Compression reinforcement ratio
ρ_l	Longitudinal tension reinforcement ratio
ρ_{lx}, ρ_{ly}	Longitudinal tension reinforcement ratios in x and y directions
ρ_r	Effective reinforcement ratio
ρ_w	Shear reinforcement ratio
$\rho_{w,min}$	Minimum shear reinforcement ratio
ρ_1, ρ_2	Principal and secondary reinforcement ratios in solid slabs
σ_{cg}	Stress in concrete adjacent to tendons due to self-weight and any other permanent actions
σ_{cp}	Average stress in concrete due to axial force
σ_{cpo}	Initial stress in concrete adjacent to tendons due to prestress
σ_{po}	Initial stress in tendons immediately after stressing (pre-tensioning) or immediately after transfer (post-tensioning)
σ_s	Stress in tension reinforcement calculated on basis of cracked section
σ_{sr}	Value of σ_s under loading conditions causing first cracking
τ_{Rd}	Basic design shear strength
ψ	Factor defining representative value of variable action
ψ_o	Value of ψ for rare load combination
ψ_1	Value of ψ for frequent loading
ψ_2	Value of ψ for quasi-permanent loading
ω	Mechanical ratio of tension reinforcement
ω'	Mechanical ratio of compression reinforcement

ω_{lim}	Limiting value of ω for singly reinforced section
Σ_j	Total vertical force applied to frame at floor j
Δ_{sl}	Anchorage slip or wedge set
ΔF_d	Variation of longitudinal force in section of flange over distance a_v
ΔH_j	Equivalent horizontal force acting on frame at floor j due to assumed imperfections
$\Delta M_{Rd,c}$	Moment of force $\Delta N_{Rd,c}$ about mid-depth of section
ΔM_{Sd}	Reduction in design moment at support
$\Delta N_{Rd,c}$	Design resistance to axial force due to concrete in area of hypothetical section lying outside actual section
ΔP_c	Average loss of prestressing force due to elastic deformation of concrete
ΔP_{sl}	Loss of prestressing force at active end of tendon due to anchorage slip
$\Delta P_t(t)$	Loss of prestressing force due to creep, shrinkage and relaxation at time t
$\Delta P_\mu(x)$	Loss of prestressing force due to friction between tendon and duct at distance x from active end of tendon
$\Delta\sigma_{pr}$	Variation of stress in tendon due to relaxation
ϕ	Bar size or duct diameter or creep coefficient
$\phi(t,t_o)$	Creep coefficient, defining creep between times t and t_o, related to elastic deformation at 28 days
$\phi(\infty,t_o)$	Final creep coefficient

2 COMPLETE DESIGN EXAMPLE

2.1 Introduction

Design calculations for the main elements of a simple in-situ concrete office block are set out. The structure chosen is the same as that used in Higgins and Rogers' *Designed and detailed (BS 8110: 1985)*[4]. Calculations are, wherever possible, given in the same order as those in Higgins and Rogers enabling a direct comparison to be made between BS 8110[2] and EC2[1] designs. For the same reason, a concrete grade C32/40 is used. This is not a standard grade recognized by EC2 or ENV 206[6], which gives grade C35/45 in Table NA.1. Some interpolation of the tables in EC2 has, therefore, been necessary.

The example was deliberately chosen to be simple and to cover a considerable range of member types. Comparison shows that, for this type of simple structure, there is very little difference between BS 8110 and EC2 in the complexity of calculation necessary or the results obtained.

2.2 Basic details of structure, materials and loading

These are summarized in Table 2.1 and Figure 2.1.

Table 2.1 Design information

Intended use			
Laboratory and office block			
Fire resistance			
1 hour for all elements			
Loading (excluding self-weight of structure)			
Roof	–	imposed (kN/m^2)	1.5
	–	finishes (kN/m^2)	1.5
Floors	–	imposed including partition allowance (kN/m^2)	4.0
	–	finishes (kN/m^2)	0.5
Stairs	–	imposed (kN/m^2)	4.0
	–	finishes (kN/m^2)	0.5
External cladding (kN/m)			5.0
Wind load			
Speed (m/sec)			40
Factors			
s_1			1.0
s_2			0.83
s_3			1.0
C_f			1.1
Exposure class			
2b (external) and 1 (internal)			
Subsoil conditions			
Stiff clay – no sulphates			
Allowable bearing pressure (kN/m^2)			200
Foundation type			
Reinforced concrete footings to columns and walls			
Materials			
Grade C32/40 concrete with 20 mm maximum aggregate			
Characteristic strength of main bars (N/mm^2)			460
Characteristic strength of links (N/mm^2)			250
Self-weight of concrete (kN/m^3)			24

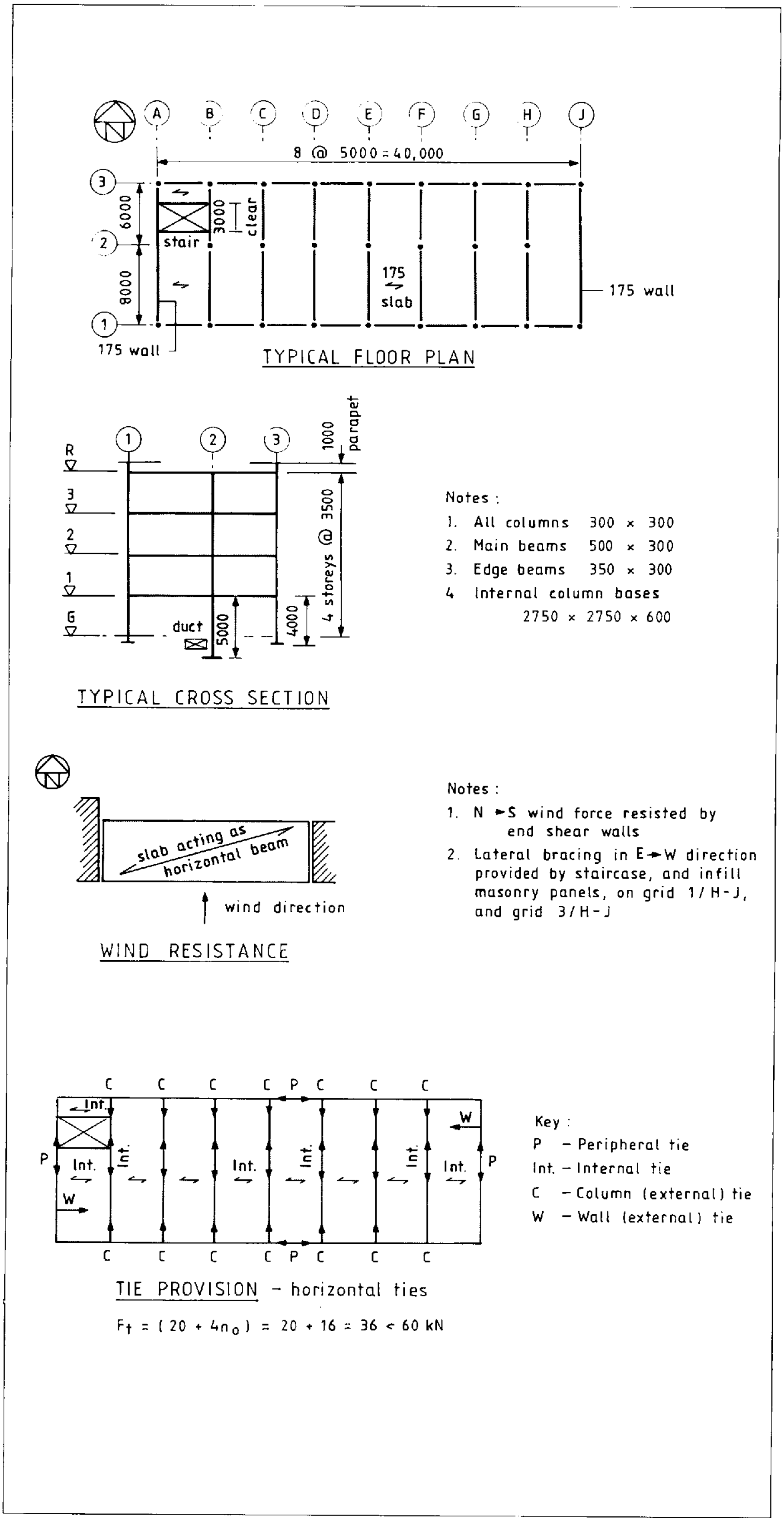


Figure 2.1 Structural details

2.3 Floor slab

2.3.1 Idealization of structure

Consider as a one-way continuous slab on knife-edge supports and design a typical 5 m interior span where

f_{ck} = 32 N/mm^2

f_{yk} = 460 N/mm^2

2.3.2 Cover for durability and fire resistance

Nominal cover for exposure class 1 (internal) is 20 mm. — NAD Table 6

Cover should not be less than the bar size when 20 mm maximum aggregate size is used. — 4.1.3.3

175 mm slab with 20 mm cover will give 1.5 hours fire resistance. . . . OK — NAD 6.1(a) & BS 8110 Table 3.5 & Figure 3.2

Use 20 mm nominal cover bottom and top

2.3.3 Loading

Self-weight of slab = 0.175 × 24 = 4.2 kN/m^2

Finishes = 0.5 kN/m^2

Characteristic permanent load (g_k) = 4.7 kN/m^2

Characteristic variable load (q_k) = 4.0 kN/m^2

Design permanent load	=	1.35 × 4.7	=	6.35 kN/m^2
Design variable load	=	1.5 × 4.0	=	6.0 kN/m^2

Table 2.2

2.3.4 Design moments and shears

Moments have been obtained using moment coefficients given in Reynolds and Steedman's *Reinforced concrete designer's handbook*[7], Table 33.

Support moment = 0.079 × 6.35 × 5^2 + 0.106 × 6.0 × 5^2
= 28.4 kNm/m

Span moment = 0.046 × 6.35 × 5^2 + 0.086 × 6.0 × 5^2 = 20.2 kNm/m

Design shear force = 0.5 × 6.35 × 5 + 0.6 × 6.0 × 5 = 33.9 kN/m

2.3.5 Reinforcement

2.3.5.1 Support

Assume effective depth = 175 − 20 − 6 = 149 mm

$$\frac{M}{bd^2 f_{ck}} = 0.040$$

$$\frac{A_s f_{yk}}{bd f_{ck}} = 0.048, \quad x/d = 0.092$$ (Section **13**, Table 13.1)

For zero redistribution, x/d should be less than 0.45 . . . . OK 2.5.3.4.2(5)

$A_s = 498\ mm^2/m$

Minimum area of reinforcement

$\frac{0.6 b_t d}{f_{yk}} \nless 0.0015\ b_t d = 224\ mm^2/m$ . . . . OK 5.4.2.1.1(1)

Use T12 @ 200 mm crs. (565 mm²/m)

2.3.5.2 Span

$\frac{M}{bd^2 f_{ck}} = 0.028$

$\frac{A_s f_{yk}}{bdf_{ck}} = 0.033, \quad x/d = 0.063$ (Section **13**, Table 13.1)

$A_s = 342\ mm^2/m$

Use T12 @ 300 mm crs. (377 mm²/m)

Note:
Reinforcement areas differ somewhat from those given by BS 8110 which permits design for the single load case of maximum load on all spans combined with 20% redistribution. EC2 requires alternate and adjacent spans to be considered. In this instance, no redistribution has been carried out but it would have been permissible to carry out 30% redistribution in the EC2 design. This would have resulted in an identical answer to that given by BS 8110 but ductility class H (as defined in prEN 10080[8]) reinforcement would need to be specified. NAD Table 5

2.3.6 Shear

Shear resistance of the slab without shear reinforcement is given by 4.3.2.3

$V_{Rd1} = \tau_{Rd} k (1.2 + 40\rho_l) b_w d$ Eqn 4.18

where

$\tau_{Rd} = 0.35\ N/mm^2$ Table 4.8

$k = 1.6 - d = 1.6 - 0.149 = 1.451$

$\rho_l = \frac{565}{1000 \times 149} = 0.0038$

Hence

$V_{Rd1} = 102.3\ kN/m > V_{Sd} = 33.9\ kN/m$ . . . . OK

No shear reinforcement required

Note:
Since shear is rarely a problem for normally loaded solid slabs supported on beams, as the calculation has shown, it is not usually necessary to check in these instances.

2.3.7 Deflection

Reinforcement ratio provided in span $= \dfrac{377}{1000 \times 149} = 0.0025$

Using NAD Table 7[1] and interpolating between 48 for 0.15% and 35 for 0.5%, a basic span/effective depth ratio of 44 is given. By modifying according to the steel stress, the ratio becomes — NAD Table 7; 4.4.3.2(4)

$$\frac{44\,(400 \times 377)}{460 \times 342} = 42.2$$

The actual span/effective depth ratio is $\dfrac{5000}{149} = 33.6$ . . . . OK

Had EC2 Table 4.14 been used instead of NAD Table 7, the basic ratio before modification would have been 35, which would not have been OK.

2.3.8 Cracking

For minimum area of reinforcement assume — 4.4.2.2

$f_{ct,eff} = 3$ N/mm²

$k_c = 0.4$

$k = 0.8$

$A_{ct} = 0.5 \times 175 \times 1000 = 87500$ mm²

Hence

$A_s = k_c k f_{ct,eff} A_{ct} / \sigma_s$ — Eqn 4.78

$= 0.4 \times 0.8 \times 3 \times 87500/460 = 183$ mm²/m

Area of reinforcement provided = 377 mm²/m . . . . OK

No further check is necessary as $h = 175 \leq 200$ mm — 4.4.2.3(1); NAD Table 3

Maximum bar spacing $= 3h \leq 500$ mm . . . . OK — 5.4.3.2.1(4)

2.3.9 Tie provisions

The NAD requires that ties are provided in accordance with BS 8110. — NAD 6.5(g)

Internal tie in E–W direction, with $F_t = 36$ kN/m width, is given by — BS 8110 3.12.3.4

$$\text{Tie force} = F_t \times \frac{(g_k + q_k)}{7.5} \times \frac{l_r}{5} = 36 \times \frac{(4.7 + 4)}{7.5} \times \frac{5}{5} = 41.8 \text{ kN/m}$$

$$\text{Minimum area} = \frac{41.8 \times 10^3}{460} = 91 \text{ mm}^2/\text{m}$$

Thus this area of the bottom reinforcement is the minimum that should be made continuous throughout the slab.

2.3.10 Reinforcement details

The reinforcement details are shown in Figure 2.2.

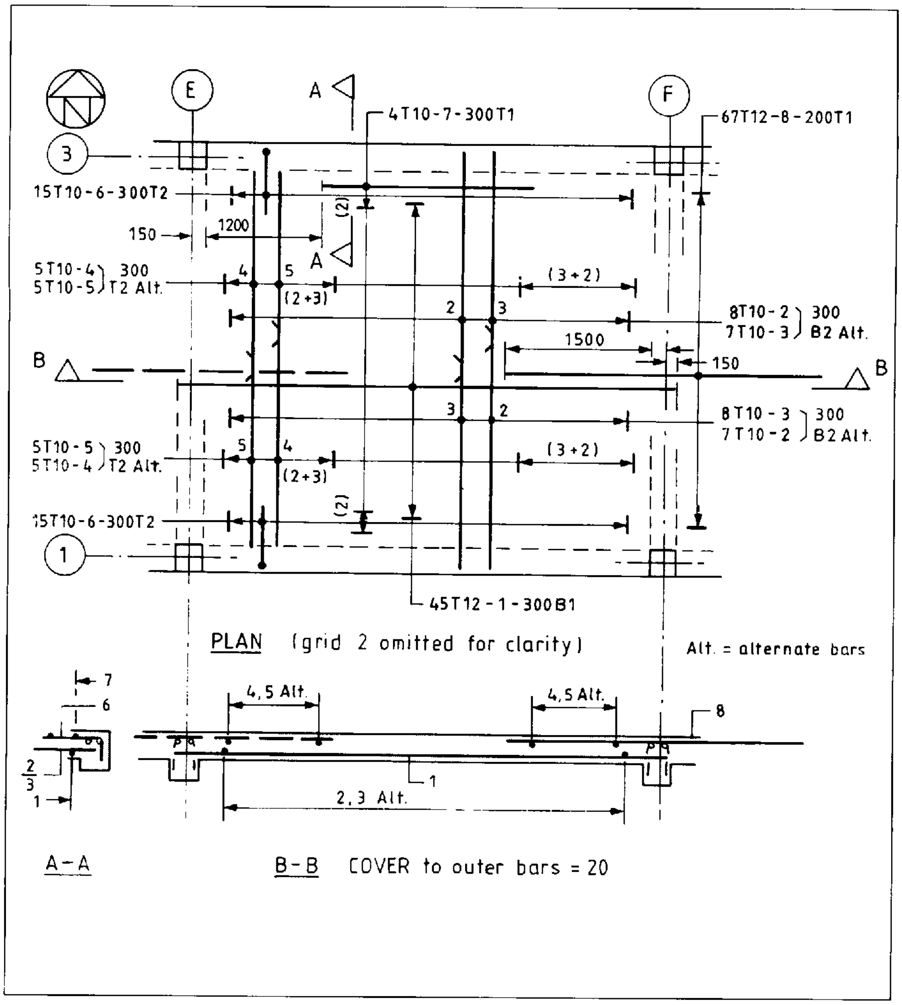


Figure 2.2 Slab reinforcement details

2.4 Main beam

2.4.1 Cover for durability and fire resistance

Nominal cover for exposure class 1 (internal) is 20 mm. NAD Table 6

Nominal cover for 1 hour fire resistance is 20 mm. BS 8110 Table 3.5

Use 20 mm nominal cover to links

2.4.2 Loading

Permanent load from slab (Section **2.3.3**) = 4.7 × 5 = 23.5 kN/m

Self-weight of beam = (0.5 − 0.175) × 0.3 × 24 = 2.3 kN/m

Characteristic permanent load (g_k) = 25.8 kN/m

Characteristic imposed load (q_k) = 5 × 4 = 20 kN/m

Maximum design load = $1.35g_k + 1.5q_k$ = 64.8 kN/m		2.3.3.1
Minimum design load = $1.35g_k$ = 34.8 kN/m		2.3.2.3.(4)

2.4.3 Analysis

2.4.3.1 Idealization of structure and load cases

The structure is simplified as a continuous beam attached to columns above and below, which are assumed to be fixed at their upper ends and pinned at the foundations, as shown in Figure 2.3. 2.5.3.3

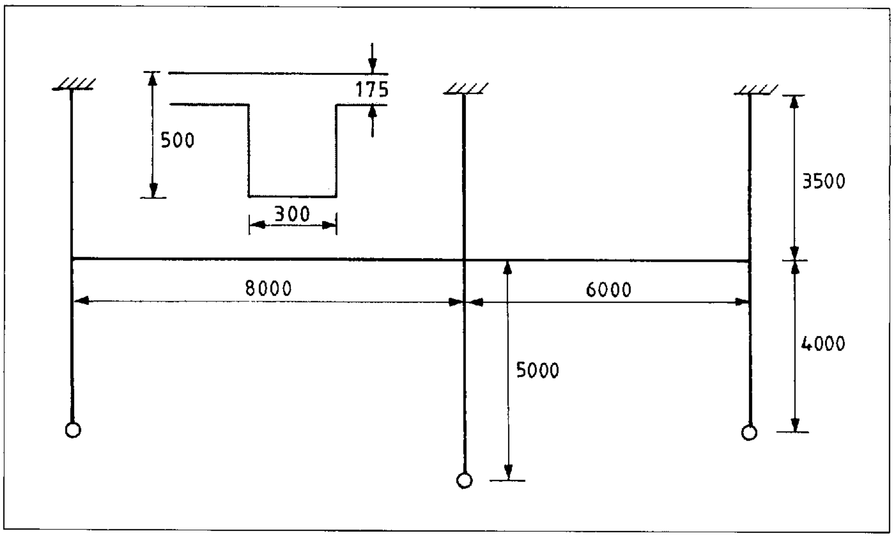


Figure 2.3 Idealization of structure

2.4.3.2 Design moments and shears

These are summarized in Table 2.2 and Figures 2.4 and 2.5.

Table 2.2 Results of frame analysis

	Load case 1	Load case 2	Load case 3
Load per m on 8 m span (kN)	64.8	64.8	34.8
Load per m on 6 m span (kN)	64.8	34.8	64.8
Upper LH column moment (kNm)	103	109	50
Lower LH column moment (kNm)	68	72	33
LH end of 8 m span moment (kNm)	−171	−180	−82
LH end of 8 m span shear (kN)	233	238	119
Middle of 8 m span moment (kNm)	242	256	116
RH end of 8 m span moment (kNm)	−382	−345	−242
RH end of 8 m span shear (kN)	286	280	159
Upper centre column moment (kNm)	33	55	3
Lower centre column moment (kNm)	18	29	2
LH end of 6 m span moment (kNm)	−331	−262	−247
LH end of 6 m span shear (kN)	240	146	223
Middle of 6 m span moment (kNm)	98	20	130
RH end of 6 m span moment (kNm)	−57	−12	−76
RH end of 6 m span shear (kN)	149	63	166
Upper RH column moment (kNm)	34	7	46
Lower RH column moment (kNm)	22	5	30

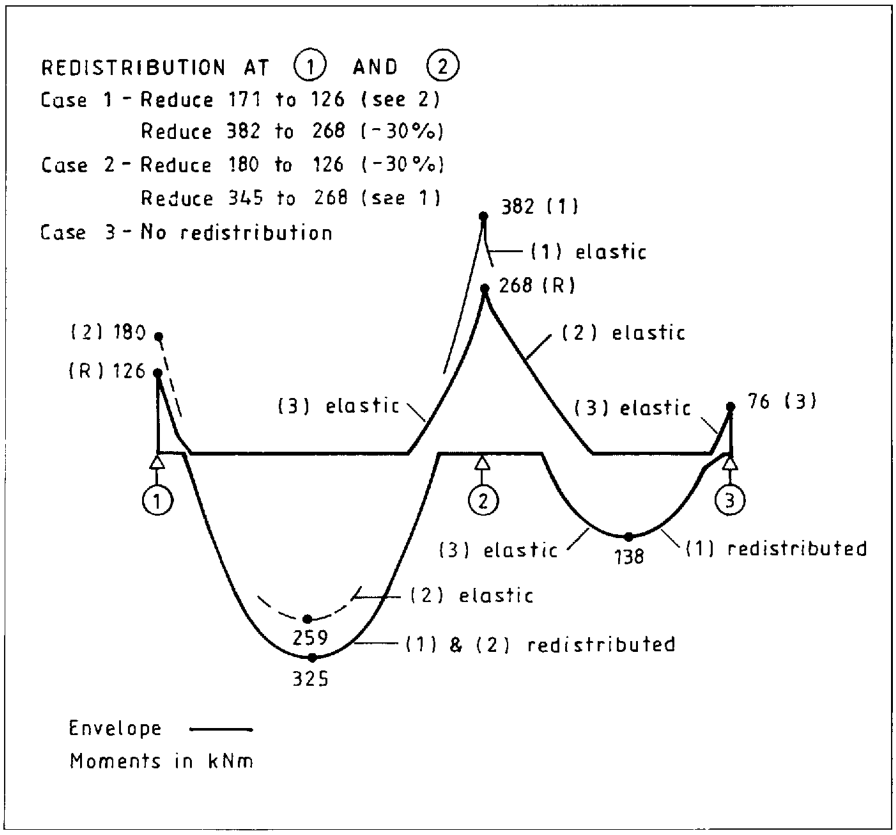


Figure 2.4 Bending moment envelope

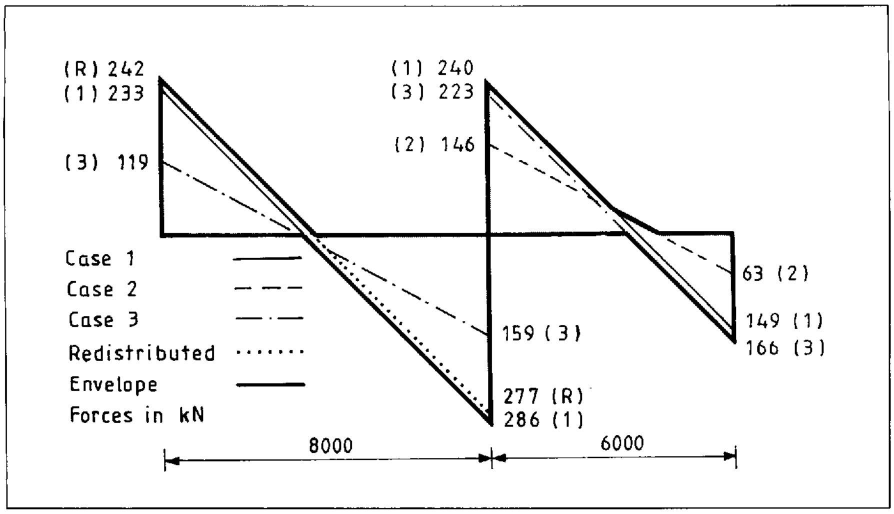


Figure 2.5 Shear force envelope

2.4.4 Reinforcement for flexure

2.4.4.1 Internal support

From bending moment envelope

$M = 268$ kNm

$\delta = 0.7$ and $x/d \leq (\delta - 0.44)/1.25 = 0.208$ 2.5.3.4.2 Eqn 2.17

$\mu_{lim} = 0.0864$ and $\omega_{lim} = 0.1084$ (Section **13**, Table 13.2)

$$\mu = \frac{M}{bd^2 f_{ck}} = \frac{268 \times 10^6}{300 \times 440^2 \times 32} = 0.1442 > \mu_{lim}$$

Therefore compression reinforcement is required

$$\omega' = \frac{A'_s f_{yk}}{bdf_{ck}} = \frac{\mu - \mu_{lim}}{0.87\,(1 - d'/d)} = \frac{0.1442 - 0.0864}{0.87\,(1 - 50/440)}$$

$$= 0.0750 \text{ (Section 13)}$$

$$\omega = \frac{A_s f_{yk}}{bdf_{ck}} = \omega_{lim} + \omega' = 0.1084 + 0.0750 = 0.1834 \text{ (Section 13)}$$

$$A_s = 0.1834 \times 300 \times 440 \times 32/460 = 1684 \text{ mm}^2$$

Since $d'/x = d'/0.208d = 0.546 > (1 - f_{yk}/805) = 0.429$

Increase ω' to $\left(\dfrac{1 - 0.429}{1 - 0.546}\right) 0.075 = 0.0943$

$$A'_s = 0.0943 \times 300 \times 440 \times 32/460 = 866 \text{ mm}^2$$

Use 4T25 (1960 mm^2) top
Use 2T25 (982 mm^2) bottom

2.4.4.2 Near middle of 8 m span

From bending moment envelope

$M = 325$ kNm

$\delta > 1.0$

Effective flange width $= 300 + 0.2 \times 0.85 \times 8000 = 1660$ mm — 2.5.2.2.1 Eqn 2.13

$$\mu = \frac{325 \times 10^6}{1660 \times 450^2 \times 32} = 0.030$$

$x/d = 0.068$ (Section **13**, Table 13.1)

Neutral axis is in flange since $x = 31 < 175$ mm

$\omega = 0.035$ (Section **13**, Table 13.1)

$A_s = 0.035 \times 1660 \times 450 \times 32/460 = 1819$ mm^2

Use 4T25 (1960 mm^2)

2.4.4.3 Left-hand end of 8 m span

From bending moment envelope

$M = 126$ kNm

$\delta = 0.7$ and $\mu_{lim} = 0.0864$ (Section **13**, Table 13.2)

$$\mu = \frac{126 \times 10^6}{300 \times 440^2 \times 32} = 0.0678 < \mu_{lim}$$

Therefore no compression reinforcement is required.

$\omega = 0.084$ (Section **13**, Table 13.1)

$A_s = 0.084 \times 300 \times 440 \times 32/460 = 772$ mm^2

Using 2T25 bent-up bars, minimum diameter of mandrel — 5.2.1.2 NAD Table 8

$= 13\phi\ (A_{s,req}/A_{s,prov}) = 10\phi$

Use 2T25 (982 mm^2) with $r = 5\phi$

2.4.4.4 Right-hand end of 6 m span

From bending moment envelope

$M = 76$ kNm

$$\mu = \frac{M}{bd^2 f_{ck}} = \frac{76 \times 10^6}{300 \times 440^2 \times 32} = 0.041$$

$\omega = 0.049$ (Section **13**, Table 13.1)

$A_s = 450$ mm^2

Use 2T25 (982 mm^2) with $r = 4\phi$ minimum

2.4.4.5 Near middle of 6 m span

From bending moment envelope

$M = 138$ kNm

Effective flange width $= 300 + 0.2 \times 0.85 \times 6000 = 1320$ mm

$$\mu = \frac{138 \times 10^6}{32 \times 450^2 \times 1320} = 0.0161$$

$\omega = 0.019$ (Section **13**, Table 13.1)

$A_s = 0.019 \times 1320 \times 450 \times 32/460 = 785$ mm^2

Use 2T25 (982 mm^2)

2.4.4.6 Minimum reinforcement

$A_s \geq k_c k f_{ct,eff} A_{ct} / \sigma_s$ 4.4.2.2 Eqn 4.78

where

$k_c = 0.4$

$k = 0.68$

$f_{ct,eff} = 3$ N/mm^2

$A_{ct} = 300 \times 325$ mm^2

$\sigma_s = 460$ N/mm^2

Therefore

$A_s \geq 173$ mm^2 . . . . OK

$\dfrac{0.6 b_t d}{f_{yk}} \nless 0.0015\, b_t d = 203$ mm^2 . . . . OK 5.4.2.1.1(1)

2.4.5 Shear reinforcement 4.3.2

2.4.5.1 Minimum links

Here, for comparison with BS 8110 design, grade 250 reinforcement will be used. 5.4.2.2

Interpolation from EC2 Table 5.5 gives

Minimum

$\rho_w = 0.0022$

$A_{sw}/s = 0.0022 \times 300 = 0.66\ \text{mm}^2/\text{mm}$

If $V_{Sd} \leq (\frac{1}{5})\ V_{Rd2}$ – refer to Section **2.4.5.3** for V_{Rd2}

s_{max} = lesser of 300 mm or 0.8d = 300 mm Eqn 5.17

Use R12 links @ 300 mm crs. ($A_{sw}/s = 0.75\ \text{mm}^2/\text{mm}$)

2.4.5.2 Capacity of section without shear reinforcement 4.3.2.3

$V_{Rd1} = \tau_{Rd} k(1.2 + 40\rho_l)\ b_w d$

Assume 2T25 effective

$\rho_l = 982/(300 \times 440) = 0.00743$

$k = 1.6 - d = 1.6 - 0.44 = 1.16$

$\tau_{Rd} = 0.35$ Table 4.8

$V_{Rd1} = 300 \times 440 \times 0.35 \times 1.16 \times (1.2 + 40 \times 0.00743) \times 10^{-3} = 80.2\ \text{kN}$

2.4.5.3 Shear reinforcement by standard method 4.3.2.4.3

Maximum capacity of section

$\upsilon = 0.7 - f_{ck}/200 = 0.7 - 32/200 = 0.54 \not< 0.5$ Eqn 4.21

$V_{Rd2} = 0.5 \times 0.54 \times (32/1.5) \times 300 \times 0.9 \times 440 \times 10^{-3} = 684\ \text{kN}$ Eqn 4.25

Design shear force is shear at a distance d from the face of the support. This is 590 mm from the support centreline. 4.3.2.2(10)

$$\frac{A_{sw}}{s} = \frac{V_{Sd} - 80.2}{0.9 \times 440 \times 0.87 \times 250} = 0.0116\ (V_{Sd} - 80.2)$$ Eqn 4.23

Design of shear reinforcement is summarized in Table 2.3.

Table 2.3 Design of shear reinforcement

Location	V_{Sd}	A_{sw}/s	s for 12 mm links	Links
8 m span LH end RH end	 203 248	 1.42 1.95	 159 116	 R12 @ 150 R12 @ 100
6 m span LH end RH end	 202 128	 1.41 min.	 160 max.	 R12 @ 150 R12 @ 300
Minimum				R12 @ 300

2.4.6 Deflection

Reinforcement percentage at centre of 8 m span 4.4.3.2

$= 100 \times 1960/(450 \times 1660) = 0.26\%$

Interpolating between 0.15 and 0.5%, basic span/effective depth ratio for end span = 40 NAD Table 7

To modify for steel stress multiply by 400/460

To modify for T section multiply by 0.8

To modify for span > 7 m multiply by 7/8

Therefore permissible ratio $= 40 \times (400/460) \times 0.8 \times 7/8 = 24.3$

Actual ratio $= 8000/450 = 17.8$ .... OK

2.4.7 Cracking

For estimate of steel stress under quasi-permanent loads 4.4.2.2

Ultimate load = 64.8 kN/m

Assuming ψ_2 = 0.3 NAD Table 1

Quasi-permanent load $= 0.3 \times 20 + 25.8 = 31.8$ kN/m

Approx. steel stress at midspan $= \dfrac{460}{1.15} \times \dfrac{31.8}{64.8} = 196$ N/mm^2

Approx. steel stress at supports allowing for 30% redistribution

$= 196/0.7 = 280$ N/mm^2

These are conservative figures since they do not allow for excess reinforcement over what is needed or for moment calculated at centreline of support rather than at face of support. Check limits on either bar size or spacing.

From EC2 Table 4.11, 25 mm bars in spans are satisfactory at any spacing since steel stress < 200 N/mm^2................................. OK 4.4.2.3

From EC2 Table 4.12, bar spacing at supports should be ≤ 150 mm with no limitation on size. As bars are located inside column bars the maximum possible spacing is 125 mm....................................... OK

2.4.8 Curtailment of reinforcement

Reinforcement must extend for a distance of $a_l + l_{b,net}$ beyond the moment envelope where 5.4.2.1.3

$$a_l = 0.9d/2 = 198 \text{ mm}$$

$$l_{b,net} = \frac{25}{4} \times \frac{460}{1.15} \times \frac{1}{3.2} = 782 \text{ mm}$$ 5.2.3.4.1

$$a_l + l_{b,net} = 980 \text{ mm}$$

Bars mark 8, which are located outside the web, must extend a further 150 mm – refer to Figure 2.8.

2.4.9 Reinforcement details

Curtailment of the main reinforcement and arrangement of the link reinforcement are shown in Figures 2.6 and 2.7. Reinforcement details are shown in Figure 2.8 and given in Table 2.4. 5.4.2.1.3

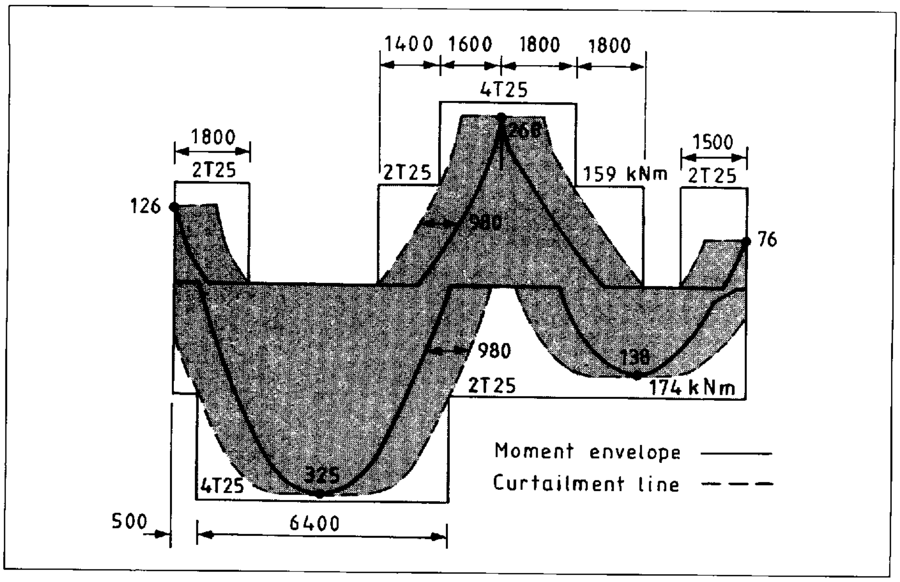


Figure 2.6 Curtailment diagram of main reinforcement

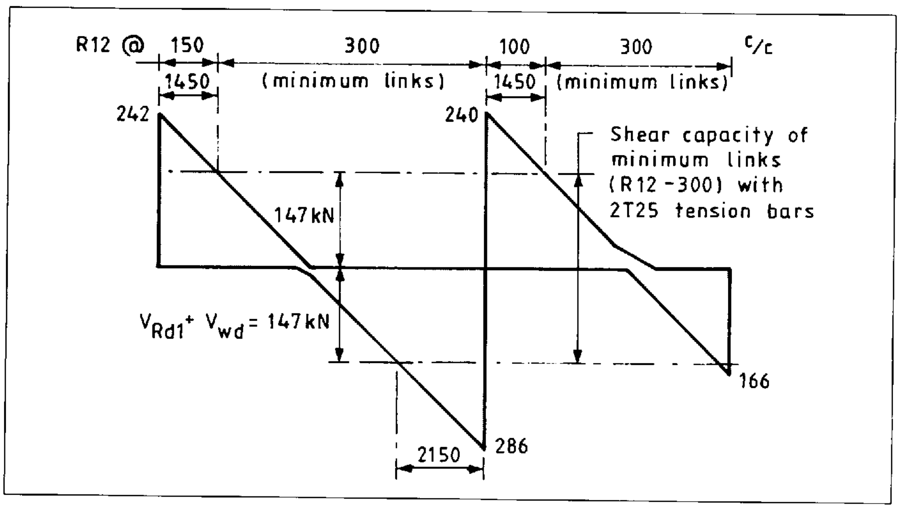


Figure 2.7 Arrangement of link reinforcement

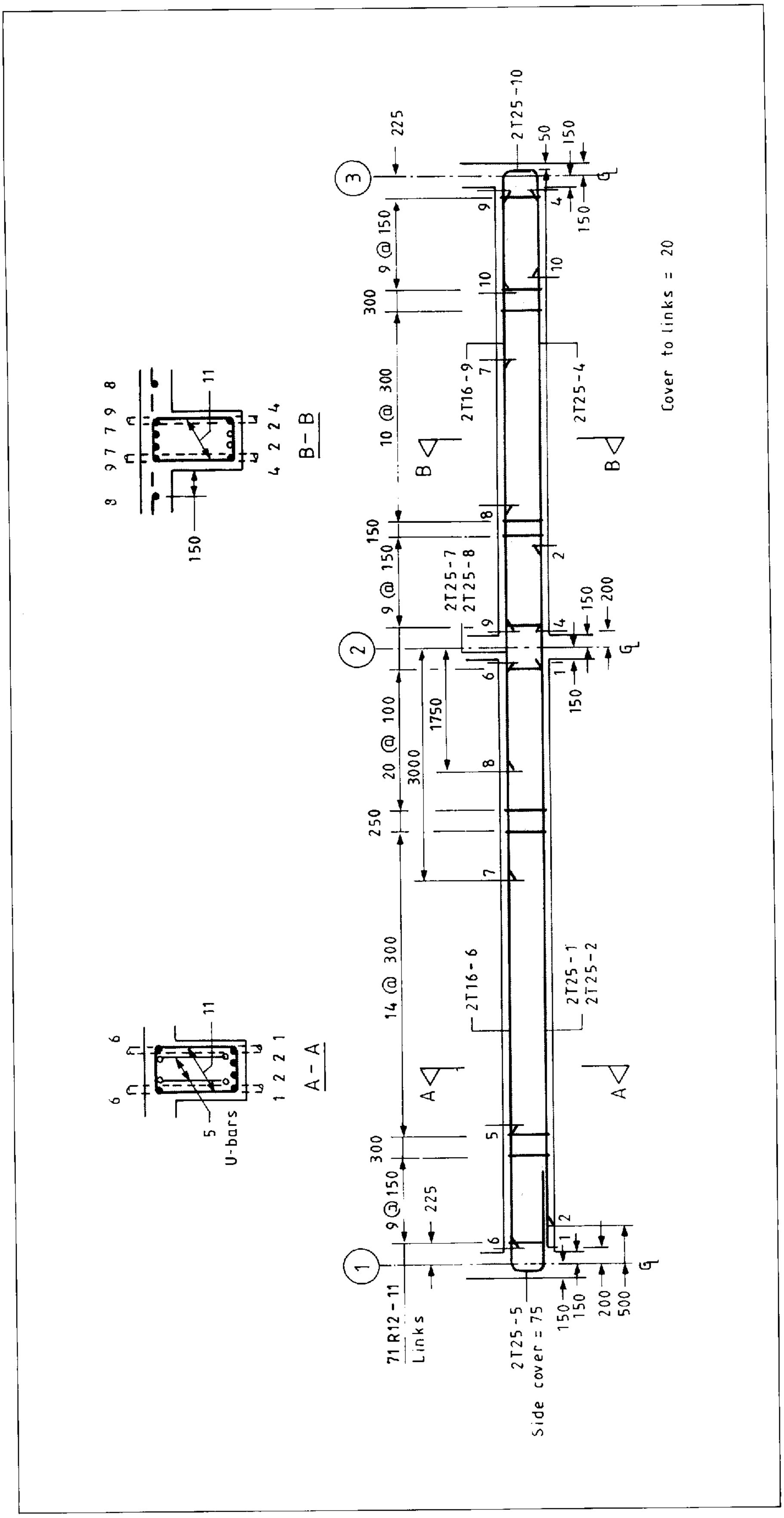


Figure 2.8 Main beam reinforcement details

Table 2.4 Commentary on bar arrangement

Bar marks	Notes	
1	Tension bars are stopped 50 mm from column face to avoid clashing with the column bars Nominal cover = 20 + 12 = 32 > 25 mm .... OK	4.1.3.3(5)
2	Remaining tension bars stopped off at LH end as shown in Figure 2.6. Bars extended at RH end to provide compression reinforcement (lap = $l_{b,net}$) and continuity for internal ties (lap = 1000 mm) Check minimum distance between bars ≥ bar size or 20 mm (300 − 32 × 2 − 4 × 25)/3 = 45 > 25 mm .... OK	BS 8110 5.2.1.1
3	Not used	
4	Similar to bar mark 1	
5,10	Loose U bars are fixed inside the column bars and provide continuity for column and internal ties Top legs project from centre line into span, minimum dimensions shown in Figure 2.6 Bottom legs are lapped 1000 mm to provide continuity for internal ties	5.4.2.1.3 BS 8110
5	Top legs = 1800 mm Bottom legs = 200 + 1000 = 1200 mm Use r = 5ϕ for both bends Note that the bottom legs are raised to avoid gap between bars being < 25 mm	5.2.1.1
10	Top legs = 1500 mm Bottom legs = 200 + 1000 = 1200 mm	
6,9	2T16 provided as link hangers are stopped 50 mm from column face	
7,8	Tension bars over the support are stopped as in Figure 2.6. Bars mark 8 are located outside the web	5.4.2.1.2(2)
11	Links are arranged in accordance with Figure 2.7 for shear. Links also provide transverse reinforcement with a spacing ≤ 150 mm at all laps	5.2.4.1.2(2)

2.5 Edge beam (interior span)

2.5.1 Cover for durability and fire resistance

Nominal cover for exposure class 2b (external) is 35 mm. NAD Table 6

Nominal cover for 1 hour fire resistance is 20 mm. BS 8110 Table 3.5

Use 40 mm nominal cover to links

2.5.2 Loading

Permanent load from slab = 4.7 × 5 × 1.25 = 29.4 kN

(assuming 1.25 m strip to be loading on edge beam)

Self-weight of beam = (0.35 − 0.175) × 0.3 × 5 × 24 = 6.3 kN

Cladding load @ 5 kN/m = 5 × 5 = 25 kN

Characteristic permanent load = 60.7 kN

Characteristic imposed load = 4 × 5 × 1.25 = 25 kN

Total design load = 1.35 × 60.7 + 1.5 × 25 = 119.5 kN

2.5.3 Design moments and shears

These are taken from the Concise Eurocode, Appendix, Table A.1[5].

2.5.3.1 Interior support

Moment = 0.10 × 119.5 × 5 = 59.8 kNm Shear = 0.55 × 119.5 = 65.7 kN

2.5.3.2 Mid-span

Moment = 0.07 × 119.5 × 5 = 41.8 kNm

2.5.4 Reinforcement for flexure

2.5.4.1 Interior support

Assume effective depth = 280 mm

$$\frac{M}{bd^2 f_{ck}} = \frac{59.8 \times 10^6}{280^2 \times 300 \times 32} = 0.079$$

$$\frac{A_s f_{yk}}{bdf_{ck}} = 0.099 \text{ (Section 13, Table 13.1)}$$

x/d = 0.189 < 0.45 . . . . OK 2.5.3.4.2(5)

A_s = 579 mm^2

Use 2T20 (628 mm^2)

2.5.4.2 Mid-span

Assume effective depth = 290 mm

Effective flange width = 300 + 0.1 × 0.7 × 5000 = 650 mm 2.5.2.2.1

$$\frac{M}{bd^2 f_{ck}} = \frac{41.8 \times 10^6}{650 \times 290^2 \times 32} = 0.024$$

$$\frac{A_s f_{yk}}{bdf_{ck}} = 0.028 \text{ (Section **13**, Table 13.1)}$$

$$A_s = 367 \text{ mm}^2$$

Use 2T20 (628 mm^2)

The cross-section is shown in Figure 2.9.

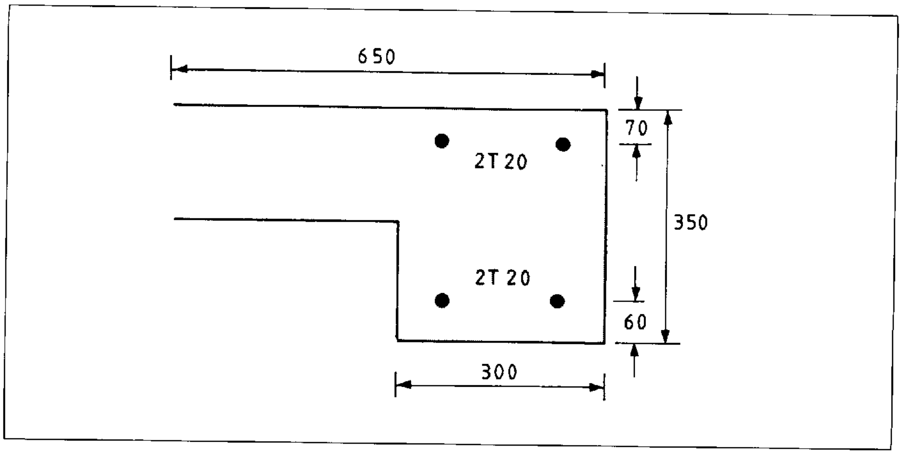


Figure 2.9 Edge beam cross-section

2.5.5 Shear reinforcement

Design shear force may be taken to be at distance *d* into the span from the face of the support. This can be calculated approximately as **4.3.2.2(10)**

$$V_{Sd} = 65.7 - 119.5\ (0.28 + 0.15)/5.0 = 55.4 \text{ kN}$$

$$V_{Rd1} = 300 \times 280 \times 0.35\ (1.6 - 0.28) \times \left(1.2 + \frac{40 \times 628}{300 \times 280}\right) = 58.2 \text{ kN}$$ **4.3.2.3(1)**

This is greater than V_{Sd}, hence only minimum links are required. **4.3.2.2(2)**

Assuming grade 250 reinforcement for links, EC2 Table 5.5 gives

$$\rho_w = 0.0022$$

Hence

$$\frac{A_{sw}}{s} = 0.0022 \times 300 = 0.66 \text{ mm}^2/\text{mm}$$

$$V_{Rd2} = 0.5 \left(0.7 - \frac{32}{200}\right) \times \frac{32}{1.5} \times 300 \times 0.9 \times 280 = 435 \text{ kN}$$ **4.3.2.3(3)**

Since

$V_{Sd} < (\frac{1}{5}) V_{Rd2}$, $s_{max} = 0.8d = 224$ mm Eqn 5.17

200 mm spacing gives $A_{sw} = 132$ mm^2

Use R10 links at 200 mm crs. ($A_{sw} = 2 \times 78.5 = 157$ mm^2)

2.5.6 Deflection

Actual span/effective depth ratio = 5000/290 = 17.2

At mid-span

$$\frac{100\,A_s}{bd} = \frac{100 \times 628}{650 \times 290} = 0.33\%$$

By interpolation from NAD Table 7, modified for $f_{yk} = 460$

Basic span/effective depth ratio = 36 Concise Eurocode Figure A.12

Note:
This can be increased allowing for use of a larger than required steel area to

$= 36 \times 628/367 = 61.6$ 4.4.3.2(4)

But not greater than 48/1.15 = 41.7 NAD Table 7 Note 2

Inspection shows this to be unnecessary.

Allowable l/d > actual l/d . . . . OK

2.5.7 Curtailment of reinforcement

Since the bending moment diagram has not been drawn, simplified curtailment rules are needed. These are given in Section 8 of the Appendix to the Concise Eurocode.

Using the rules, the 20 mm bars in the top may be reduced to 12 mm bars at a distance from the face of support

$= 0.1l + 32\phi + 0.45d = 500 + 32 \times 20 + 0.45 \times 280$

$= 1266$ mm from the column face

2.5.8 Reinforcement details

The reinforcement details are shown in Figure 2.10.

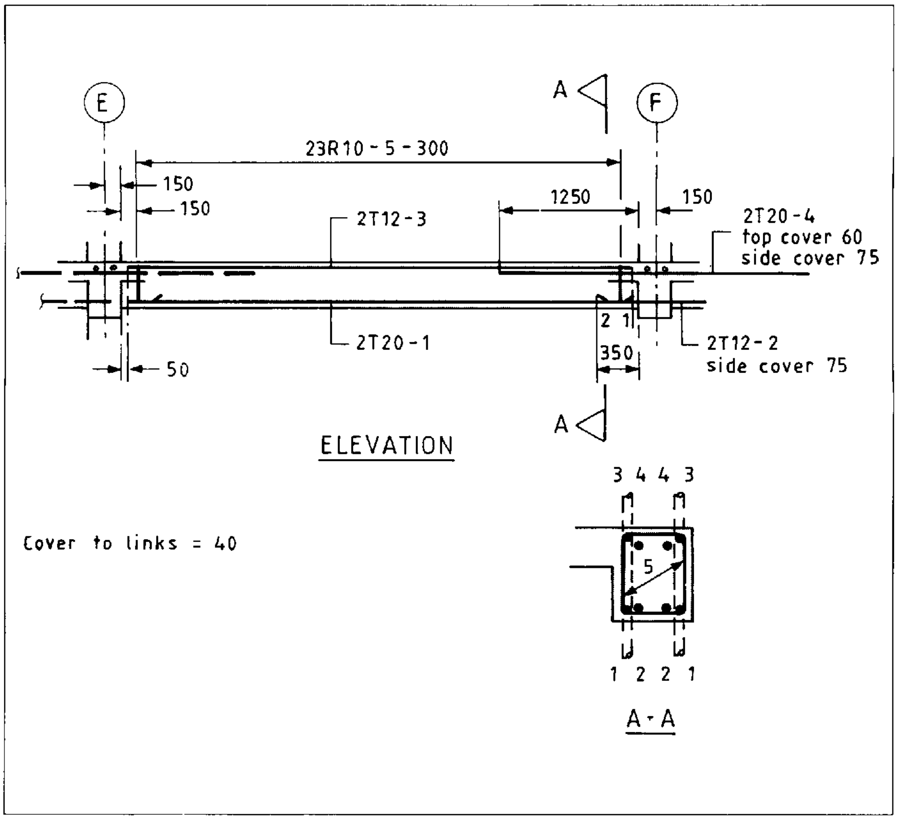


Figure 2.10 Edge beam reinforcement details

2.6 Columns

2.6.1 Idealization of structure

The simplification assumed for the design of the main beam is shown in Figure 2.3.

2.6.2 Analysis

Moments and column loads at each floor are taken from the analysis for the main beam given in Section **2.4.3.**

2.6.3 Cover for durability and fire resistance

Nominal cover for interior columns (exposure class 1) is 20 mm. NAD Table 6

Nominal cover for exterior columns (exposure class 2b) is 35 mm.

Nominal cover for 1 hour fire resistance is 20 mm.

Use 20 mm (interior) and 40 mm (exterior) nominal cover to links

2.6.4 Internal column

2.6.4.1 Loading and moments at various floor levels

These are summarized in Table 2.5.

Table 2.5 Loading and moments for internal column

	Beam loads (kN)		Column design loads (kN)				Column moments (kNm)			
	Total		Imposed		Dead		Top		Bottom	
Load case	1	2	1	2	1	2	1	2	1	2
Roof 8 m	240	238	53	51	187	187	32	42		
6 m	202	165	43	6	159	159				
Self-weight					9	9				
			96	57	355	355			30	39
3rd floor 8 m	283	278	131	126	152	152	33	49		
6 m	236	143	110	17	126	126				
Self-weight					9	9				
			337	200	642	642			30	55
2nd floor 8 m	283	278	131	126	152	152	33	49		
6 m	236	143	110	17	126	126				
Self-weight					9	9				
			578	343	929	929			33	55
1st floor 8 m	286	280	132	126	154	154	18	29		
6 m	240	146	111	17	129	129				
Self-weight					14	14				
Foundations			821	486	1226	1226				

2.6.4.2 Design for column between first floor and foundation

Effective height in N – S direction

$$k_{top} = \left(\frac{0.5 \times 675 \times 10^6}{5000} + \frac{675 \times 10^6}{3500}\right) \div \left(\frac{3125 \times 10^6}{8000} + \frac{3125 \times 10^6}{6000}\right)$$

Eqn 4.60

$= 0.28$ but take not less than 0.4

$$k_{bottom} = \infty$$

Hence

$\beta = 0.8$ — Figure 4.27

Effective height $= 0.8 \times 5000 = 4000$ mm

Load case 1 gives worst condition (by inspection). — BS 6399: Part 1, Reduction factor

Imposed load $= 0.7 \times 821 = 575$ kN

Dead load = 1226 kN

N_{Sd} = 1801 kN

M_{Sd} = 18 kNm (top), 0 (bottom)

$$\nu_u = \frac{1801 \times 10^3 \times 1.5}{300^2 \times 32} = 0.94$$ 4.3.5.3.5(2)

$15/\sqrt{\nu_u}$ = 14.5 < 25

Hence

λ_{min} = 25

$$\lambda = l_o/i = \frac{4000\sqrt{12}}{300} = 46$$

Note: $l_o/i = (l_o/h) \times \sqrt{12}$

λ > 25, hence column is slender in N–S direction

The slenderness in the E–W direction will be found to be approximately the same.

The structure is braced and non-sway (by inspection), hence the Model Column Method may be used with the column designed as an isolated column.

$\lambda_{crit} = 25(2 - e_{o1}/e_{o2})$ = 50 in both E–W and N–S directions 4.3.5.5.3 Eqn 4.62

Slenderness ratios in both directions are less than λ_{crit}, hence it is only necessary to ensure that the column can withstand an end moment of at least

$N_{Sd}h/20$ = 1801 × 0.3/20 = 27.0 kNm 4.3.5.5.3 Eqn 4.64

This exceeds the first order moments.

Hence N_{Sd} = 1801 kN and M_{Sd} = 27.0 kNm

$$\frac{N_{Sd}}{bhf_{ck}} = 0.62$$

$$\frac{M_{Sd}}{bh^2f_{ck}} = \frac{27.0 \times 10^6}{300^3 \times 32} = 0.031$$

Assume

d' = 45 mm

Then

$$d'/h = 45/300 = 0.15$$

$$\frac{A_s f_{yk}}{bhf_{ck}} = 0.16 \text{ (Section 13, Figure 13.2(c))}$$

Hence

$$A_s = 1002 \text{ mm}^2$$

Use 4T20 (1260 mm²)

Note:
In the design by Higgins and Rogers, the slenderness ratio exceeds the equivalent of λ_{crit} but the design moment is still $Nh/20$. EC2 requires less reinforcement due to the smaller design load and the assumption of a smaller cover ratio. If the same cover ratio is used in the Higgins and Rogers design, 4T20 are sufficient in both cases.

2.6.5 External column

2.6.5.1 Loading and moments at various levels

These are summarized in Table 2.6.

Table 2.6 Loading and moments for external column

	Beam loads (kN)		Column design loads (kN)				Column moments (kNm)			
	Total		Imposed		Dead		Top		Bottom	
Load case	1	2	1	2	1	2	1	2	1	2
Roof										
Main	184	186	39	41	145	145	104	107		
Edge	55	55			55	55				
Self-weight					9	9				
			39	41	209	209			93	98
3rd floor										
Main	235	240	109	114	126	126	93	98		
Edge	55	55			55	55				
Self-weight					9	9				
			148	155	399	399			93	98
2nd floor										
Main	235	240	109	114	126	126	93	98		
Edge	55	55			55	55				
Self-weight					9	9				
			257	269	589	589			103	109
1st floor										
Main	233	238	108	113	125	125	68	72		
Edge	55	55			55	55				
Self-weight					9	9				
Foundations			365	382	778	778				

2.6.5.2 Design for column between first floor and foundation

$$k_{top} = \left(\frac{675 \times 10^6 \times 0.5}{4000} + \frac{675 \times 10^6}{3500}\right) \div \left(\frac{3125 \times 10^6}{8000}\right) = 0.71$$

$$k_{bottom} = \infty$$

Hence

$$\beta = 0.85$$ Figure 4.27

Effective height = 0.85 × 4000 = 3400 mm

Slenderness ratio $= l_o/i = \frac{3400\sqrt{12}}{300} = 39.3$

ν_u will be small so $15/\sqrt{\nu_u}$ will be less than 25

Hence

$$\lambda_{min} = 25$$

$\lambda > 25$, therefore column is slender in N–S direction

Calculate λ_{crit}

$$\frac{e_{o1}}{e_{o2}} = \frac{\text{bottom moment}}{\text{top moment}} = \frac{0}{85} = 0$$ 4.3.5.5.3

Hence

$$\lambda_{crit} = 25\ (2 + 0) = 50$$

Slenderness ratios in the E–W and N–S directions are both less than 50, hence it is only necessary to ensure that the end moment is at least $Nh/20$.

The worst condition occurs with load case 2 at section just above the first floor, where M_{Sd} is greatest.

$$N_{Sd} = 589 + 0.8 \times 269 = 804 \text{ kN}$$

$$\frac{Nh}{20} = \frac{804 \times 0.3}{20} = 12.0 \text{ kNm}$$

Design end moment = 109 > 12 kNm

Hence N_{Sd} = 804 kN and M_{Sd} = 109 kNm

2.6.6 Reinforcement details

Maximum spacing of links for internal column — 5.4.1.2.2(3)

Generally	12 × 20 = 240 mm	NAD
Above and below floor	0.67 × 240 = 160 mm	Table 3

Maximum spacing of links for external column — 5.4.1.2.2(4)

Generally	12 × 25 = 300 mm
At lap and below floor	0.67 × 300 = 200 mm

The reinforcement details are shown in Figure 2.11.

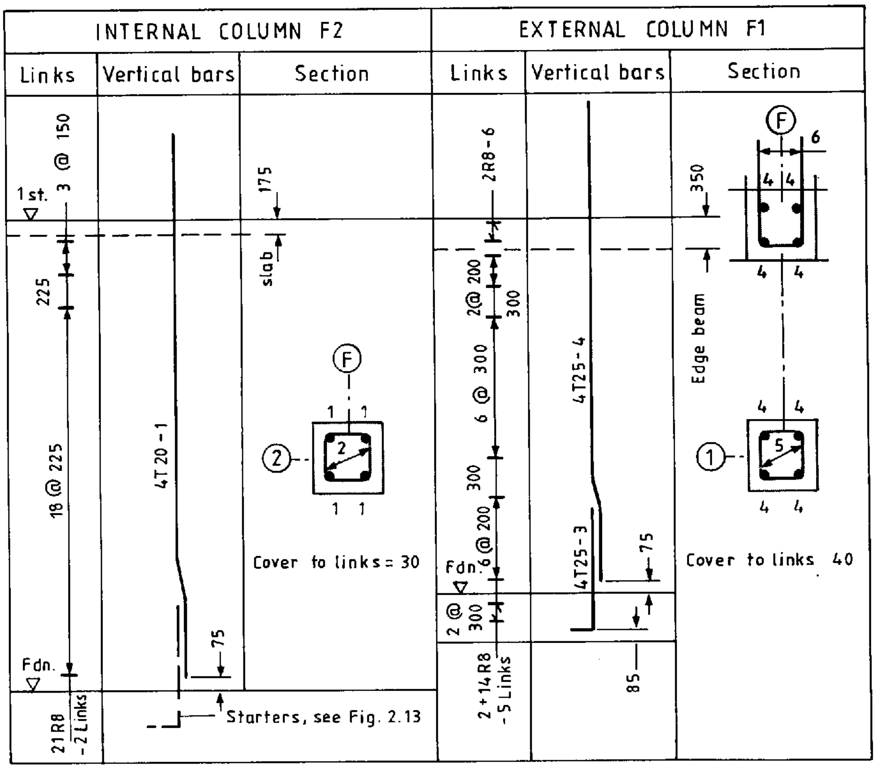


Figure 2.11 Column reinforcement details

2.7 Foundation

Design typical pad footing for internal column.

2.7.1 Cover

Use 50 mm nominal cover against blinding

4.1.3.3(9)

BS 8110 specifies a nominal cover of not less than 40 mm against blinding. EC2 specifies a minimum cover greater than 40 mm. This implies a nominal cover greater than 45 mm, hence the choice of 50 mm.

2.7.2 Loading

Taken from internal column design.

Ultimate design loads:	Dead	=	1226
	Imposed	=	575
	Total	=	1801 kN

Hence service loads:	Dead	=	908
	Imposed	=	383
	Total	=	1291 kN

The assumption is made that the base takes no moment. Also it is assumed that the dead weight of the base less the weight of soil displaced is 10 kN/m^2 over the area of the base.

2.7.3 Size of base

Since, at the time of publication, EC7: *Geotechnical design*[9] and EC2, Part 3: *Concrete foundations*[10] have not been finalized, the approach used here is based on current UK practice.

Use 2.75 m × 2.75 m × 0.6 m deep pad

Bearing pressure under service loads

$$= \frac{1291}{2.75^2} + 10 = 181 < 200 \text{ kN/m}^2 \ldots\ldots\ldots\ldots \text{OK}$$

Design pressure at ultimate limit state $= \dfrac{1801}{2.75^2} = 238$ kN/m^2

2.7.4 Flexural reinforcement

Moment at face of column = 238 × 2.75 × 1.225^2/2 = 491 kNm

Average effective depth = 600 − 50 − 25 = 525 mm

$$\frac{M_{Sd}}{bd^2 f_{ck}} = \frac{491 \times 10^6}{2750 \times 525^2 \times 32} = 0.020$$

$$\frac{A_s f_{yk}}{bdf_{ck}} = 0.023 \text{ (Section 13, Table 13.1)}$$

Hence

$$A_s = 0.023 \times 2750 \times 525 \times 32/460 = 2310 \text{ mm}^2$$

Use 9T20 @ 300 mm crs. each way (2830 mm^2)

2.7.5 Shear

2.7.5.1 Shear across base

Shear force may be calculated at a critical section distance d from the face of the column. 4.3.2.2(10)

$$\text{Design shear } (V_{Sd}) = 238 \times 2.75 \times \left[\frac{(2.75 - 0.3)}{2} - 0.525\right] = 458 \text{ kN}$$

In calculating V_{Rd1}, the influence of the reinforcement will be ignored since, if straight bars are used, they will not extend $d + l_{b,net}$ beyond the critical section. 4.3.2.3(1)

V_{Rd1} = 0.35 × 1.075 × 1.2 × 2750 × 525/1000 = 652 kN Eqn 4.18

$V_{Rd1} > V_{Sd}$, hence no requirement for shear reinforcement

2.7.5.2 Punching shear

The critical perimeter is shown in Figure 2.12.

Design load on base = 1801 kN

Length of critical perimeter

u = [4 × 300 + π (2 × 1.5 × 525)]/1000 = 6.15 m

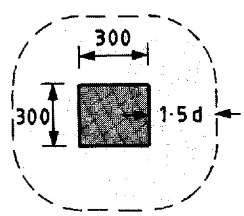


Figure 2.12 Critical perimeter for punching

V_{Rd1} = 0.35 × 1.075 × 1.2 × 525 × 6.15 = 1458 kN 4.3.4.5.1

Area within perimeter = 2.98 m^2 Area of base = 7.56 m^2

Design shear (V_{Sd}) = (7.56 − 2.98) × 238 = 1090 kN 4.3.4.1(5)

$V_{Sd} < V_{Rd1}$, hence no requirement for shear reinforcement

2.7.6 Cracking

Approximate steel stress under quasi-permanent loads

$$= \frac{460}{1.15} \times \frac{(908 + 0.3 \times 383)}{1801} \times \frac{2310}{2830} = 186 \text{ N/mm}^2$$

From EC2 Table 4.11 bar size should not exceed 25 > 20 mm used. 4.4.2.3

Hence cracking .... OK Table 4.11

2.7.7 Reinforcement details

The reinforcement details are shown in Figure 2.13 and given in Table 2.7.

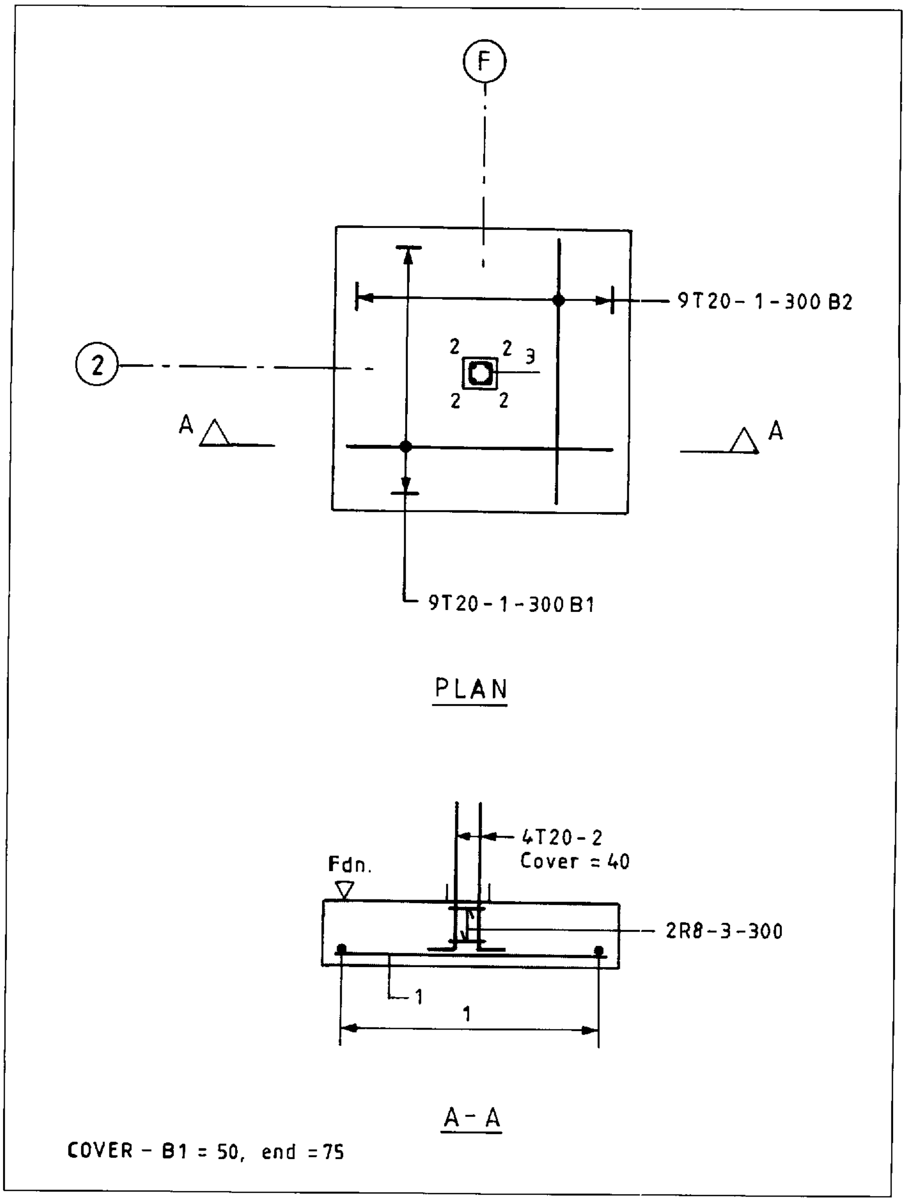


Figure 2.13 Base reinforcement details

Table 2.7 Commentary on bar arrangement

Bar marks	Notes
1	Straight bars extend full width of base less end cover of 75 mm. Bars should extend an anchorage length beyond the column face Anchorage length = 32 × 20 = 640 mm Actual extension = 1150 mm
2	Column starter bars wired to bottom mat Minimum projection above top of base is a compression lap + kicker = 32 × 20 + 75 = 715 mm
3	Links are provided to stabilize and locate the starters during construction

4.1.3.3(9)
5.2.3.4.1

5.2.4.1.3

2.8 Shear wall

2.8.1 Structure

The structure is shown in Figure 2.14.

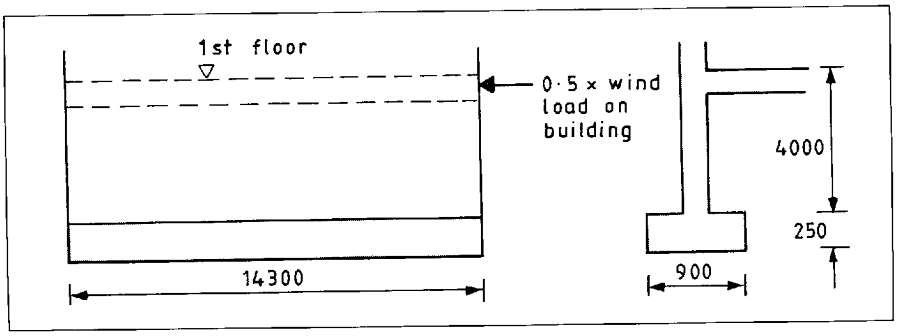


Figure 2.14 Shear wall structure

2.8.2 Loading at foundation level

Dead load from first to third floors and roof

$= 0.5\,(3 \times 23.5 + 28.5) = 49.5$ kN/m

Self-weight $= 0.175 \times 24 \times 15.5 = 65.1$ kN/m

Characteristic dead load $= 49.5 + 65.1 = 114.6$ kN/m

Characteristic imposed load from slabs

$= 2.5\,(1.5 + 3 \times 4) \times 0.7 = 23.6$ kN/m

Wind loading is taken as 90% of value obtained from CP3: Ch V: Part 2[11]. NAD 4(c)

Total wind load on building in N–S direction $= 0.9 \times 449 = 404$ kN

Wind load on wall $= 404/2 = 202$ kN

Moment in plane of wall $= 202 \times 8 = 1616$ kNm

Hence

Maximum force per unit length due to wind moment

$$= \pm \frac{M \times 6}{l^2}$$

$$= \pm \frac{1616 \times 6}{14.2^2} = \pm 47.4 \text{ kN/m}$$

2.8.3 Vertical design load intensities at ultimate limit state

Dead load + imposed load

$= 1.35 \times 114.6 + 1.5 \times 23.6 = 190.1$ kN/m Eqn 2.8(a)

Dead load + wind load

$= 1.35 \times 114.6 + 1.5 \times 47.4 = 225.8$ kN/m; or Eqn 2.8(a)

$= 1.0 \times 114.6 - 1.5 \times 47.4 = 43.5$ kN/m

Dead load + wind load + imposed load

$= 1.35 \times 114.6 + 1.35 \times 23.6 \pm 1.35 \times 47.4$ — Eqn 2.8(b)

$= 250.6$ kN/m or 122.6 kN/m — NAD 4(c)

Therefore maximum design load = 250.6 kN/m From analysis of slab (not presented), maximum moment perpendicular to plane of wall = 11.65 kNm/m

2.8.4 Slenderness ratio

$$k_A = \frac{\frac{0.5}{4} + \frac{1}{3.5}}{\frac{1}{5}} = 2.05$$ Eqn 4.60

$$k_B = \infty$$

Hence

$$\beta = 0.94$$ Figure 4.27

$$l_o = \beta l_{col} = 0.94 \times 4 = 3.76 \text{ m}$$

$$l_o/i = \frac{3.76 \times 1000 \times \sqrt{12}}{175} = 74.4$$

Hence wall is slender

2.8.5 Vertical reinforcement

Higgins and Rogers design the shear wall as unreinforced. Plain concrete walls will be covered in EC2 Part 1A which, at the time of publication, has not yet been finalized. The wall will, therefore, be designed here as a reinforced wall. As will be seen, the result is the same.

Eccentricity due to applied loads

$$e_{o1} = 0$$

$$e_{o2} = 11.65 \times 1000/250.6 = 46.5 \text{ mm}$$

Hence

$$e_e = 0.6 \times 46.5 + 0 = 27.9 \text{ mm}$$ Eqn 4.66

Accidental eccentricity

$$e_a = \frac{1}{200} \times \frac{3760}{2} = 9.4 \text{ mm}$$ Eqn 4.61

Second order eccentricity

$$e_2 = \frac{3760^2}{10} \times 2 \times \frac{460}{1.15 \times 200000} \times \frac{1}{0.9 \times 122} \times K_2$$ Eqns 4.72 & 4.69

$$= 51.5K_2$$

Assuming $K_2 = 1$

Design eccentricity = 27.9 + 9.4 + 51.5 = 88.8 mm

Design ultimate load = 250.6 kN/m

Design ultimate moment = 88.8 × 250.6/1000 = 22.3 kNm/m

$$\frac{M}{bh^2 f_{ck}} = 0.023$$

$$\frac{N}{bhf_{ck}} = 0.045$$

$$\frac{A_s f_{yk}}{bhf_{ck}} = 0.01$$ (Section **13**, Figure 13.2(d))

Hence

A_s = 122 mm²/m or 61 mm²/m in each face

Minimum area of reinforcement

= 0.004 × 1000 × 175 = 700 mm²/m 5.4.7.2

This exceeds the calculated value. Hence the minimum governs.

Use T12 @ 300 mm crs. in each face (754 mm²/m)

2.8.6 Shear

Design horizontal shear = 1.5 × 202 = 303 kN

Shear stress = $\frac{303 \times 1000}{14300 \times 175}$ = 0.12 N/mm² .... OK

Note:
V_{Rd1} is not calculated since it must be > $0.12 b_w d$ by quick inspection of EC2 Eqn 4.18.

2.8.7 Horizontal reinforcement

Minimum at 50% of vertical reinforcement provided 5.4.7.3

A_s = 188 mm²/m (EF)

Minimum for controlled cracking due to restraint of early thermal contraction 4.4.2.2

$A_s = k_c k f_{ct.eff} A_{ct}/\sigma_s$ — Eqn 4.78

$k_c = 1.0$

$k = 0.8$

$f_{ct,eff} = 1.9$ N/mm^2 (assuming concrete strength to be equivalent to C16/20 at time of cracking) — Table 3.1

$\sigma_s = 360$ N/mm^2 (assuming 10 mm bars) — Table 4.11

$A_s = 1.0 \times 0.8 \times 1.9 \times 175 \times 1000/360 = 739$ mm^2/m

Use T10 @ 200 mm crs. in each face (785 mm^2/m)

2.8.8 Tie provisions at first floor

NAD 6.5(g)

According to the NAD, these should follow the rules in BS 8110. — BS 8110 3.12.3

$F_t = 36$ kN

2.8.8.1 Peripheral tie

$$A_s = \frac{36 \times 10^3}{460} = 78 \text{ mm}^2$$

Use 1T10 (78.5 mm^2)

2.8.8.2 Internal tie force

$$\text{Force} = \frac{2.5 \times 36\,(4.7 + 4.0)}{7.5} \times \frac{14.3}{5} = 299 \text{ kN}$$

Hence

$$A_s = \frac{299 \times 10^3}{460} = 650 \text{ mm}^2$$

Use 5T10 in each face (785 mm^2)

Hence T10 @ 200 mm crs. horizontal reinforcement in wall 0.5 m above and below slab is adequate.

2.8.8.3 Wall tie

Take the greater of (a) and (b)

(a) Lesser of $2.0F_t$ or $\dfrac{l_s F_t}{2.5} = 72$ or 48 kN

(b) 3% of total vertical load = 0.03 × 190.1 = 5.7 kN

Hence

Tie force = 48 kN

$$A_s = \frac{48 \times 10^3}{460} = 104 \text{ mm}^2$$

Therefore reinforcement in slab will suffice

2.8.9 Strip footing

EC2, Part 3: *Concrete foundations*, at the time of publication, has not yet been drafted, hence current UK practice is adopted.

Maximum pressure due to characteristic dead, imposed and wind loads

$$= 114.6 + 23.6 + 47.4/0.9 = 191 \text{ kN/m}$$

For 900 mm wide strip, pressure $= \dfrac{191}{0.9} = 212 \text{ kN/m}^2$

Allow extra 10 kN/m^2 for ground floor loads and weight of concrete displacing soil in foundations. This gives 222 kN/m^2.

Allowable pressure = 1.25 × 200 = 250 > 222 kN/m^2........ OK

Use 900 mm wide strip

Calculate reinforcement for flexure

$$\text{Moment} = 250.6 \times \frac{(0.9 - 0.175)^2}{8} = 16.5 \text{ kNm/m}$$

A_s = 209 mm^2/m

Minimum area = 0.0015*bd* 5.4.2.1.1

= 0.0015 × 1000 × 200 = 300 mm^2/m

Use T12 @ 300 mm crs. (377 mm^2/m)

2.8.10 Reinforcement details

The reinforcement details are shown in Figure 2.15 and given in Table 2.8.

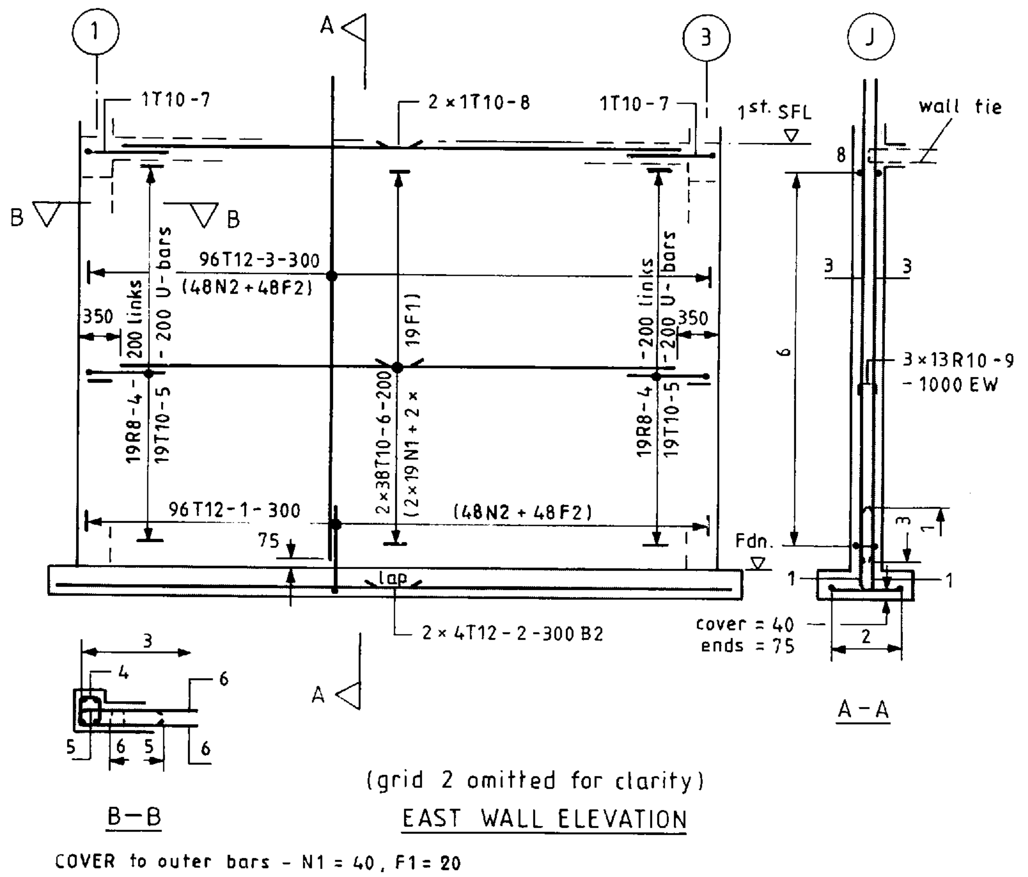


Figure 2.15 Shear wall reinforcement details

Table 2.8 Commentary on bar arrangement

Bar marks	Notes	
1	Wall starters match vertical reinforcement The projection of the horizontal legs beyond the face of the wall form the tension reinforcement in the footing This extension must be at least a tension anchorage length $= \frac{12}{4} \times \frac{460}{1.15 \times 3.2} \times \frac{209}{377} = 208$ mm .... OK The minimum projection above the top of the base is a compression lap + 75 mm kicker $= 32 \times 12 + 75 = 459$ mm This is detailed at 525 mm .... OK	5.2.2.2 5.2.2.3 5.2.3.4.1
2	Minimum longitudinal reinforcement provided	
4,5,6	Minimum horizontal reinforcement provided	5.4.7.3 4.4.2.2
7,8	Peripheral tie at floor	BS 8110 3.12.3.5
9	Wall spacers to maintain location of reinforcement	

2.9 Staircase

2.9.1 Idealization

The idealization of the staircase is shown in Figure 2.16.

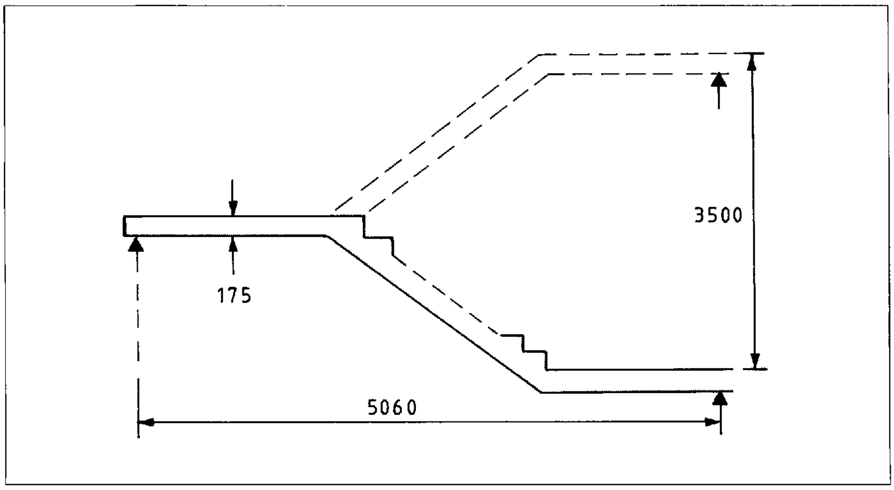


Figure 2.16 Idealization of staircase

Design as end span of a continuous beam. Calculations will be given for 1 m width.

2.9.2 Durability and fire resistance

As for floor slab, Section **2.3**, 20 mm nominal cover will be satisfactory.

2.9.3 Loading

Average slab thickness on plan	=	250 mm
Self-weight = 0.25 × 24	=	6.0 kN/m
Finishes	=	0.5
Characteristic dead load	=	6.5 kN/m
Characteristic imposed load	=	4.0 kN/m
Design ultimate load	=	1.35 × 6.5 + 1.5 × 4 = 14.78 kN/m

2.9.4 Analysis

Using coefficients in the Concise Eurocode

Moment at interior support	=	$0.11 \times 14.78 \times 5.06^2 = 41.6$ kNm	Concise Eurocode Table A.1
Moment near mid-span	=	$0.09 \times 14.78 \times 5.06^2 = 34.1$ kNm	
Shear	=	$0.6 \times 14.78 \times 5.06 = 44.9$ kN	

2.9.5 Reinforcement for flexure

Effective depth = 175 − 20 − 6 = 149 mm

Interior support, $\dfrac{M}{bd^2 f_{ck}} = \dfrac{41.6 \times 10^6}{10^3 \times 149^2 \times 32} = 0.059$

From Section **13**, Table 13.1

$$\frac{A_s f_{yk}}{bdf_{ck}} = 0.072$$

Hence

$$A_s = 746 \text{ mm}^2/\text{m}$$

Use T12 @ 150 mm crs. (754 mm²/m)

Span

$$\frac{M}{bd^2 f_{ck}} = 0.048$$

$$\frac{A_s f_{yk}}{bdf_{ck}} = 0.058$$

Hence

$$A_s = 601 \text{ mm}^2/\text{m}$$

Use T12 @ 150 mm crs. (754 mm²/m)

2.9.6 Shear

Reinforcement ratio $= \dfrac{754}{1000 \times 149} = 0.0051$

Near support

$V_{Rd1} = 0.35 \times (1.6 - 0.175) \times (1.2 + 40 \times 0.0051) \times 149 = 104.3$ kN — 4.3.2.3 Eqn 4.18

$V_{Rd1} > V_{Sd} = 44.9$ kN, hence no shear reinforcement required

2.9.7 Deflection

Reinforcement ratio at mid-span = 0.51%

Concrete is lightly stressed, hence basic span/effective depth ratio is 32. — Table 4.14

Since f_{yk} = 460, this should be modified to:

32 × 400/460 × 754/601 = 34.9 — 4.4.3.2(4)

Actual span/effective depth ratio = 5060/149 = 34 < 34.9. . . . . . . OK

2.9.8 Cracking

As for floor slab in Section **2.3.8**

Minimum area of reinforcement = 183 mm²/m — 4.4.2.2

Thickness of waist = 175 < 200 mm — 4.4.2.3 (1)

No further check is necessary.

2.9.9 Tie provisions

E–W internal tie, the minimum area required = 91 mm²/m — BS 8110 3.12.3.2

(see Section **2.3.9**)

Total area for staircase = 91 × 3 = 273 mm²

Provide 2T12 tie bars each side of staircase in adjacent slab

2.9.10 Reinforcement details

The reinforcement details are shown in Figure 2.17.

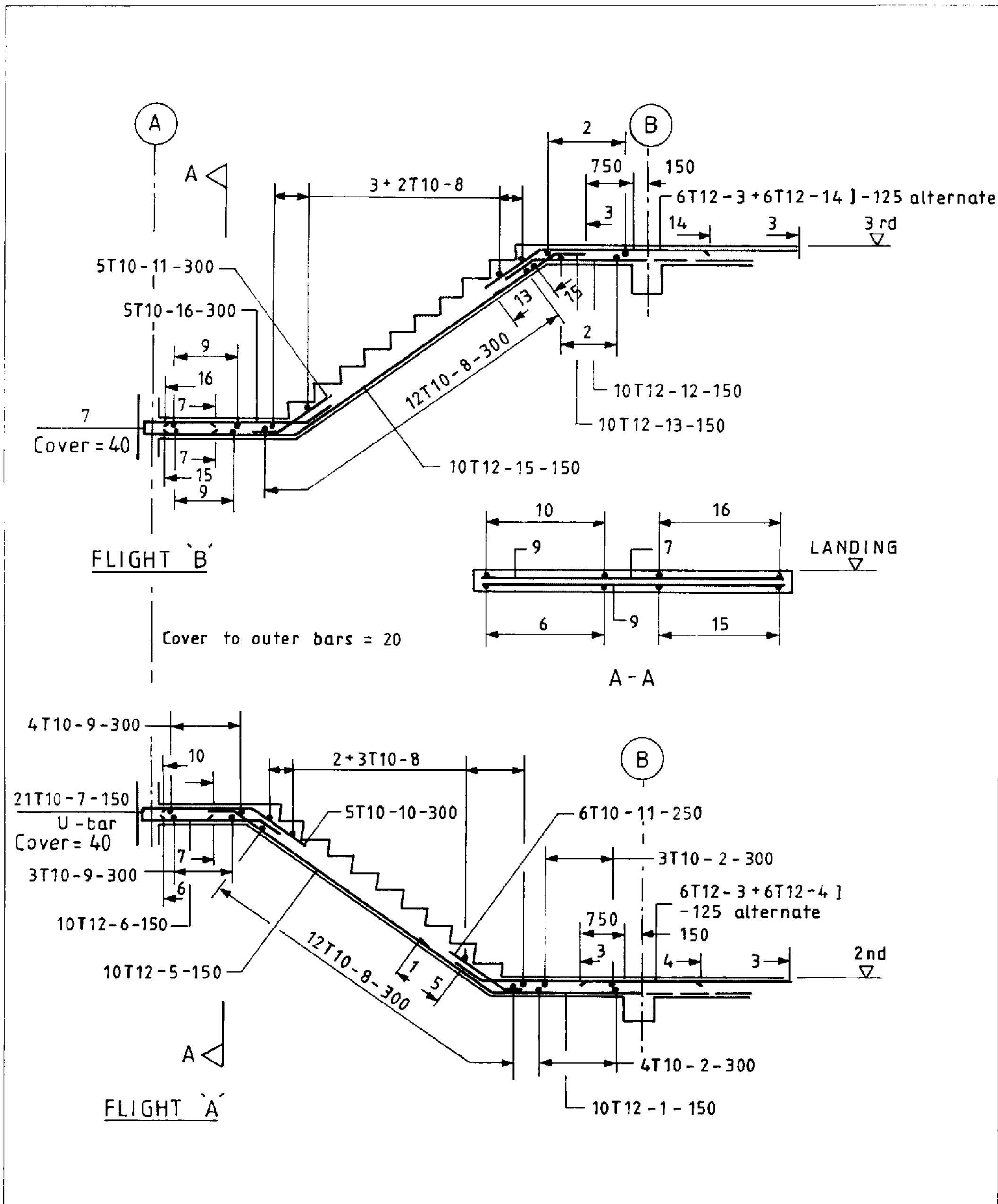


Figure 2.17 Staircase reinforcement details

3.1 Introduction

This Section covers the design of beams for shear and torsion, and supplements the examples given in Section 2. The requirements for adequate safety against lateral buckling are also examined.

3.2 Design methods for shear

3.2.1 Introduction

EC2[1] differs from BS 8110[2] because the truss assumption used in shear design is explicit. Leading on from this, two alternative methods are given in the Code.

(1) Standard
(2) Variable Strut Inclination (VSI).

The standard method assumes a concrete strut angle of 45° ($\cot\Theta = 1$) and that the direct shear in the concrete, V_{cd}, is to be taken into account. This contrasts with the VSI method which permits the designer to choose strut angles between the limits set in the NAD[1], as shown in Figure 3.1, but ignores the direct shear in the concrete.

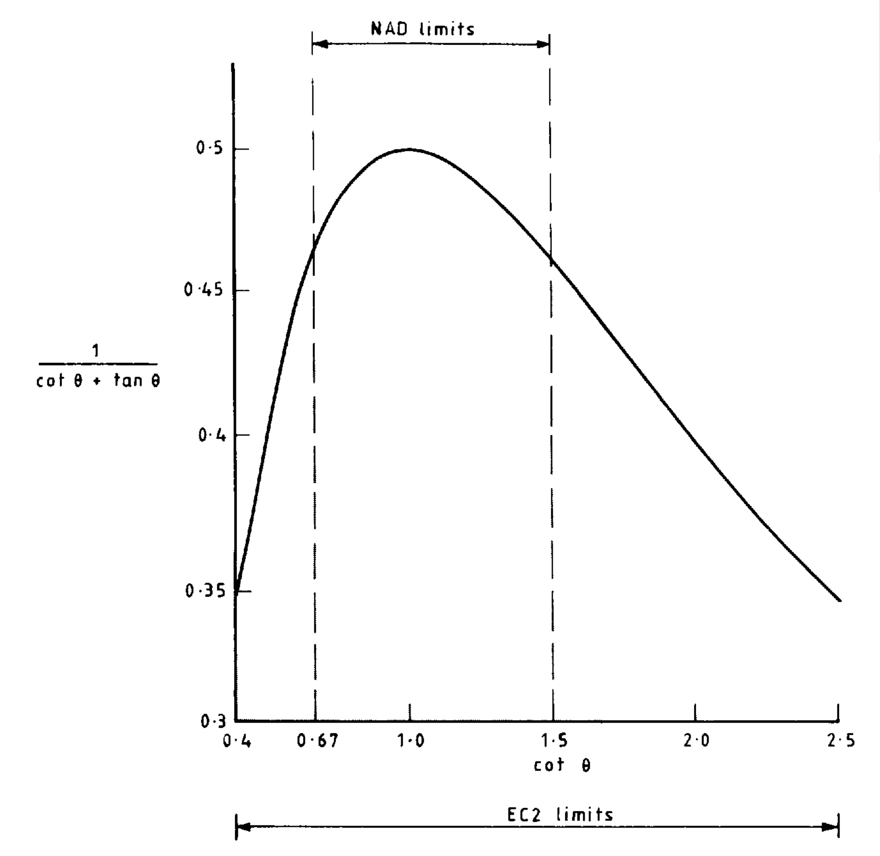


Figure 3.1 Limits of $\cot\Theta$ (VSI method)

Because the direct shear in the concrete is not taken into account in the VSI method, no savings in shear reinforcement can be achieved until the applied shear exceeds three times the concrete shear ($V_{Sd} > 3V_{cd}$).

A further disadvantage of this method is that with increasing values of $\cot\Theta$, i.e., reductions in the concrete strut angle, the forces in the tension reinforcement increase significantly and may well outweigh any notional savings in shear reinforcement. These forces are, it should be noted, explicitly checked in EC2 but not in BS 8110. Given special circumstances the VSI method may be required but for most practical situations, the standard method will provide the most economic design.

3.2.2 Example 1 – uniformly distributed loading

The beam shown in Figures 3.2 and 3.3 is to be designed for shear.

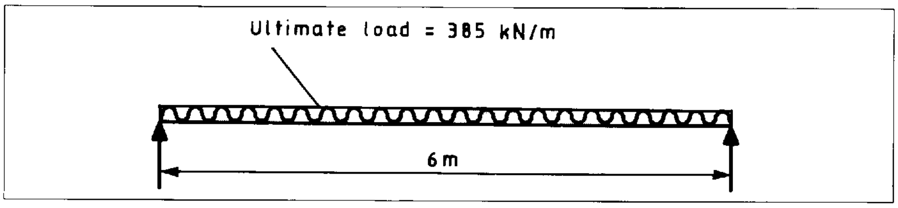


Figure 3.2 Beam span and loading – example 1

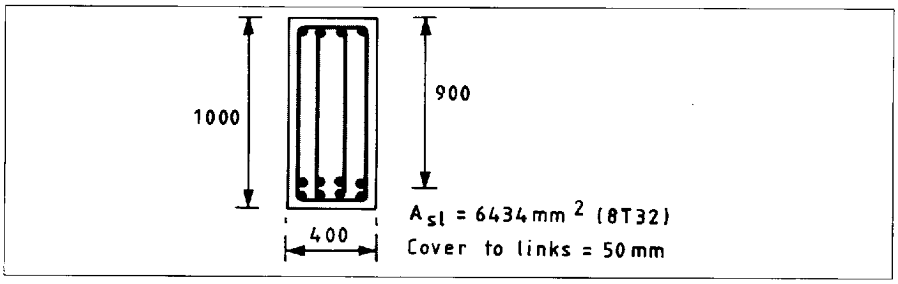


Figure 3.3 Typical section – example 1

The material strengths are

f_{ck} = 30 N/mm^2 (concrete strength class C30/37)

f_{ywk} = 250 N/mm^2 (characteristic yield strength of links)

The beam will be checked for shear reinforcement at three locations using both the standard and VSI methods for comparison. These are 4.3.2.4.3 4.3.2.4.4

(1) d from support 4.3.2.2(10)
(2) Where V_{Sd} = V_{Rd1}, i.e., the point beyond which only minimum shear reinforcement is required 4.3.2.2(2)
(3) An intermediate point between 1 and 2.

3.2.2.1 Standard method 4.3.2.4.3

The shear force diagram is shown in Figure 3.4.

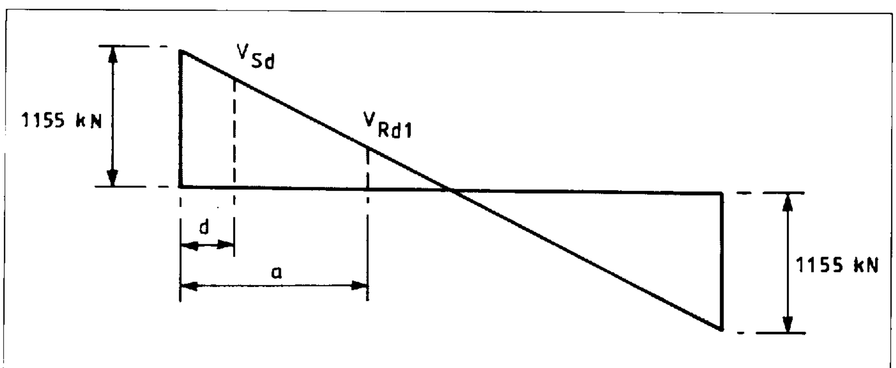


Figure 3.4 Shear force diagram – example 1

The design shear resistance of the section, V_{Rd1}, is given by 4.3.2.3(1)

$V_{Rd1} = [\tau_{Rd} k\,(1.2 + 40\rho_l) + 0.15\sigma_{cp}]\, b_w d$ Eqn 4.18

$\tau_{Rd} = 0.34\ \text{N/mm}^2 \text{ for } f_{ck} = 30\ \text{N/mm}^2$ Table 4.8

$k = 1.6 - d \not< 1 = 1$

$$\rho_l = \frac{A_{sl}}{b_w d} = \frac{6434}{400 \times 900} = 0.018 \not> 0.02$$

(assuming 8T32 throughout span)

$$\sigma_{cp} = \frac{N_{Sd}}{A_c} = 0$$

$V_{Rd1} = 0.34 \times 1\ (1.2 + 40 \times 0.018) \times 400 \times 900 = 235\ \text{kN}$

3.2.2.1.1 Position 1 – at *d* from support

$V_{Sd} = 1155 - 0.9 \times 385 = 808.5\ \text{kN}$

$V_{Sd} > V_{Rd1}$, shear reinforcement is required 4.3.2.4.3

The shear resistance of a section with shear reinforcement is given by

$V_{Rd3} = V_{cd} + V_{wd}$ Eqn 4.22

$V_{cd} = V_{Rd1} = 235\ \text{kN}$

$$V_{wd} = \frac{A_{sw}}{s}(0.9d)f_{ywd}$$ Eqn 4.23

where

A_{sw} = area of shear reinforcement

s = spacing of shear reinforcement

$f_{ywd} = 250/1.15 = 217.4\ \text{N/mm}^2$

For $V_{Rd3} \geq V_{Sd}$

$V_{wd} \geq V_{Sd} - V_{cd}$; or

$$\frac{A_{sw}}{s}(0.9d)f_{ywd} \geq V_{Sd} - V_{cd}$$

Therefore

$$\frac{A_{sw}}{s} = \frac{(808.5 - 235) \times 10^3}{0.9 \times 900 \times 217.4} = 3.25\ \text{mm}^2/\text{mm}$$

Try R12 links @ 140 mm crs. (4 legs), $A_{sw}/s = 3.23\ \text{mm}^2/\text{mm}$

Check crushing of compression struts

$V_{Rd2} = (\frac{1}{2})\nu f_{cd} b_w 0.9d(1 + \cot\alpha)$ Eqn 4.25

For vertical links, $\cot\alpha = 0$

$\nu = 0.7 - \frac{f_{ck}}{200} = 0.55 \not< 0.5$ Eqn 4.21

$f_{cd} = \frac{30}{1.5} = 20 \text{ N/mm}^2$

Therefore

$V_{Rd2} = (\frac{1}{2}) \times 0.55 \times 20 \times 400 \times 0.9 \times 900 \times 1$

$= 1782 \text{ kN} > V_{Sd,\,max} = 1155 \text{ kN}$ .... OK

Check maximum spacing of links 4.4.2.3

$\rho_w = \frac{A_{sw}}{s b_w \sin\alpha} = \frac{452}{140 \times 400} = 0.0081$ Eqn 4.79

$\frac{V_{Sd} - 3V_{cd}}{\rho_w b_w d} = \frac{(808.5 - 3 \times 235) \times 10^3}{0.0081 \times 400 \times 900} = 35 \text{ N/mm}^2$ Table 4.13

Maximum spacing for crack control = 300 mm

Since $(\frac{1}{5}) V_{Rd2} < V_{Sd} \leq (\frac{2}{3}) V_{Rd2}$ 5.4.2.2(7) Eqn 5.18

$s_{max} = 0.6d \not> 300 \text{ mm}$

140 mm spacing .... OK

Check minimum value of ρ_w Table 5.5

Concrete strength class C30/37

Steel class S250

By interpolation from EC2 Table 5.5

$\rho_{w,min} = 0.0022 < 0.0081$ proposed

Use R12 links @ 140 mm crs. (4 legs)

Note:
Using the standard method, the increase in force in the tension reinforcement is best covered by using the shift rule. 4.3.2.1P(6) 5.4.2.1.3

It will, however, be calculated in this example to provide a comparison with the values obtained in the subsequent examples using the VSI method.

Force in tension reinforcement

$T_d = \frac{M_{Sd}}{z} + (\frac{1}{2}) V_{Sd}(\cot\Theta - \cot\alpha)$ Eqn 4.30

M_{Sd} = 884 kNm, z = 0.9d = 810 mm

V_{Sd} = 808.5 kN

cotΘ = 1, cotα = 0 for vertical links 4.3.2.4.3(5)

Therefore T_d = 1091 + 404 = 1495 kN

3.2.2.1.2 Position 2 – where V_{Sd} = V_{Rd1} = 235 kN

From Figure 3.4

V_{Sd} = 1155 – a × 385 = 235 kN

a = 2.39 m from support

From Section **3.2.2.1.1**, V_{Rd2} > $V_{Sd,\ max}$ .... OK

The amount of shear reinforcement provided should be greater than $\rho_{w,min}$ Table 5.5

$\rho_{w,min}$ = 0.0022

Re-arranging EC2 Eqn 5.16 in terms of $\frac{A_{sw}}{s}$ gives

$$\frac{A_{sw}}{s} = \rho_w b_w \sin\alpha$$

For vertical links sinα = 1

Hence

$$\frac{A_{sw}}{s} = 0.0022 \times 400 \times 1 = 0.88 \text{ mm}^2/\text{mm}$$

Maximum longitudinal spacing (s_{max}) is given by EC2 Eqns 5.17–5.19.

V_{Sd} = 235 kN

V_{Rd2} = 1782 kN from Section **3.2.2.1.1**

Since

V_{Sd} ≤ ($\frac{1}{5}$) V_{Rd2}, EC2 Eqn 5.17 applies

s_{max} = 0.8d $\not>$ 300 mm Eqn 5.17

A_{sw} = 0.88 × 300 = 264 mm², 4R10 = 314 mm²

Use R10 links @ 300 mm crs. (4 legs)

3.2.2.1.3 Position 3 – at 1.65 m from support

This is a point intermediate between the section at d from support and the point at which shear reinforcement is no longer required.

V_{Sd} = 1155 – 1.65 × 385 = 520 kN

V_{Rd1} = 235 kN

Since $V_{Sd} > V_{Rd1}$, shear reinforcement is required

Re-arranging EC2 Eqn 4.23

$$\frac{A_{sw}}{s} = \frac{V_{Sd} - V_{cd}}{0.9df_{ywd}} = \frac{(520 - 235) \times 10^3}{0.9 \times 900 \times 217.4} = 1.62 \text{ mm}^2/\text{mm}$$

Try R12 links @ 250 mm crs. (4 legs) = 1.81 mm²/mm

Check maximum spacing of links 4.4.2.3

$$\rho_w = \frac{A_{sw}}{sb_w \sin\alpha}$$ Eqn 4.79

For vertical links $\sin\alpha = 1$

Hence

$$\rho_w = \frac{452}{250 \times 400} = 0.0045$$

$$\frac{V_{Sd} - 3V_{cd}}{\rho_w b_w d} = \frac{(520 - 3 \times 235) \times 10^3}{0.0045 \times 400 \times 900} = -114 \text{ N/mm}^2$$

Maximum spacing for crack control = 300 mm .... OK Table 4.13

Since

$$(\tfrac{1}{5})V_{Rd2} < V_{Sd} \leq (\tfrac{2}{3})V_{Rd2}$$ 5.4.2.2(7) Eqn 5.18

$$s_{max} = 0.6d \not> 300 \text{ mm}$$

From Section **3.2.2.1.1**

$V_{Rd2} > V_{Sd,max}$ .... OK

Provide R12 links @ 250 mm crs (4 legs)

To optimize link spacing, check the point at which shear reinforcement is satisfied by R12 @ 200 mm crs. (4 legs).

$$\frac{A_{sw}}{s} = \frac{452}{200} = 2.26 \text{ mm}^2/\text{mm}$$

$$V_{wd} = \frac{A_{sw}}{s}(0.9d)f_{ywd} = 2.26 \times 0.9 \times 900 \times 217.4 = 398 \text{ kN}$$

$$V_{Rd3} = V_{cd} + V_{wd}$$

Equating

$V_{Rd3} = V_{Sd}$ and noting that $V_{cd} = V_{Rd1}$

$$V_{Sd} = V_{Rd1} + V_{wd} = 235 + 398 = 633 \text{ kN}$$

$$\text{Distance of point from support} = \frac{1155 - 633}{385} = 1.36 \text{ m}$$

The proposed link arrangement is shown in Figure 3.5.

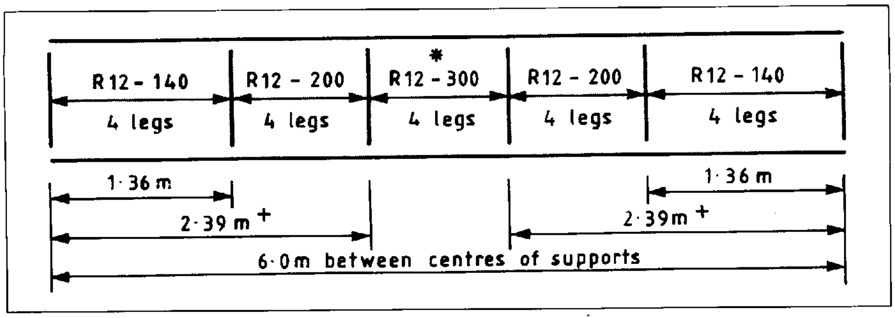


Figure 3.5 Link arrangement (standard method) – example 1

Note:
In the centre portion of the beam R10 links are required by calculations but R12 (*) are shown to avoid the possible misplacement on site. Distance from the support (+) could be reduced to 1.70 m in this case.

3.2.2.2 Variable strut inclination method 4.3.2.4.4

This method allows the angle of the concrete compression strut to be varied at the designer's discretion within limits stated in the Code.

It can give some economy in shear reinforcement but will require the provision of additional tension reinforcement. In most cases the standard method will suffice.

This reduced shear reinforcement will only be obtained at high levels of design shear and is counter-balanced by increased tension reinforcement. This can be seen by a comparison of EC2 Eqns 4.22 and 4.23 in the standard method and EC2 Eqn 4.27 in the variable strut inclination method.

The standard method gives

$$V_{Rd3} = V_{cd} + V_{wd} \quad \text{Eqn 4.22}$$

$$V_{wd} = \frac{A_{sw}}{s}(0.9d)f_{ywd} \quad \text{Eqn 4.23}$$

Re-arranging gives

$$\frac{A_{sw}}{s} = \frac{V_{Rd3} - V_{cd}}{(0.9d)f_{ywd}}$$

The VSI method gives

$$V_{Rd3} = \frac{A_{sw}}{s}(0.9d)f_{ywd}\cot\Theta \quad \text{Eqn 4.27}$$

Re-arranging gives

$$\frac{A_{sw}}{s} = \frac{V_{Rd3}}{(0.9d)f_{ywd}\cot\Theta}$$

Note:
In the above equation the contribution of the concrete, V_{cd}, to the shear resistance of the section is not taken into account.

With cotΘ = 1.5 which is the maximum value permitted in the NAD, reductions in shear reinforcement will only occur when

$$\frac{V_{Rd3}}{(0.9d)f_{ywd} \times 1.5} < \frac{V_{Rd3} - V_{cd}}{(0.9d)f_{ywd}} \text{ ; or}$$

$$V_{Rd3} < 1.5(V_{Rd3} - V_{cd})$$

Putting $V_{Sd} = V_{Rd3}$ gives $V_{Sd} > 3V_{cd}$

If $V_{Sd} > 3V_{cd}$, then the VSI method will allow a reduction in shear reinforcement.

If this inequality is not satisfied, use of the variable strut inclination method will produce an uneconomic amount of shear reinforcement. In this case the standard method should be used.

For elements with vertical shear reinforcement, V_{Rd2} is given by

$$V_{Rd2} = \frac{b_w z \nu f_{cd}}{\cot\Theta + \tan\Theta}$$ Eqn 4.26

Putting $V_{Sd} = V_{Rd2}$ and re-arranging gives

$$\frac{V_{Sd}}{b_w z \nu f_{cd}} = \frac{1}{\cot\Theta + \tan\Theta}$$

Figure 3.1 shows cotΘ plotted against 1/(cotΘ + tanΘ) together with the EC2 and NAD limits for cotΘ. Hence for a given V_{Sd}, the limits for cotΘ can be found.

Increasing the value of cotΘ will reduce the shear reinforcement required but increase the force in the tension reinforcement.

In this example, cotΘ will be chosen to minimize the shear reinforcement.

3.2.2.2.1 Position 1 – at *d* from support

From above

$$\frac{V_{Sd}}{b_w z \nu f_{cd}} = \frac{1}{\cot\Theta + \tan\Theta}$$

$$b_w = 400 \text{ mm}$$

$$z = 0.9 \times 900 = 810 \text{ mm}$$

$$\nu = 0.7 - \frac{f_{ck}}{200} = 0.55 \not< 0.5$$ Eqn 4.21

$$f_{cd} = \frac{30}{1.5} = 20 \text{ N/mm}^2$$

$$V_{Sd} = 808.5 \text{ kN}$$

Therefore

$$\frac{1}{\cot\Theta + \tan\Theta} = \frac{808.5 \times 10^3}{400 \times 810 \times 0.55 \times 20} = 0.22$$

From Figure 3.1, this lies under the curve. Therefore, $\cot\Theta = 1.5$ can be chosen which is the maximum value allowed under the NAD limits.

$$V_{Rd3} = \left(\frac{A_{sw}}{s}\right) z f_{ywd} \cot\Theta$$ Eqn 4.27

Now equating V_{Rd3} to V_{Sd} and re-arranging

$$\frac{A_{sw}}{s} = \frac{V_{Sd}}{z f_{ywd} \cot\Theta} = \frac{808.5 \times 10^3}{810 \times 217.4 \times 1.5} = 3.06 \text{ mm}^2\text{/mm}$$

Check

$$\frac{A_{sw} f_{ywd}}{b_w s} = 1.66 \leq (\tfrac{1}{2})\nu f_{cd} = 5.5$$ .... OK

Try R12 links @ 150 mm crs. (4 legs), $A_{sw}/s = 3.01 \text{ mm}^2\text{/mm}$

Check maximum spacing of links. 4.4.2.3

$$\rho_w = \frac{A_{sw}}{s b_w \sin\alpha} = 0.0075$$ Eqn 4.79

$$\frac{V_{Sd} - 3V_{cd}}{\rho_w b_w d} = \frac{(808.5 - 3 \times 235) \times 10^3}{0.0075 \times 400 \times 900} = 38.3 \text{ N/mm}^2$$

Maximum spacing for crack control = 300 mm Table 4.13

$\rho_w = 0.0075 > \rho_{w,min} = 0.0022$ .... OK Table 5.5

Check s_{max} 5.4.2.2(7)

$$V_{Sd} = 808.5 \text{ kN}$$

$$V_{Rd2} = \frac{b_w z \nu f_{cd}}{\cot\Theta + \tan\Theta} = \frac{400 \times 810 \times 0.55 \times 20}{2.167} = 1644 \text{ kN}$$

Since $(\tfrac{1}{5})V_{Rd2} < V_{Sd} \leq (\tfrac{2}{3})V_{Rd2}$ Eqn 5.18

$$s_{max} = 0.6d \ngtr 300 \text{ mm}$$

Use R12 links @ 150 mm crs. (4 legs)

Check additional force in tension reinforcement.

$$T_d = \frac{M_{Sd}}{z} + (\tfrac{1}{2})\ V_{Sd}(\cot\Theta - \cot\alpha) = 1091 + 606 = 1697 \text{ kN}$$ Eqn 4.30

This compares with T_d = 1495 kN using the standard method.

Note:
Although not permitted by the NAD, values of cotΘ up to 2.5 are given in EC2.

A check on shear reinforcement using cotΘ = 2.5 is now given to illustrate the effect of increasing values of Θ on shear and tension reinforcement.

$$\frac{A_{sw}}{s} = \frac{V_{Sd}}{zf_{ywd}\cot\Theta} = \frac{808.5 \times 10^3}{810 \times 217.4 \times 2.5} = 1.84 \text{ mm}^2/\text{mm}$$

Try R12 @ 225 mm crs. (4 legs), A_{sw}/s = 2.01 mm²/mm

Check maximum spacing of links

$$\rho_w = 0.005$$

$$\frac{V_{Sd} - 3V_{cd}}{\rho_w b_w d} = 57.5 \text{ N/mm}^2$$

Maximum spacing for crack control = 250 mm . . . . OK Table 4.13

s_{max} = 0.6d ⊁ 300 mm . . . . OK Eqn 5.18

Use R12 links @ 225 mm crs. (4 legs)

Check additional force in tension reinforcement.

$$T_d = \frac{M_{Sd}}{z} + (\tfrac{1}{2})V_{Sd}\ (\cot\Theta - \cot\alpha) = 1091 + 1011 = 2102 \text{ kN}$$

This compares with T_d = 1495 kN using the standard method.

3.2.2.2.2 Position 2 – where $V_{Sd} = V_{Rd1}$

Since only minimum shear reinforcement is required this case is identical to that shown in Section **3.2.2.1.2**.

3.2.2.2.3 Position 3 – at 1.65 m from support

$$V_{Sd} = 520 \text{ kN}$$

$$\frac{A_{sw}}{s} = \frac{V_{Sd}}{zf_{ywd}\cot\Theta} = \frac{520 \times 10^3}{810 \times 217.4 \times 1.5} = 1.96 \text{ mm}^2/\text{mm}$$

Try R12 links @ 225 mm crs. (4 legs), A_{sw}/s = 2.01 mm²/mm

From Section **3.2.2.2.1** spacing is satisfactory.

Use R12 links @ 225 mm crs. (4 legs)

As in Section **3.2.2.1.3**, check the point at which the shear requirement is satisfied by R12 @ 200 mm crs. (4 legs).

$$\frac{A_{sw}}{s} = \frac{452}{200} = 2.26 \text{ mm}^2/\text{mm}$$

$$V_{Rd3} = \left(\frac{A_{sw}}{s}\right) z f_{ywd} \cot\Theta = 2.26 \times 810 \times 217.4 \times 1.5 = 597 \text{ kN}$$ Eqn 4.27

$$\text{Distance from support} = \frac{1155 - 597}{385} = 1.45 \text{ m}$$

The proposed link arrangement is shown in Figure 3.6.

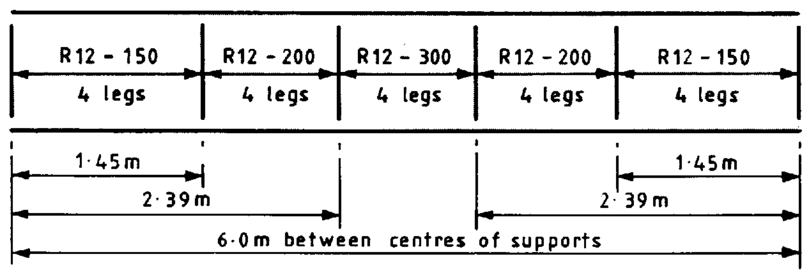


Figure 3.6 Link arrangement (VSI method) – example 1

Comparing this with the arrangement in Figure 3.5 obtained using the standard method, it can be seen that less reinforcement is required near the support but this needs to be carried further along the beam. There is little overall saving in this case.

3.3 Shear resistance with concentrated loads close to support

3.3.1 Introduction

Where concentrated loads are located within 2.5d of a support, the value τ_{Rd} may be modified by a factor β when calculating V_{Rd1}. This enhancement only applies when the section is resisting concentrated loads and the standard method is used. For a uniformly distributed load, an unmodified value of V_{Rd1} should be used. 4.3.2.2(9)

3.3.2 Example 2 – concentrated loads only

The beam shown in Figures 3.7 and 3.8 is to be designed for shear.

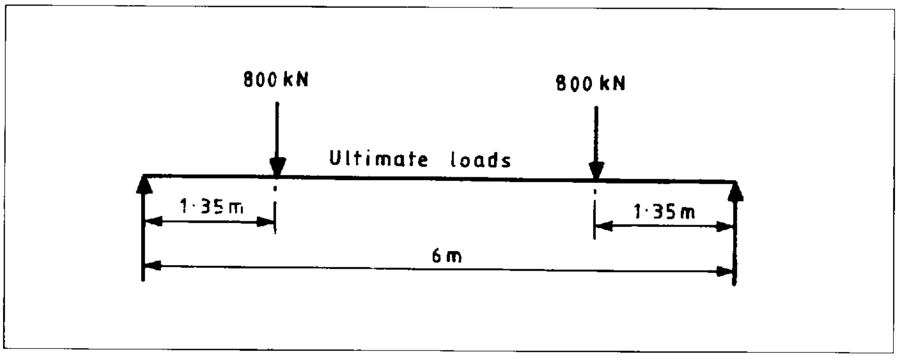


Figure 3.7 Beam span and loading – example 2

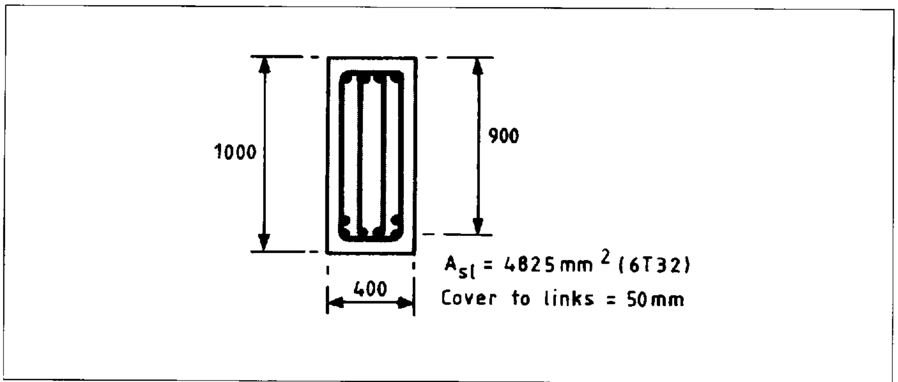


Figure 3.8 Typical section – example 2

The materials strengths are

f_{ck} = 30 N/mm² (concrete strength grade, C30/37)

f_{ywk} = 250 N/mm² (characteristic yield strength of links)

In the example, V_{Rd1} will be calculated at positions between the support and 2.5*d* away at intervals of 0.5*d*. This is done to illustrate the effect even though the critical section will normally be at the position of the concentrated load.

3.3.2.1 Shear reinforcement

The shear force diagram is shown in Figure 3.9.

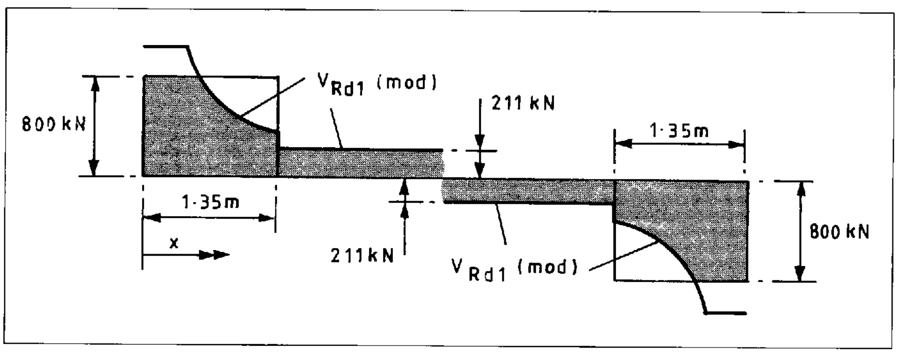


Figure 3.9 Shear force diagram – example 2

The basic design shear resistance of the section, V_{Rd1}, is given by 4.3.2.3(1)

$$V_{Rd1} = [\tau_{Rd}\, k\, (1.2 + 40\rho_l) + 0.15\, \sigma_{cp}] b_w d$$ Eqn 4.18

$$\tau_{Rd} = 0.34 \text{ N/mm}^2 \text{ for } f_{ck} = 30 \text{ N/mm}^2$$ Table 4.8

For concentrated loads within 2.5d of the face of the support, an enhancement of shear resistance is permitted. τ_{Rd} may be multiplied by a factor β when determining V_{Rd1}.

$$\beta = 2.5d/x \text{ with } 1.0 \leq \beta \leq 5.0$$ Eqn 4.17

Taking values of x between 0.5d and 2.5d gives values of $\beta\tau_{Rd}$ shown in Table 3.1.

Table 3.1 Design shear strength $\beta\tau_{Rd}$

x (m)	β	$\beta\tau_{Rd}$ (N/mm^2)
0.45	5	1.7
0.90	2.5	0.85
1.35	1.67	0.57
1.80	1.00*	0.34
2.25	1.00*	0.34

* No enhancement taken, see Figure 3.9

The equation for V_{Rd1} can be modified to give a range of values corresponding to the distance from the support.

$$V_{Rd1}(x) = [\beta\tau_{Rd} k\, (1.2 + 40\rho_l) + 0.15\sigma_{cp}] b_w d$$ Eqn 4.18 (mod)

$$k = 1.6 - d \not< 1 = 1$$

$$\rho_l = \frac{A_{sl}}{b_w d} = \frac{4825}{400 \times 900} = 0.013$$

$$\sigma_{cp} = \frac{N_{Sd}}{A_c} = 0$$

Values of design shear resistance, V_{Rd1}, are given in Table 3.2.

Table 3.2 Design shear resistance V_{Rd1}

x (m)	V_{Rd1} (kN)
0.45	1052
0.90	526
1.35	353
1.80	211
2.25	211

Shear reinforcement is required when $V_{Sd} > V_{Rd1}$. 4.3.2.4

From Figure 3.9, V_{Sd} = 800 kN from x = 0 to x = 1.35 m

Using the standard method 4.3.2.4.3

$$V_{Rd3} = V_{cd} + V_{wd} \qquad \text{Eqn 4.22}$$

Putting $V_{Rd3} = V_{Sd}$ and $V_{cd} = V_{Rd1}$ gives

$$V_{Sd} = V_{Rd1} + V_{wd}$$

Values of design shear resistance to be provided by shear reinforcement, V_{wd}, are given in Table 3.3.

Table 3.3 Design shear resistance V_{wd}

V_{Rd1} (kN)	V_{Sd} (kN)	$V_{Sd} - V_{Rd1} = V_{wd}$ (kN)
1052	800	< 0
526	800	274
353	800	447
211	0	< 0
211	0	< 0

Therefore maximum shear reinforcement is required when V_{Rd1} = 353 kN, i.e., when x = 1.35 m.

This should be provided over the entire length from x = 0 to x = 2.25 m ($0 < x < 2.5d$). 4.3.2.2(9)

Note: 4.3.2.2(9)

If a concentrated load is positioned close to a support, it is possible that using β to modify V_{Rd1} may lead to only minimum shear reinforcement being provided throughout the beam. In this case, the designer may wish to base the shear resistance on the unmodified V_{Rd1}.

This can be illustrated by taking the example above but placing the point load at 0.5d from the support.

The modified shear force diagram is shown in Figure 3.10.

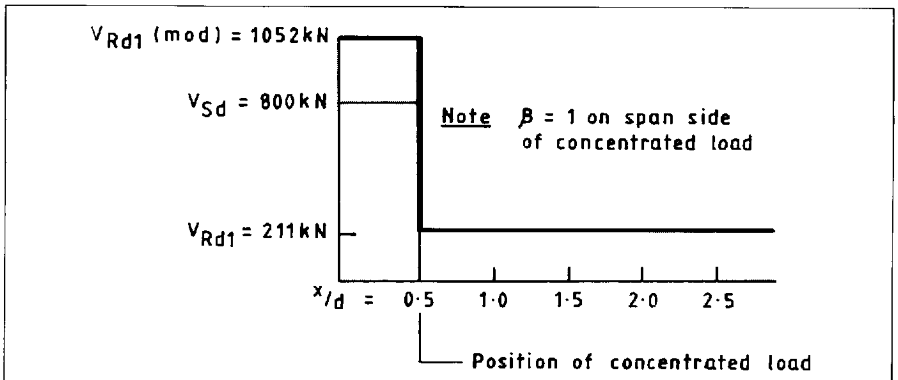


Figure 3.10 Shear force diagram (load at 0.5d) – example 2 modified

In this case it would be prudent to check the shear resistance on the unmodified V_{Rd1} = 211 kN. The required shear reinforcement should be provided from $x = 0$ to $x = 0.5d$

Check area of shear reinforcement required in example 2.

Re-arranging the equation for V_{wd} gives — Eqn 4.23

$$\frac{A_{sw}}{s} = \frac{V_{wd}}{0.9df_{ywd}} = \frac{447 \times 10^3}{0.9 \times 900 \times 217.4} = 2.54 \text{ mm}^2/\text{mm}$$

Try R12 links @ 175 mm crs. (4 legs), $A_{sw}/s = 2.58 \text{ mm}^2/\text{mm}$

Check crushing of compression strut

$$V_{Rd2} = (\tfrac{1}{2})\,\nu f_{cd} b_w 0.9d\,(1 + \cot\alpha)$$ Eqn 4.25

For vertical links, $\cot\alpha = 0$

$$\nu = 0.7 - \frac{f_{ck}}{200} = 0.55$$ Eqn 4.21

$$f_{cd} = \frac{30}{1.5} = 20 \text{ N/mm}^2$$

Therefore

$$V_{Rd2} = (\tfrac{1}{2}) \times 0.55 \times 20 \times 400 \times 0.9 \times 900 \times 1$$

$$= 1782 \text{ kN} > V_{Sd} = 800 \text{ kN}$$ .... OK

Check maximum spacing of links. — 4.4.2.3

$$\rho_w = \frac{A_{sw}}{sb_w \sin\alpha}$$ Eqn 4.79

For vertical links $\sin\alpha = 1$

$$\rho_w = \frac{452}{175 \times 400} = 0.0064 > \rho_{w,min} = 0.0022$$ .... OK — Table 5.5

$$\frac{V_{Sd} - 3V_{cd}}{\rho_w b_w d} = \frac{(800 - 3 \times 353) \times 10^3}{0.0064 \times 400 \times 900} < 0$$

Maximum spacing for crack control = 300 mm — Table 4.13

By inspection, EC2 Clause 5.4.2.2(7) is satisfied. — 5.4.2.2(7)

Use R12 links @ 175 mm crs. (4 legs) for $0 < x < 2.25$ m

3.3.3 Example 3 – combined loading

The revised loading and shear force diagrams are shown in Figures 3.11 and 3.12 respectively.

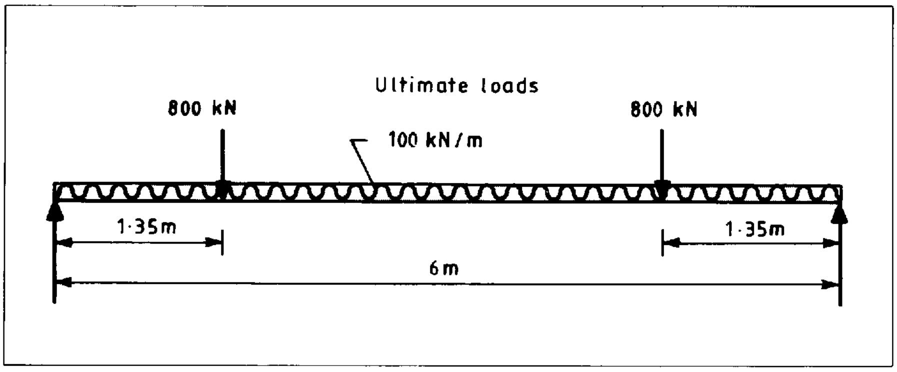


Figure 3.11 Beam span and loading – example 3

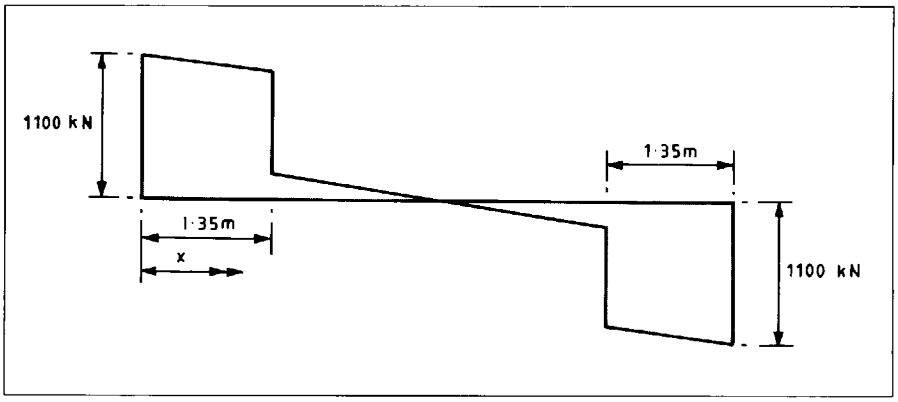


Figure 3.12 Shear force diagram – example 3

The basic design shear resistance of the section, V_{Rd1}, is given by 4.3.2.3(1)

$$V_{Rd1} = [\tau_{Rd}k(1.2 + 40\rho_l) + 0.15\sigma_{cp}]b_w d$$ Eqn 4.18

For concentrated loads within 2.5d of the face of the support, τ_{Rd} may be increased as in Section **3.3.2**. However, no similar enhancement is permitted for uniformly distributed loads. 4.3.2.2(9)

β must be reduced depending on the proportion of concentrated loads to total design load. β can then be written as

$$\beta_{red} = 1 + (\beta - 1)\frac{V_{Sd(conc)}}{V_{Sd(tot)}} \quad \text{with } 1.0 \le \beta \le 5.0$$

$V_{Sd(conc)}$ = design shear force due to concentrated loads

$V_{Sd(tot)}$ = design shear force due to total loads

Values of the concentrated load ratio and the resulting design shear strength are given in Tables 3.4 and 3.5.

Table 3.4 Concentrated load ratio $V_{Sd(conc)}/V_{Sd(tot)}$

x (m)	$V_{Sd(conc)}$ (kN)	$V_{Sd(ud1)}$ (kN)	$V_{Sd(tot)}$ (kN)	$V_{Sd(conc)}/V_{Sd(tot)}$
0.45	800	255	1055	0.76
0.90	800	210	1010	0.79
1.35	800	165	965	0.83
1.80	0	120	120	0
2.25	0	75	75	0

Table 3.5 Design shear strength $\beta_{red}\tau_{Rd}$

x (m)	β	β_{red}	$\beta_{red}\tau_{Rd}$ (N/mm²)
0.45	5	4.04	1.37
0.90	2.5	2.19	0.75
1.35	1.67	1.56	0.53
1.80	1.0	1.00	0.34
2.25	1.0	1.00	0.34

The equation for V_{Rd1} can be modified to give a range of values corresponding to the distance from the support.

$$V_{Rd1}(x) = [\beta_{red}\tau_{Rd}k(1.2 + 40\rho_l) + 0.15 \quad \sigma_{cp}]\, b_w d$$

Eqn 4.18 (mod)

As in Section **3.3.2.1**

$k = 1, \quad \rho_l = 0.013, \quad \sigma_{cp} = 0$

Values of design shear resistance, V_{Rd1}, and design shear resistance to be provided by shear reinforcement, V_{wd}, are given in Tables 3.6 and 3.7.

Table 3.6 Design shear resistance (V_{Rd1})

x (m)	V_{Rd1} (kN)
0.45	848
0.90	464
1.35	328
1.80	211
2.25	211

Table 3.7 Design shear resistance (V_{wd})

V_{Rd1} (kN)	V_{Sd} (kN)	$V_{Sd} - V_{Rd1} = V_{wd}$ (kN)
848	1055	207
464	1010	546
328	965	637
211	120	< 0
211	75	< 0

Therefore maximum shear reinforcement is required when

$V_{Rd1} = 328$ kN, i.e., when $x = 1.35$ m

This should be provided from $x = 0$ to $x = 2.25$ m $(0 < x < 2.5d)$

Check area of shear reinforcement required.

Re-arranging the equation for V_{wd} Eqn 4.23

$$\frac{A_{sw}}{s} = \frac{V_{wd}}{(0.9d)f_{ywd}} = \frac{637 \times 10^3}{0.9 \times 900 \times 217.4} = 3.61 \text{ mm}^2/\text{mm}$$

Try R12 links @ 125 mm crs. (4 legs), $A_{sw}/s = 3.62$ mm²/mm

Check crushing of compression strut

From example 2, $V_{Rd2} = 1782$ kN $> V_{Sd} = 1100$ kN .... OK

Check maximum spacing of links 4.4.2.3

By comparison with example 2, requirements are satisfied .... OK 5.4.2.2(7)

Use R12 links @ 125 mm crs. (4 legs) for $0 < x < 2.25$ m

For the remainder of the beam beyond $x = 2.5d$ (2.25 m) provide minimum reinforcement as example given in Section **3.2.2**.

3.4 Design method for torsion

3.4.1 Introduction

The edge beam shown in Figure 3.13 carries the ends of simply supported floor slabs seated on the lower flange. The beam is fully restrained at its ends.

The example chosen is the same as that used in Allen's *Reinforced concrete design to BS 8110: Simply explained*[12].

Analysis of the structure and the design of the section for flexure is not included.

The section will be checked for shear, torsion and the combination of both.

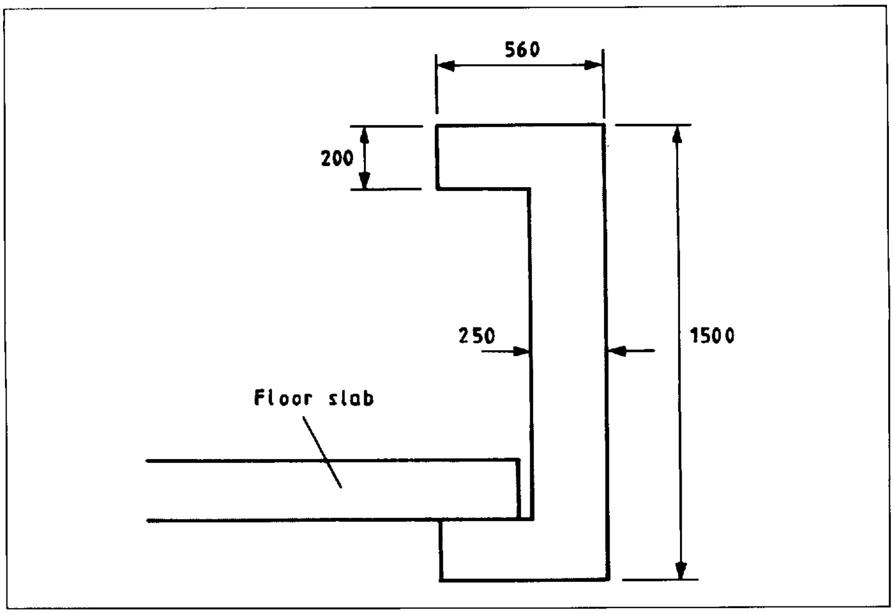


Figure 3.13 Beam section

3.4.2 Design data

Design torsional moment (T_{Sd}) = 120 kNm Design shear (V_{Sd}) = 355 kN

Concrete strength grade is C30/37, f_{ck} = 30 N/mm^2 3.1.2.4 Table 3.1

Nominal cover to links is 35 mm. 4.1.3.3 NAD Table 6

Assuming 25 mm bars and 10 mm links

$$d = 1500 - 35 - 10 - \frac{25}{2} = 1441.5 \text{ say } 1440 \text{ mm}$$

Assume 0.25% tensile reinforcement for flexure

3.4.3 Shear resistance

Shear will be taken as acting on the web of the section only.

When combined shear and torsion effects are to be considered, shear is to be checked using the variable strut inclination method. The angle Θ of the equivalent concrete struts is to be the same for both torsion and shear design. 4.3.3.2.2(4)

The design shear resistance, V_{Rd1}, with zero axial load is given by 4.3.2.3(1)

$$V_{Rd1} = \tau_{Rd}k(1.2 + 40\rho_l)b_w d$$ Eqn 4.18

$$\tau_{Rd} = 0.34 \text{ N/mm}^2 \text{ for } f_{ck} = 30 \text{ N/mm}^2$$ Table 4.8

$$k = 1.6 - d = 1.6 - 1.44 = 0.16 \not< 1.0$$

Assuming 0.25% tensile reinforcement, ρ_l = 0.0025 $\not>$ 0.02

$$V_{Rd1} = 0.34 \times 1(1.2 + 40 \times 0.0025) \times 250 \times 1440 \times 10^{-3}$$
$$= 159.1 \text{ kN} < 355 \text{ kN}$$

Therefore shear reinforcement required.

Use the variable strut inclination method. The maximum design shear force, V_{Rd2}, to avoid web crushing is given by

$$V_{Rd2} = \frac{b_w z \nu f_{cd}}{(\cot\Theta + \tan\Theta)}$$

4.3.2.4.4(2)
Eqn 4.26

Re-arranging gives

$$\frac{V_{Rd2}}{b_w z \nu f_{cd}} = \frac{1}{\cot\Theta + \tan\Theta}$$

$$V_{Sd} = 355 \text{ kN}$$

$$b_w = 250 \text{ mm}$$

$$z = 0.9d = 0.9 \times 1440 = 1296 \text{ mm}$$

$$\nu = 0.7 - \frac{f_{ck}}{200} = 0.7 - \frac{30}{200} = 0.55 \not< 0.5$$

4.3.2.4.2(3)

$$f_{cd} = \frac{f_{ck}}{\gamma_c} = \frac{30}{1.5} = 20 \text{ N/mm}^2$$

Therefore

$$\frac{V_{Sd}}{b_w z \nu f_{cd}} = \frac{355 \times 10^3}{250 \times 1296 \times 0.55 \times 20} = 0.1$$

$$\frac{1}{\cot\Theta + \tan\Theta} \text{ should be} \geq 0.1$$

By reference to Figure 3.1, it will be seen that the value of cotΘ may be taken anywhere between the limits of 0.67 to 1.5.

NAD
Table 3
4.3.2.4.4(1)

To minimize link reinforcement, take $\cot\Theta = 1.5$

Design shear resistance, V_{Rd3}, for shear reinforcement is given by

$$V_{Rd3} = \left(\frac{A_{sw}}{s}\right) z f_{ywd} \cot\Theta$$

4.3.2.4.4(2)
Eqn 4.27

Re-arranging gives

$$\frac{A_{sw}}{s} = \frac{V_{Rd3}}{zf_{ywd}\cot\Theta}$$

Putting V_{Rd3} equal to V_{Sd}

$$\frac{A_{sw}}{s} = \frac{V_{Sd}}{zf_{ywd}\cot\Theta}$$

Using high yield reinforcement

$$f_{ywd} = \frac{f_{ywk}}{\gamma_s} = \frac{460}{1.15} = 400 \text{ N/mm}^2$$

Therefore

$$\frac{A_{sw}}{s} = \frac{355 \times 10^3}{0.9 \times 1440 \times 400 \times 1.5} = 0.46 \text{ mm}^2\text{/mm}$$

$$\frac{A_{sw}f_{ywd}}{b_w s} = 0.46 \times \frac{400}{250} = 0.74 \leq \frac{\upsilon f_{cd}}{2} = 0.55 \times \frac{20}{2} = 5.5 \text{ N/mm}^2 \ldots \text{OK}$$ 4.3.2.4.4(2) Eqn 4.27

Before choosing the reinforcement, the effects of torsion will be considered and the results combined.

The force in the longitudinal reinforcement, T_d, ignoring flexure, is given by

$$T_d = (\tfrac{1}{2})V_{Sd}(\cot\Theta - \cot\alpha)$$ 4.3.2.4.4(5) Eqn 4.30

For vertical links, $\cot\alpha = 0$

$$T_d = \frac{355}{2} \times 1.5 = 266.3 \text{ kN}$$

Additional area of longitudinal reinforcement

$$\frac{T_d}{f_{ywd}} = \frac{266.3 \times 10^3}{400} = 666 \text{ mm}^2$$

This area of reinforcement must be combined with the tension reinforcement required for flexure together with the longitudinal reinforcement required for torsion.

3.4.4 Torsional resistance

Torsional resistance is calculated on the basis of a thin-walled closed section. Solid sections are replaced by an idealized equivalent thin-walled section. Sections of complex shape are divided into sub-sections with each sub-section treated as an equivalent thin-walled section. The torsional resistance is taken as the sum of the torsional resistances of the sub-sections.

The torsional moment, carried by each sub-section according to elastic theory, may be found on the basis of the St Venant torsional stiffness. Division of the section into sub-sections should be so arranged as to maximize the calculated stiffness.

For this example the section will be divided into the sub-sections shown in Figure 3.14.

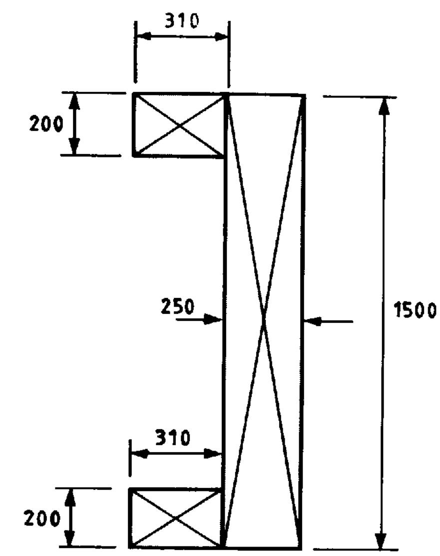


Figure 3.14 Dimensions of sub-sections

3.4.4.1 St Venant torsional stiffnesses

$$J = \beta h^3_{min} h_{max}$$

BS 8110: Part 2 2.4.3 Eqn 1

3.4.4.1.1 Top and bottom flanges

$h_{max} = 310$ mm, $h_{min} = 200$ mm

$$\frac{h_{max}}{h_{min}} = \frac{310}{200} = 1.55$$

From which $\beta = 0.203$

BS 8110: Part 2 2.4.3 Table 2.2

Therefore

$$J = 0.203 \times 200^3 \times 310 = 0.5 \times 10^9 \text{ mm}^4$$

3.4.4.1.2 Web

$h_{max} = 1500$ mm, $h_{min} = 250$ mm

$$\frac{h_{max}}{h_{min}} = \frac{1500}{250} = 6$$

From which $\beta = 0.33$

BS 8110: Part 2 2.4.3 Table 2.2

Therefore

$$J = 0.33 \times 250^3 \times 1500 = 7.7 \times 10^9 \text{ mm}^4$$

3.4.4.1.3 Total stiffness

$J_{tot} = [(2 \times 0.5) + 7.7] \times 10^9 = 8.7 \times 10^9 \text{ mm}^4$

3.4.4.2 Thicknesses of equivalent thin-walled sections

$t = \frac{A}{u} \ngtr$ the actual wall thickness 4.3.3.1(6)

where

u = outer circumference of the section

A = total area within the outer circumference

3.4.4.2.1 Top and bottom flanges

$u = (310 + 200)2 = 1020 \text{ mm}$

$A = 310 \times 200 = 62 \times 10^3 \text{ mm}^2$

Therefore

$$t = \frac{62 \times 10^3}{1020} = 61 \text{ mm}$$

t may not be less than twice the cover, c, to the longitudinal bars. Hence, with 10 mm links 4.3.3.1(6)

$t_{min} = 2(35 + 10) = 90 \text{ mm}$

3.4.4.2.2 Web

$u = (1500 + 250)2 = 3500 \text{ mm}$

$A = 1500 \times 250 = 375 \times 10^3 \text{ mm}^2$

Therefore

$$t = \frac{375 \times 10^3}{3500} = 107 \text{ mm} > 2c$$ .... OK

Values of t between the limits of A/u and $2c$ may be chosen provided that the design torsional moment, T_{Sd}, does not exceed the torsional moment that can be resisted by the concrete compression struts.

3.4.4.3 Torsional moments

$T_{Sd,tot} = 120 \text{ kNm}$

This total moment is shared between the flanges and web in proportion to their torsional stiffness.

Therefore

$$T_{Sd,fl} = 120 \times \frac{0.5}{8.7} = 6.9 \text{ kNm}$$

$$T_{Sd,w} = 120 \times \frac{7.7}{8.7} = 106 \text{ kNm}$$

T_{Sd} must satisfy the following two conditions 4.3.3.1(5)

$T_{Sd} \leq T_{Rd1}$ and $\leq T_{Rd2}$ Eqn 4.38
Eqn 4.39

3.4.4.4 Torsion in flanges

$$T_{Rd1} = \frac{2\nu f_{cd} t A_k}{\cot\Theta + \tan\Theta}$$ 4.3.3.1(6) Eqn 4.40

Re-arranging gives

$$\frac{T_{Rd1}}{2\nu f_{cd} t A_k} = \frac{1}{\cot\Theta + \tan\Theta}$$

Putting T_{Rd1} equal to T_{Sd}

$$\frac{T_{Sd}}{2\nu f_{cd} t A_k} = \frac{1}{\cot\Theta + \tan\Theta}$$

$T_{Sd,fl}$ = 6.9 kNm

$$\nu = 0.7\left(0.7 - \frac{f_{ck}}{200}\right)$$ 4.3.3.1(6) Eqn 4.41

$$= 0.7\left(0.7 - \frac{30}{200}\right) = 0.385 \not< 0.35$$

f_{cd} = 20 N/mm^2

t = 90 mm

A_k = area enclosed within the centre line of the thin-wall section

= (310 − 90) × (200 − 90) = 24.2 × 10^3 mm^2

Therefore

$$\frac{T_{Sd}}{2\nu f_{cd} t A_k} = \frac{6.9 \times 10^6}{2 \times 0.385 \times 20 \times 90 \times 24.2 \times 10^3} = 0.206$$

By reference to Figure 3.1 it may be seen that the value of cotΘ may be taken anywhere between the limits of 0.67 to 1.5. NAD Table 3 4.3.3.1(6)

To minimize link reinforcement take cotΘ = 1.5.
Note that this value must be consistent with the value taken for normal shear.

$$T_{Rd2} = 2A_k\left(f_{ywd}\frac{A_{sw}}{s}\cot\Theta\right)$$ 4.3.3.1(7) Eqn 4.43

Re-arranging gives

$$\frac{A_{sw}}{s} = \frac{T_{Rd2}}{2A_k f_{ywd}\cot\Theta}$$

Putting T_{Rd2} equal to T_{Sd}

$$\frac{A_{sw}}{s} = \frac{T_{Sd}}{2A_k f_{ywd} \cot\Theta}$$

Using mild steel reinforcement

$$f_{ywd} = \frac{f_{ywk}}{\gamma_s} = \frac{250}{1.15} = 217 \text{ N/mm}^2$$

Therefore

$$\frac{A_{sw}}{s} = \frac{6.9 \times 10^6}{2 \times 24.2 \times 10^3 \times 217 \times 1.5} = 0.44 \text{ mm}^2\text{/mm}$$

The spacing of torsion links should not exceed $\frac{u_k}{8}$ 5.4.2.3(3)

where

u_k = the circumference of the area A_k 4.3.3.1(7)

$= 2[(310 - 90) + (200 - 90)] = 660$ mm

Therefore

$$s_{max} = \frac{660}{8} = 82.5 \text{ mm, say 80 mm}$$

$$A_{sw} = 0.44 \times 80 = 35.2 \text{ mm}^2$$

Use R8 links at 80 mm crs.

The additional area of longitudinal steel for torsion is given by

$$A_{sl} f_{yld} = \left(T_{Rd2} \frac{u_k}{2A_k}\right) \cot\Theta$$ Eqn 4.44

Re-arranging and putting T_{Rd2} equal to T_{Sd}

$$A_{sl} = \frac{\left(T_{Sd} \frac{u_k}{2A_k}\right) \cot\Theta}{f_{yld}}$$

Using high yield reinforcement

$$f_{yld} = \frac{460}{1.15} = 400 \text{ N/mm}^2$$

Therefore

$$A_{sl} = \frac{6.9 \times 10^6 \times 660 \times 1.5}{400 \times 2 \times 24.2 \times 10^3} = 353 \text{ mm}^2$$

Use 4T12 bars

Reinforcement will also be required in the bottom flange to cater for flexure of the flange acting as a continuous nib.

3.4.4.5 Torsion in web

$T_{Sd,w} = 106$ kNm

$A_k = (1500 - 107) \times (250 - 107) = 199.2 \times 10^3$ mm^2

Therefore

$$\frac{T_{Sd}}{2\upsilon f_{cd} t A_k} = \frac{106 \times 10^6}{2 \times 0.385 \times 20 \times 107 \times 199.2 \times 10^3} = 0.32$$

Again by reference to Figure 3.1, cotΘ should fall within the limits of 0.67 to 1.5.

Similarly use cotΘ = 1.5

As the web is subject to shear and torsion, the combined effects should now be checked to satisfy the condition

$$\left(\frac{T_{Sd}}{T_{Rd1}}\right)^2 + \left(\frac{V_{Sd}}{V_{Rd2}}\right)^2 \leq 1$$ 4.3.3.2.2(3) Eqn 4.47

$T_{Sd} = 106$ kNm

$$T_{Rd1} = \frac{2\upsilon f_{cd} t A_k}{\cot\Theta + \tan\Theta}$$ 4.3.3.1(6) Eqn 4.40

$$= \frac{2 \times 0.385 \times 20 \times 107 \times 199.2 \times 10^3}{1.5 + (\frac{1}{1.5})} = 151.5 \text{ kNm}$$

$V_{Sd} = 355$ kN

$$V_{Rd2} = \frac{b_w z \upsilon f_{cd}}{\cot\Theta + \tan\Theta}$$ 4.3.2.4.4(2) Eqn 4.26

$$= \frac{250 \times 1300 \times 0.55 \times 20}{1.5 + (\frac{1}{1.5})} = 1650 \text{ kN}$$

Therefore

$$\left(\frac{T_{Sd}}{T_{Rd1}}\right)^2 + \left(\frac{V_{Sd}}{V_{Rd2}}\right)^2$$

$$= \left(\frac{106}{151.5}\right)^2 + \left(\frac{355}{1650}\right)^2 = 0.54 < 1.0$$ . . . . OK

Where the entire section is used to resist normal shear, each sub-section should be checked to satisfy the above interaction condition.

3.4.5 Reinforcement in web

Link reinforcement for torsion

Using high yield links

$$\frac{A_{sw}}{s} = \frac{106 \times 10^6}{2 \times 199.2 \times 10^3 \times 400 \times 1.5} = 0.44 \text{ mm}^2/\text{mm}$$

Note that A_{sw} for torsion relates to a single leg in the wall of the section.

Link reinforcement required for shear

$$\frac{A_{sw}}{s} = 0.46 \text{ mm}^2/\text{mm from Section } \mathbf{3.4.3}$$

Note that A_{sw} for shear relates to the total shear link legs.

Assuming single links, total area for one leg

$$A_{sw} = \frac{0.46s}{2} + 0.44s = 0.67s \text{ mm}^2$$

Using T12 links

$$0.67s = 113 \text{ mm}^2$$

$$s = 168 \text{ mm, say } 160 \text{ mm}$$

Maximum link spacing for shear

$$(\tfrac{1}{5})V_{Rd2} < V_{Sd} \leq (\tfrac{2}{3})V_{Rd2}$$ 5.4.2.2(7) Eqn 5.18

$$s_{max} = 0.6d = 864 \not> 300 \text{ mm}$$

Therefore

$$s_{max} = 300 \text{ mm}$$

For cracking

$$\frac{V_{Sd} - 3V_{cd}}{\rho_w b_w d} \leq 50 \text{ N/mm}^2$$ 4.4.2.3(5) Table 4.13

Therefore s_{max} = 300 mm

For torsion

$$s_{max} = \frac{u_k}{8}$$

$$u_k = 2[(1500 - 107) + (250 - 107)] = 3072 \text{ mm}$$

Therefore

$$s_{max} = \frac{3072}{8} = 384 \text{ mm}$$

Maximum spacing to suit all conditions is 300 mm.

Use T12 links @ 160 mm crs.

Additional area of longitudinal steel for torsion in web Eqn 4.44

$$A_{sl} = \frac{106 \times 10^6 \times 3072 \times 1.5}{400 \times 2 \times 199.2 \times 10^3} = 3065 \text{ mm}^2$$

Use 16T16 bars

The bars in the tension face of the web will need to be increased to provide for the additional longitudinal steel required for shear and combined with the reinforcement required for flexure.

Area required in tension face for combined torsion and shear

$$= \frac{(3065 \times 2)}{16} + 666 = 1049 \text{ mm}^2$$

Use 3T25 bars

3.4.6 Summary of reinforcement

Top flange
4T12 longitudinal bars
R8 links @ 80 mm crs.

Bottom flange
4T12 longitudinal bars
R8 links @ 80 mm crs.
Plus reinforcement for flexure of the nib

Web
3T25 longitudinal bars in tension face
7T16 bars in each side face
T12 links @ 160 mm crs.
Plus reinforcement for flexure

The reinforcement details are shown in Figure 3.15

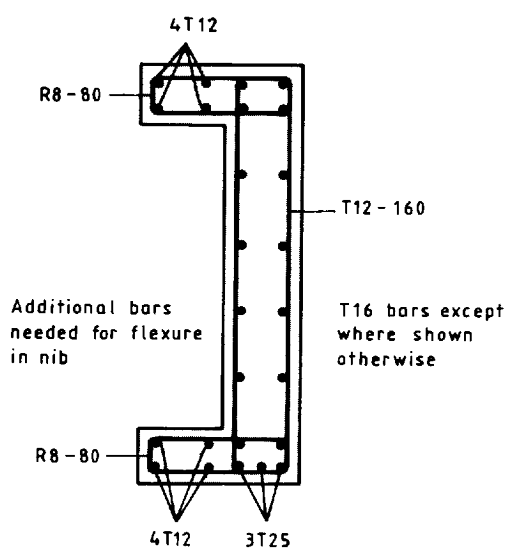


Figure 3.15 Beam reinforcement details

It will be seen from this example that choosing the upper limit value of cotΘ, to minimize the link reinforcement, results in substantial additional longitudinal reinforcement being required. In practice the value of cotΘ should be chosen so as to optimize the total reinforcement in the section.

3.5 Slenderness limits 4.3.5.7

The Code requires that a beam has an adequate factor of safety against buckling.

Providing that the following requirements are met, the safety against lateral buckling may be assumed to be adequate 4.3.5.7(2)

l_{ot} < $50b$; and Eqn 4.77

h < $4b$ NAD

where

b = width of the compression flange, which can be taken as b_{eff} for T and L beams 2.5.2.2.1(3)

h = total depth of the beam

l_{ot} = unrestrained length of the compression flange taking lateral bracing into account 2.5.2.2.1(4)

For example, consider the beam shown in Figure 3.16.

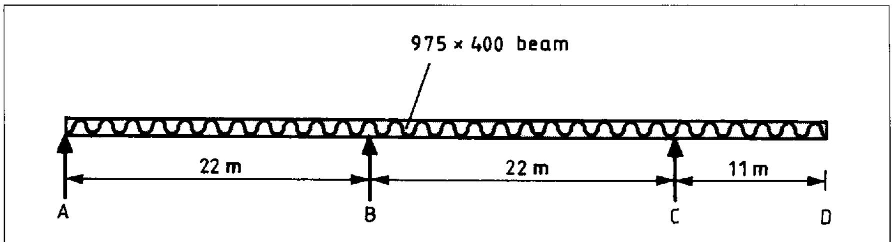


Figure 3.16 Beam spans and loading for slenderness check

In this example the top of the beam is loaded but unrestrained (for instance, the beam is carrying a wall).

The second requirement is satisfied i.e. $h < 4b = 1600$ mm

In calculating l_{ot}, the unrestrained length of the compression flange can be taken as the distance between points of contraflexure.

These distances, which need to be $< 50b = 20$ m, can be obtained from EC2 Figure 2.3. Figure 2.3

$l_{ot}(A-B) = 0.85l(A-B) = 0.85 \times 22 = 18.7$ m

$l_{ot}(B-C) = 0.7l(B-C) = 0.7 \times 22 = 15.4$ m

$l_{ot}(C-D) = 2l(C-D) = 2 \times 11 = 22$ m

Spans A–C are satisfactory but span C–D is not. It is too slender and the width will need to be increased, or additional lateral retraint will need to be provided.

4.1 Solid and ribbed slabs

4.1.1 One-way spanning solid slabs

Example of a one-way spanning slab is given in Section 2.

4.1.2 Two-way spanning solid slabs

EC2[1] permits the use of elastic analysis, with or without redistribution, or plastic analysis for ultimate limit state design. 2.5.1.1(5)

Elastic analyses are commonly employed for one-way spanning slabs and for two-way spanning slabs without adequate provision to resist torsion at the corners of the slab and prevent the corners from lifting. Plastic analyses are commonly used in other situations. 2.5.3.5.1(2)

Tabulated results for moments and shears from both types of analysis are widely available. BS 8110 Tables 3.14 & 3.15

Care is necessary in subsequent design to ensure that adequate ductility is present. Where redistribution has been performed, the necessary checks should be carried out. 2.5.3.2.2(5) 2.5.3.4.2(3) 2.5.3.5.5(2)

4.1.2.1 Design example of a simply-supported two-way spanning solid slab

Design a solid slab, spanning in two directions and simply-supported along each edge on brickwork walls as shown in Figure 4.1. The slab is rectangular on plan and measures 5 m by 6 m between the centre of the supports.

In addition to self-weight, the slab carries a characteristic dead load of 0.5 kN/m^2 and an imposed load of 5.0 kN/m^2.

The slab is in an internal environment with no exposure to the weather or aggressive conditions.

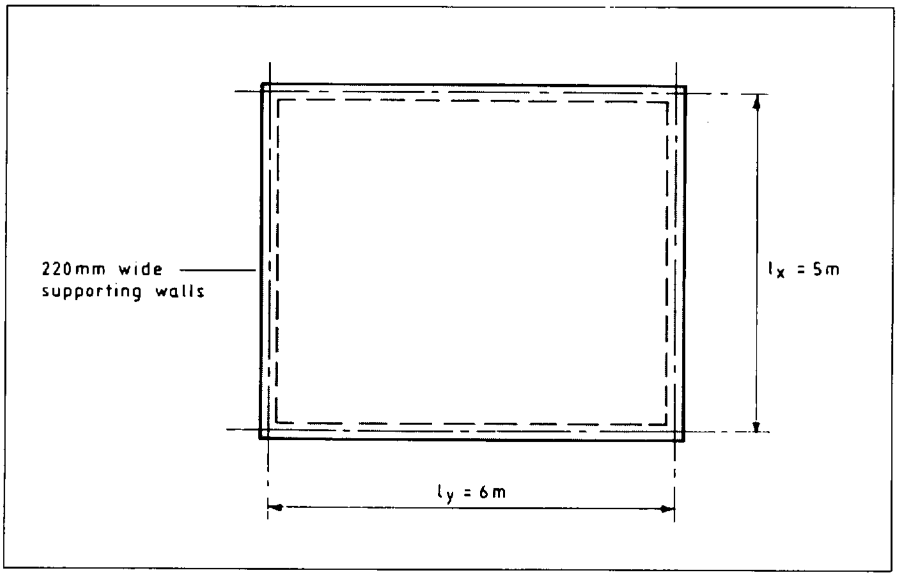


Figure 4.1 Layout of slab

4.1.2.1.1 Durability

For a dry environment, exposure class is 1.
Minimum concrete strength grade is C25/30.

Table 4.1 ENV 206 Table NA.1

For cement content and w/c ratio, refer to ENV 206 Table 3[6].

Minimum cover to reinforcement	= 15 mm	NAD Table 6
Assume nominal aggregate size	= 20 mm	
Assume maximum bar size	= 12 mm	
Nominal cover	≥ 20 mm	NAD 6.4(a)

Use nominal cover = 25 mm

Note:
20 mm nominal cover is sufficient to meet the NAD[1] requirements in all respects. — NAD Table 3, 4.1.3.3(8)

Check requirements for fire resistance to BS 8110: Part 2[2]. — NAD 6.1(a)

4.1.2.1.2 Materials

Type 2 deformed reinforcement

$f_{yk} = 460 \text{ N/mm}^2$ — NAD 6.3(a)

$$f_{yd} = \frac{f_{yk}}{\gamma_s} = \frac{460}{1.15} = 400 \text{ N/mm}^2$$ — 2.2.3.2, Table 2.3

C25/30 concrete with 20 mm maximum aggregate size

4.1.2.1.3 Loading

Assume 200 mm thick slab

$G_k = 4.8 + 0.5 = 5.3 \text{ kN/m}^2$

$Q_k = 5.0 \text{ kN/m}^2$

$\gamma_G = 1.35$ — Table 2.2

$\gamma_Q = 1.5$

Ultimate load = $\gamma_G G_k + \gamma_Q Q_k$ = 14.66 kN/m² — Eqn 2.8(a), NAD 6.2(d)

4.1.2.1.4 Flexural design

Bending moment coefficients for simply-supported two-way spanning slabs, without torsional restraint at the corners or provision to resist uplift at the corners, based on the Grashof-Rankine Formulae, are widely published and are reproduced in BS 8110. — BS 8110 Table 3.14

$$M_{Sdx} = \alpha_{sx} n l_x^2$$
$$M_{Sdy} = \alpha_{sy} n l_x^2$$

For $\dfrac{l_y}{l_x} = 1.2$

$\alpha_{sx} = 0.084, \quad \alpha_{sy} = 0.059$

Giving

M_{Sdx} = 30.8 kNm/m

M_{Sdy} = 21.6 kNm/m

For short span with reinforcement in bottom layer

$$d = 200 - 25 - \frac{12}{2} = 169 \text{ mm}$$

$$\frac{M_{Sdx}}{bd^2f_{ck}} = 0.043$$

$$\frac{x}{d} = 0.099 < 0.45 \text{ .... OK}$$ 2.5.3.4.2(5)

$$\frac{A_sf_{yk}}{bdf_{ck}} = 0.052$$

Therefore A_s = 478 mm^2/m

Use T12 @ 200 mm crs. (566 mm^2/m) in short span

For longer span

$$d = 200 - 25 - 12 - 6 = 157 \text{ mm}$$

$$\frac{M_{Sdy}}{bd^2f_{ck}} = 0.035$$

$$\frac{A_sf_{yk}}{bdf_{ck}} = 0.042$$

Therefore A_s = 359 mm^2/m

Use T12 @ 300 mm crs. (377 mm^2/m) in long span

4.1.2.1.5 Shear

4.3.2

$$V_{Sdx} = 8\alpha_{sx}\left(\frac{nl_x}{2}\right) = 24.6 \text{ kN/m}$$

$$V_{Sdy} = 8\alpha_{sy}\left(\frac{nl_x^2}{2l_y}\right) = 14.4 \text{ kN/m}$$

The shear resistance with no axial load:

$$V_{Rd1} = \tau_{Rd}k(1.2 + 40\rho_l)b_wd$$

4.3.2.3
Eqn 4.18

Where

τ_{Rd} = 0.3 N/mm^2 Table 4.8

Assume $\not>$ 50% of reinforcement curtailed at support

$k \quad = \quad 1.6 - d \quad = \quad 1.431 \not< 1$

Assume

$$\rho_l = \frac{A_{sl}}{b_w d} = 0 \not> 0.02$$ Figure 4.12

Hence

$V_{Rd1} \quad = \quad 87.0 \text{ kN/m} \quad > V_{Sdx} \quad = \quad 24.6 \text{ kN/m}$

No shear reinforcement required

4.3.2.1P(2)
4.3.2.2

4.1.2.1.6 Serviceability – deflection

Control by limiting span/effective depth ratio based on the shorter span for a two-way spanning slab. 4.4.3.2

$A_{s,prov} \quad = \quad 566 \text{ mm}^2\text{/m}, \qquad \rho \quad = \quad 0.0033$

NAD Table 7 gives basic span/effective depth ratios which are assumed to be based on $f_{yk} \quad = \quad 400 \text{ N/mm}^2$. 4.4.3.2(4)

Note 2 to NAD Table 7 states that modification to the tabulated values for nominal reinforced concrete should not be carried out to take into account service stresses in the steel (refer to EC2 Clause 4.4.3.2(4)). However, it is assumed that the correction ought to be made for concrete with $0.15\% \leq \rho < 0.5\%$ but that the resulting values should not exceed those tabulated in the NAD for nominally reinforced concrete.

Basic limiting span/effective depth ratios are:

Concrete lightly stressed ($\rho = 0.5\%$):	25	NAD
Concrete nominally reinforced ($\rho = 0.15\%$):	34	6.4(e)&(f)
By interpolation at $\rho = 0.33\%$:	29.4	Table 7

The actual service steel stress modification factor is

$$\frac{250}{\sigma_s} = \frac{400}{f_{yk}(A_{s,req}/A_{s,prov})} = \frac{400}{460 \times 478/566} = 1.03$$

Therefore, permissible span/effective depth ratio

$$= 1.03 \times 29.4 = 30.3 \leq 34$$

Since span $\not>$ 7 m, no further adjustment is required. 4.4.3.2(3)

$$\text{Actual span/effective depth ratio} = \frac{5000}{169} = 29.6 < 30.3 \text{ .... OK}$$

Note:
No modification to the longer span reinforcement is required in cases where short span reinforcement is increased to comply with deflection requirements. BS 8110 3.5.7

4.1.2.1.7 Serviceability – cracking

For a slab with $h \leq 200$ mm, no further measures to control cracking are necessary if the requirements of EC2 Clause 5.4.3 have been applied. 4.4.2.3(1)

4.1.2.1.8 Detailing

Detailing requirements for cast in situ solid slabs, including two-way slabs 5.4.3

Slab thickness, $h = 200 > 50$ mm .... OK 5.4.3.1(1)

For the short span, use alternately staggered bars and anchor 50% of the mid-span reinforcement at the supports. 5.4.3.2.2(1)

Anchorage force, $F_s = V_{Sd}\left(\frac{a_l}{d}\right) + N_{Sd}$ 5.4.3.2.1(1) 5.4.2.1.4(2) Eqn 5.15

$N_{Sd} = 0$

$a_l = d$ 5.4.3.2.1(1)

Therefore

$F_s = V_{Sd} = 24.6$ kN/m

$$A_{s,req} = \frac{F_s}{f_{yd}} = \frac{24.6 \times 10^3}{400} = 61.5 \text{ mm}^2/\text{m}$$

$A_{s,prov} = 283$ mm²/m .... OK

Net bond length, $l_{b,net} = \dfrac{\alpha_a l_b A_{s,req}}{A_{s,prov}} \nless l_{b,min}$ 5.2.3.4.1(1) Eqn 5.4

$\alpha_a = 1.0$ for straight bars 5.2.3.4.1

$$l_b = \frac{\phi}{4} \times \frac{f_{yd}}{f_{bd}}$$ 5.2.2.3 Eqn 5.3

All bars in slabs with $h \leq 250$ mm may be assumed to have good bond. 5.2.2.1 Table 5.3

$f_{bd} = 2.7$ N/mm²

$$l_b = \frac{12}{4} \times \frac{400}{2.7} = 444 \text{ mm}$$

$l_{b,min} = 0.3 l_b \nless 10\phi$ or 100 mm $= 133$ mm 5.2.3.4.1(1) Eqn 5.5

In calculating $l_{b,net}$ take $A_{s,req}$ as mid-span reinforcement/4 giving NAD 6.5(c) 5.4.2.1.4(3)

$l_{b,r.et} = 1.0 \times 444 \times \frac{1}{2} = 222$ mm $> l_{b,min}$ .... OK Eqn 5.4

For a direct support, the anchorage length required is — 5.4.2.1.4(3) Figure 5.12(a)

$(2/3)l_{b,net}$ = 148 mm

The reinforcement details are shown in Figure 4.2.

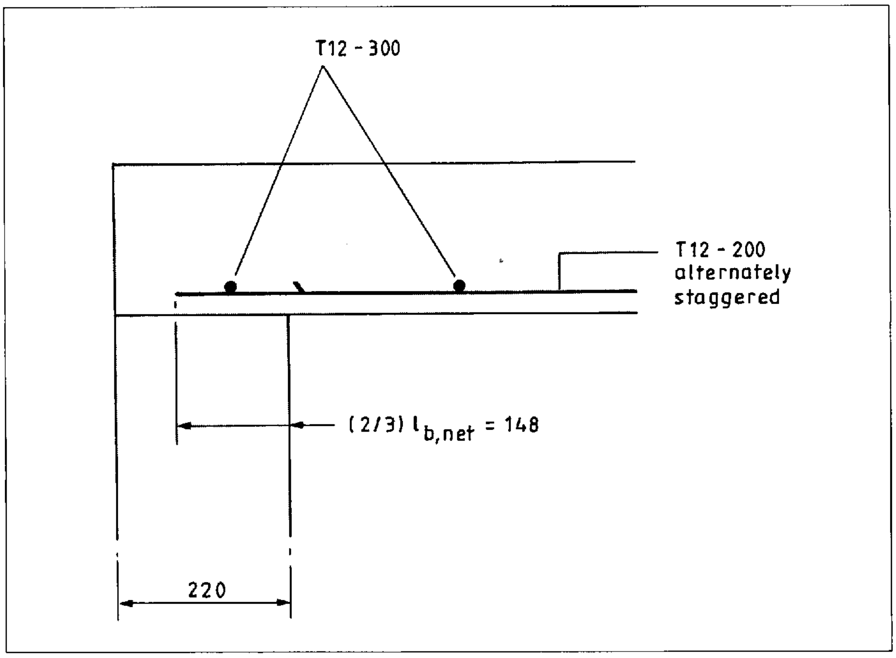


Figure 4.2 Section through short span support

The use of $(2/3)l_{b,net}$ at a direct support is an allowance for the transverse compression due to the support reaction.

Minimum area of reinforcement

$$A_s \nless \frac{0.6b_td}{f_{yk}} \nless 0.0015b_td = 254 \text{ mm}^2/\text{m}$$ 5.4.3.2.1(3) 5.4.2.1.1(1)

Minimum area provided (T12 @ 400 mm crs.) near support

= 283 mm²/m .... OK

Maximum bar spacing = $3h \ngtr 500$ mm — NAD Table 3 5.4.3.2.1(4)

Maximum spacing used = 400 mm near support .... OK

4.1.2.2 Design example of a continuous two-way spanning solid slab

Design a solid slab spanning between beams, as shown in Figure 4.3.

In addition to self-weight, the slab carries a characteristic dead load of 1.0 kN/m^2 and an imposed load of 5.0 kN/m^2.

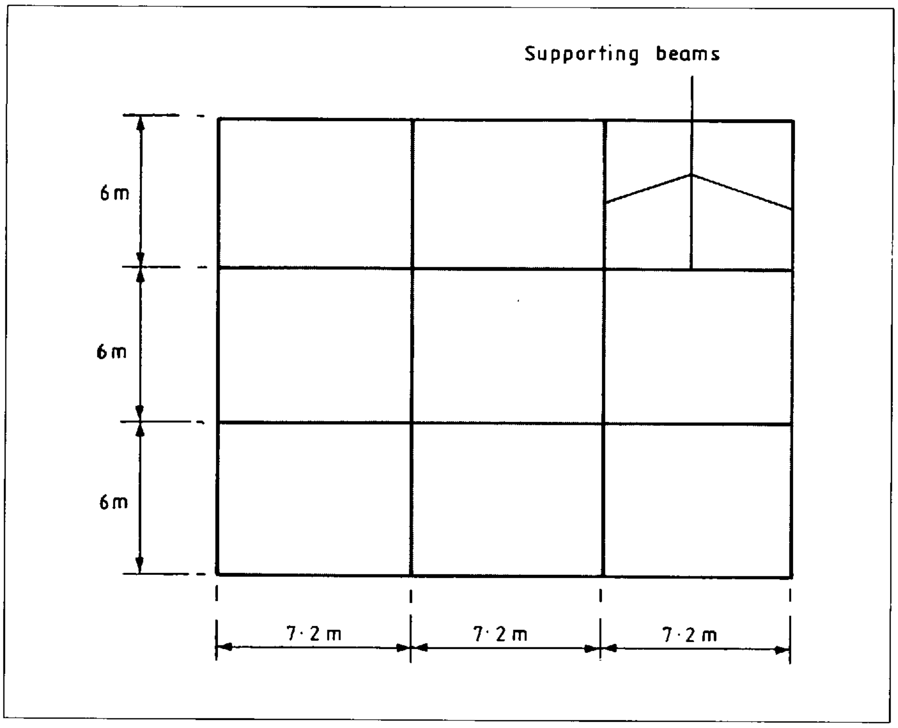


Figure 4.3 Layout of slab

4.1.2.2.1 Durability

For a dry environment, exposure class is 1. — Table 4.1
Minimum concrete strength grade is C25/30. — ENV 206

For cement content and w/c ratio, refer to ENV 206 Table 3. — Table NA.1

Minimum cover to reinforcement	=	15 mm	NAD
Assume nominal aggregate size	=	20 mm	Table 6
Assume maximum bar size	=	12 mm	
Nominal cover	≥	20 mm	NAD 6.4(a)

Use nominal cover = 25 mm

Note:
20 mm nominal cover is sufficient to meet the NAD requirements in all respects. — NAD Table 3 4.1.3.3(8)

Check requirements for fire resistance to BS 8110: Part 2. — NAD 6.1(a)

4.1.2.2.2 Materials

Type 2 deformed reinforcement, f_{yk} = 460 N/mm^2

C25/30 concrete with 20 mm maximum aggregate size.

4.1.2.2.3 Loading

Assume 200 mm thick slab

$G_k = 4.8 + 1.0 = 5.8$ kN/m^2

$Q_k = 5.0$ kN/m^2

$\gamma_G = 1.35$ or 1.0 — Table 2.2

$\gamma_Q = 1.5$ or 0.0

For non-sensitive structures, a single design value for permanent actions may be applied throughout the structure, i.e. $\gamma_G = 1.35$ throughout. — 2.3.2.3

Maximum ultimate load	$= 1.35 \times 5.8 + 1.5 \times 5.0 = 15.33$ kN/m^2
Minimum ultimate load	$= 1.35 \times 5.8 = 7.83$ kN/m^2

4.1.2.2.4 Load cases

For continuous beams and slabs in buildings without cantilevers subjected to dominantly uniformly distributed loads, it will generally be sufficient to consider only the following load cases. — 2.5.1.2(4)

(a) Alternate spans carrying the design variable and permanent load ($\gamma_Q Q_k + \gamma_G G_k$), other spans carrying only the design permanent load, $\gamma_G G_k$.

(b) Any two adjacent spans carrying the design variable and permanent load ($\gamma_Q Q_k + \gamma_G G_k$). All other spans carrying only the design permanent load, $\gamma_G G_k$.

4.1.2.2.5 Flexural design

Bending moment coefficients for two-way spanning slabs supported on four edges, with provision for torsion at the corners, have been calculated based on both elastic and yield line theory. The coefficients published in BS 8110: Part 1, Table 3.15, are based on yield line analysis and are used in this example. — BS 8110 Table 3.15

For continuous slabs the effects of rotational restraint from the supports can be ignored. — 2.5.3.3(3)

Yield line methods can only be used for very ductile structural elements. Use high ductility steel Class H to prEN 10080[8]. — 2.5.3.2.2(5) NAD Table 5

No direct check on rotational capacity is required if high ductility steel is used. — 2.5.3.5.5(3)

The area of steel should not exceed a value corresponding to — 2.5.3.5.5(2)

$$\frac{x}{d} = 0.25 \text{ which is equivalent to } \frac{M}{bd^2 f_{ck}} = 0.102$$

For the yield line (kinematic) method, a variety of possible mechanisms should be considered. This is assumed in the use of the published bending moment coefficients. — 2.5.3.5.5(4)

The ratio of moments at a continuous edge to the span moment should be between 0.5 and 2.0. This is true for the published coefficients. 2.5.3.5.5(5)

Consider the design of the corner panel, D, in Figure 4.4. 2.5.1.2

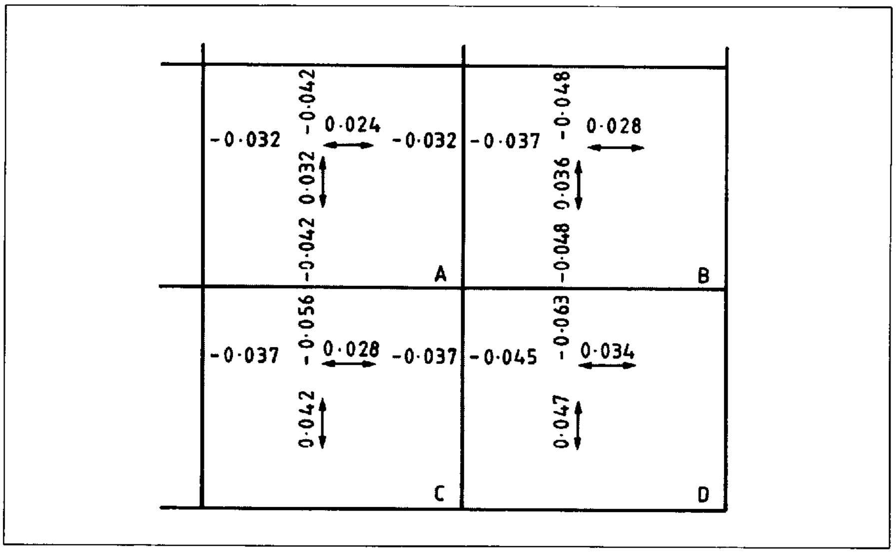


Figure 4.4 Bending moment coefficients l_y/l_x = 1.2

Using the coefficients shown in Figure 4.4 and the method described in BS 8110 to adjust moments for adjacent panels with unequal conditions, the following moments and shears can be calculated for this panel: BS 8110 3.5.3.6

In the 6 m direction, M_{sup} = 29.7 kNm/m

M_{span} = 28.5 kNm/m

In the 7.2 m direction, M_{sup} = 21.0 kNm/m

M_{span} = 20.6 kNm/m

The support moments calculated can be further reduced by an amount ΔM_{Sd} 2.5.3.3(4)

$$\Delta M_{Sd} = F_{Sd,sup} \times b_{sup}/8$$ Eqn 2.16

where

$F_{Sd,sup}$ = design support reaction compatible with the analysis moments.

In the 6 m direction, $F_{Sd,sup}$ = 81.9 kN/m

In the 7.2 m direction, $F_{Sd,sup}$ = 69.9 kN/m

For a 300 mm wide supporting beam:

In the 6 m direction, ΔM_{Sd} = 3.1 kNm/m

In the 7.2 m direction, ΔM_{Sd} = 2.6 kNm/m

Therefore, the design support moments are:

In the 6 m direction, M_{sup} = 26.6 kNm/m

In the 7.2 m direction, M_{sup} = 18.4 kNm/m

For the short span, with the reinforcement in the first layer

$$d = 200 - 25 - \frac{12}{2} = 169 \text{ mm}$$

$$\frac{M_{sup}}{bd^2 f_{ck}} = 0.038$$

$$\frac{x}{d} = 0.087 < 0.25 \text{ ..... OK}$$ 2.5.3.5.5(2)

$$\frac{A_s f_{yk}}{bdf_{ck}} = 0.045$$

$$A_s = 414 \text{ mm}^2\text{/m}$$

Use T12 @ 250 mm crs. (452 mm^2/m) T in short span The span moment is similar to that over the support and the same reinforcement may be used in the bottom

For the long span, with the reinforcement in the second layer

$$d = 200 - 25 - 12 - \frac{12}{2} = 157 \text{ mm}$$

$$\frac{M_{sup}}{bd^2 f_{ck}} = 0.030$$

$$\frac{x}{d} = 0.068 < 0.45 \text{ ..... OK}$$

$$\frac{A_s f_{yk}}{bdf_{ck}} = 0.035$$

$$A_s = 297 \text{ mm}^2\text{/m}$$

Use T12 @ 300 mm crs. (377 mm^2/m) T in long span The span moment is again similar to that over the support and the same reinforcement may be used in the bottom

For arrangements of reinforcement in middle and edge strips use BS 8110. The NAD directs the use of BS 8110 where torsion reinforcement is required in the corners of panels.

BS 8110 3.5.3.5
NAD 6.5(e) 5.4.3.2.2

4.1.2.2.6 Shear

4.3.2

Use forces consistent with the analysis moments.

In the 6 m direction:
At internal beam, V_{int} = 0.47 × 15.33 × 6 = 43.2 kN/m
At edge, V_{ext} = 0.31 × 15.33 × 6 = 28.5 kN/m

In the 7.2 m direction:
At internal beam, V_{int} = 0.4 × 15.33 × 6 = 36.8 kN/m
At edge, V_{ext} = 0.26 × 15.33 × 6 = 23.9 kN/m

$V_{Rd1} = [\tau_{Rd} k(1.2 + 40\rho_l) + 0.15\sigma_{cp}]\, b_w d$ — 4.3.2.3 Eqn 4.18

$\tau_{Rd} = 0.3\ \text{N/mm}^2$ — Table 4.8

Assume $\not>$ 50% of the bottom reinforcement curtailed at edge support. — 5.4.3.2.2

$k = 1.6 - 0.169 = 1.431$

$$\rho_l = \frac{A_{sl}}{b_w d} = 0.00134 \not> 0.02$$

Note: Ensure detailing provides necessary anchorage to A_{sl}. See EC2 Figure 4.12 for definition of A_{sl}.

$\sigma_{cp} = 0$

Therefore

$V_{Rd1} = 91.0\ \text{kN/m} > V_{Sd} = 28.5\ \text{kN/m}$ at edge support

It is also clear that $V_{Rd1} > V_{Sd} = 43.2$ kN/m at the internal beam.

No shear reinforcement required

4.3.2.1P(2)
4.3.2.2(2)

4.1.2.2.7 Serviceability – deflection

Control by limiting span/effective depth ratio based on the shorter span for a two-way spanning slab. — 4.4.3.2, 4.4.3.2(5)

$$\text{Actual span/effective depth ratio} = \frac{6000}{169} = 35.5$$

For a corner panel use structural system 2.

It may be normally assumed that slabs are lightly stressed ($\rho \leq 0.5\%$). — Table 4.14, 4.4.3.2(5)

NAD 6.4(e) and (f) allows the basic span/effective depth ratio to be interpolated, according to the reinforcement provided, for values in the range $0.15\% < \rho < 0.5\%$.

Basic span/effective depth ratio ($\rho = 0.5\%$) = 32 — NAD
($\rho = 0.15\%$) = 44 — Table 7

For the span moment $A_{s,req} = 441\ \text{mm}^2/\text{m}$

$A_{s,prov} = 452\ \text{mm}^2/\text{m}$, $\rho = 0.27\%$

Basic span/effective depth ratio ($\rho = 0.27\%$) = 39.9

Using reinforcement with $f_{yk} > 400$ N/mm², this value should be multiplied to reflect the actual service steel stress by the factor 4.4.3.2(4)

$$\frac{250}{\sigma_s} = \frac{400}{f_{yk} \times A_{s,req}/A_{s,prov}} = \frac{400 \times 452}{460 \times 441} = 0.89$$

Therefore, permissible span/effective depth ratio

$= 0.89 \times 39.9 = 35.5$ .... OK

Note 2 to NAD Table 7 is taken to mean that the resulting span/effective depth ratio, after the service stress modification, is limited to the value tabulated for nominally reinforced concrete. In this case the value is 44.

4.1.2.2.8 Serviceability – cracking

For a slab with $h \leq 200$ mm no further measures are required to control cracking, provided the requirements of EC2 Clause 5.4.3 have been applied. 4.4.2.3(1)

4.1.2.2.9 Detailing 5.4.3

Slab thickness, $h = 200$ mm > 50 mm .... OK 5.4.3.1(1)

For the short span, use alternately staggered bars and anchor 50% of the mid-span reinforcement at the external support. 5.4.3.2.2(1)

Anchorage force (at external support)

$$F_s = V_{Sd} \times \frac{a_l}{d} + N_{Sd}$$ 5.4.2.1.4(2) Eqn 5.15

$N_{Sd} = 0$

$a_l = d$ 5.4.3.2.1(1)

$F_s = V_{Sd} = 28.5$ kN/m

$$A_{s,req} = \frac{F_s}{f_{yd}} = \frac{28.5 \times 10^3}{400} = 71 \text{ mm}^2/\text{m}$$

$A_{s,prov} = 226$ mm²/m .... OK

Net bond length

$$l_{b,net} = \alpha_a l_b \times \frac{A_{s,req}}{A_{s,prov}} \geq l_{b,min}$$ 5.2.3.4.1(1) Eqn 5.4

$\alpha_a = 0.7$ for curved bars

$$l_b = \frac{\phi \times f_{yd}}{4 f_{bd}}$$ 5.2.2.3 Eqn 5.3

For all bars in slabs with $h \leq 250$ mm, good bond may be assumed. 5.2.2.1

$f_{bd} = 2.7$ N/mm^2 Table 5.3

$$l_b = \frac{12}{4} \times \frac{400}{2.7} = 444 \text{ mm}$$

In calculating $l_{b,net}$ take $A_{s,req}$ as mid-span reinforcement/4. NAD 6.5(c) 5.4.2.1.4(3)

$$l_{b,net} = 0.7 \times 444 \times \frac{1}{2} = 156 \text{ mm} > l_{b,min} \text{ .... OK}$$

Bars to extend into support for a distance

$$\frac{b}{3} + l_{b,net} = 256 \text{ mm}$$ Figure 5.12(b)

giving sufficient end cover in 300 mm wide section .... OK

4.1.2.2.10 Top reinforcement at edge beam

Design moment = $M_{span}/4$ = 7.125 kNm/m 5.4.3.2.2(2)

$$\frac{M}{bd^2 f_{ck}} = 0.01$$

$$\frac{A_s f_{yk}}{bd f_{ck}} = 0.012$$

A_s = 110 mm^2/m $\not<$ $A_{s,min}$

Minimum area of reinforcement 5.4.3.2.1(3)

$$A_s \not< \frac{0.6 b_t d}{f_{yk}} \not< 0.0015 b_t d = 254 \text{ mm}^2/\text{m}$$

Use T10 @ 250 mm crs. bars extending 0.2*l* from inner face of support into span

5.4.3.2.2(2)

The reinforcement details are shown in Figure 4.5.

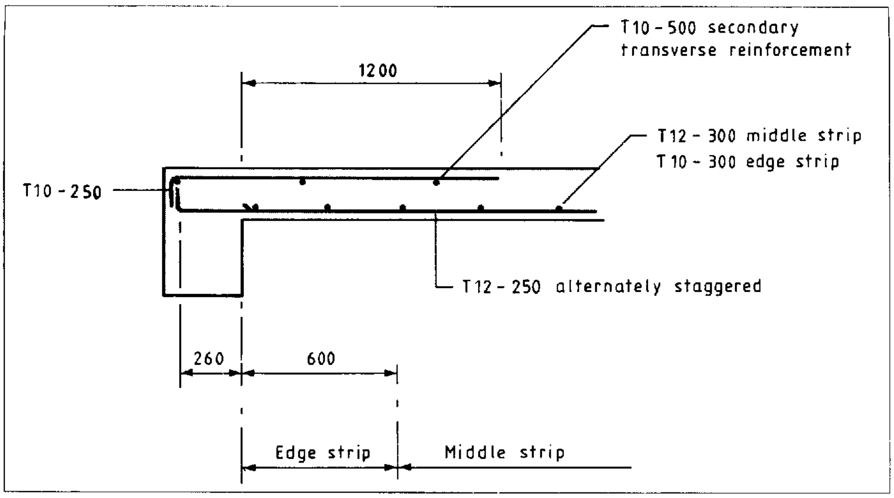


Figure 4.5 Detail at edge beam

4.1.2.2.11 Secondary transverse reinforcement – top

Principal reinforcement, T10 @ 250 mm crs., A_s = 314 mm²/m

Secondary reinforcement, A_s = 0.2 × 314 = 63 mm²/m — 5.4.3.2.1(2)

Maximum spacing = 500 mm — NAD Table 3

Use T10 @ 500 mm crs. (157 mm²/m)

5.4.3.2.1(4)

4.1.2.2.12 Corner reinforcement — 5.4.3.2.3

Use the detailing guidance given in BS 8110. — NAD 6.5(e) 5.4.3.2.3

4.1.2.2.13 Anchorage of bottom reinforcement at intermediate supports — 5.4.2.1.5

Retain not less than a quarter of mid-span reinforcement at support and provide not less than 10ϕ anchorage. — 5.4.2.1.4(1)

Provide continuity bars lapped with bottom reinforcement as shown in Figure 4.6. — Figure 5.13(b)

Using alternately staggered bars with continuity for 50% of the mid-span reinforcement.

Minimum lap, $l_{b,net}$ = $1.4 \times 444 \times \frac{1}{2}$ = 310 mm

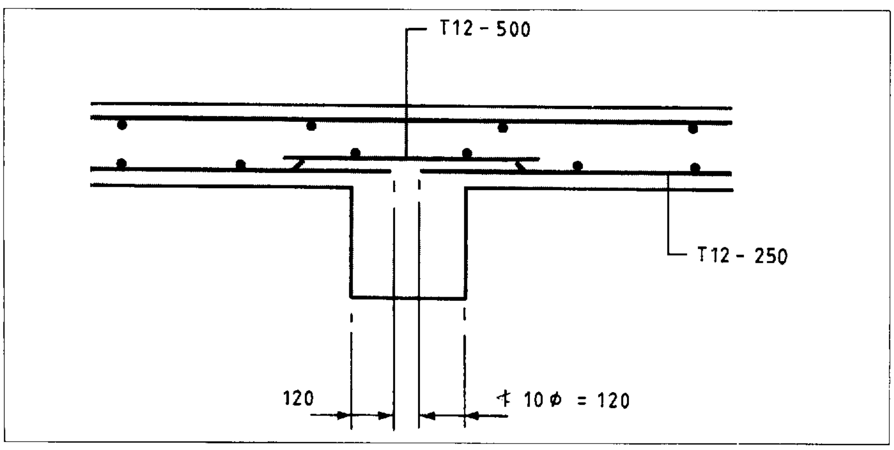


Figure 4.6 Detail at interior support

4.1.2.2.14 Transverse reinforcement at laps

No requirement for slabs. NAD 6.5(b) 5.2.4.1.2

4.1.3 Ribbed slabs

EC2 permits ribbed slabs to be treated as solid slabs for the purposes of analysis, provided that the flange and transverse ribs have sufficient torsional stiffness. 2.5.2.1(5)

4.1.3.1 Design example of a ribbed slab

Design a ribbed slab spanning between beams as shown in Figure 4.7.

In addition to self-weight, the slab carries a characteristic dead load of 1.0 kN/m^2 and an imposed load of 5.0 kN/m^2.

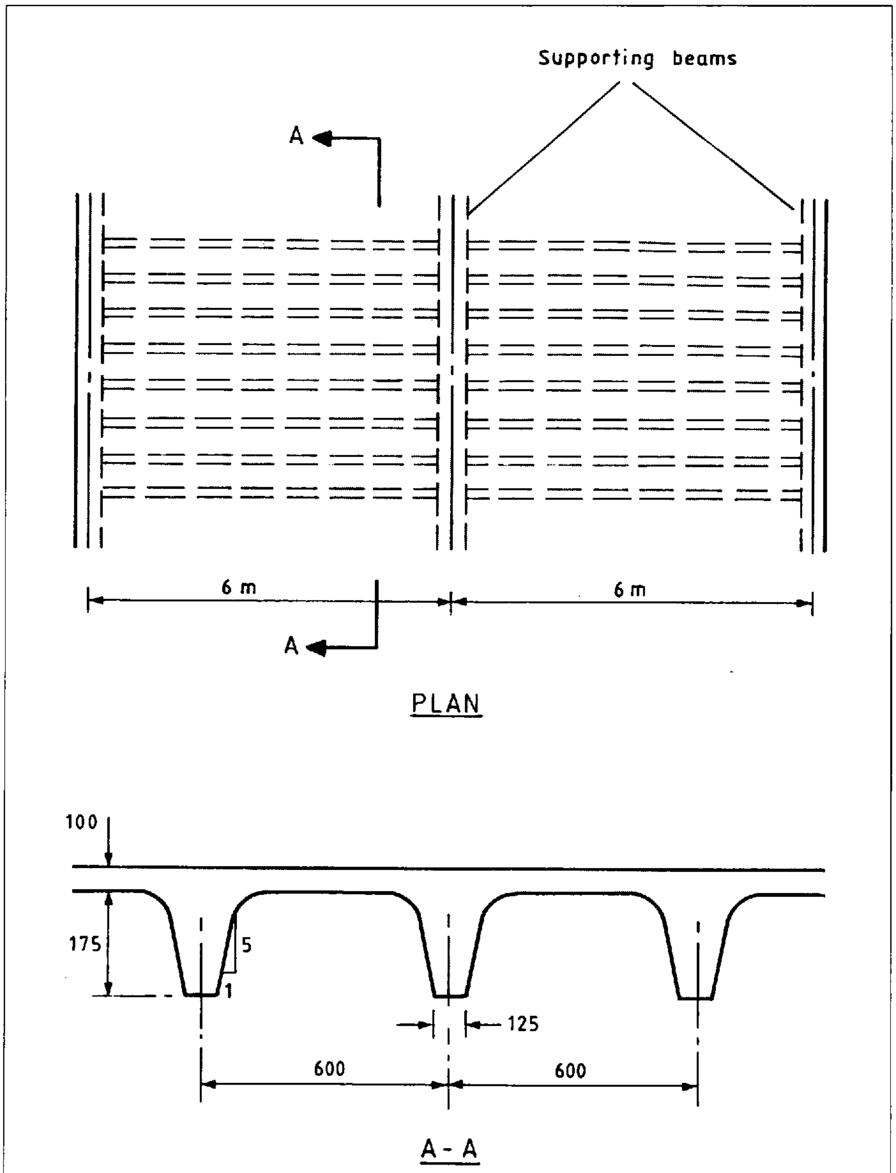


Figure 4.7 Ribbed slab spanning between beams

4.1.3.1.1 Durability

For a dry environment, exposure class is 1. Table 4.1
Minimum concrete strength grade is C25/30. ENV 206 Table NA.1

For cement content and w/c ratio, refer to ENV 206 Table 3.

Minimum cover to reinforcement	=	15 mm	NAD Table 6
Assume nominal aggregate size	=	20 mm	
Assume maximum bar size	=	20 mm	
Nominal cover	≥	20 mm	NAD 6.4(a)

Use nominal cover	=	25 mm

Note:
20 mm nominal cover is sufficient to meet the NAD requirements in all respects. NAD Table 3 4.1.3.3(8)

Check requirements for fire resistance to BS 8110: Part 2. NAD 6.1(a)

4.1.3.1.2 Materials

Type 2 deformed reinforcement, f_{yk} = 460 N/mm^2

$$f_{yd} = \frac{f_{yk}}{\gamma_s} = \frac{460}{1.15} = 400 \text{ N/mm}^2$$ 2.2.3.2P(1) Table 2.3

C25/30 concrete with 20 mm maximum aggregate size

4.1.3.1.3 Analysis model

Span ≥ 4 × slab depth 2.5.2.1(3)
6 m ≥ 4 × 0.275 = 1.1 m .... OK

Rib spacing = 600 ≤ 1500 mm .... OK 2.5.2.1(5)
Rib depth = 175 ≤ 4 × rib width = 500 mm .... OK

Flange depth = 100 mm

$$\geq \frac{1}{10} \times \text{clear spacing between ribs} \geq 50 \text{ mm}$$ ... OK

Transverse ribs (at supports only)

Spacing = 6 m > 10 × slab depth = 2.75 m

Hence the ribbed slab may not be treated as a solid slab in the analysis under the terms of this clause unless intermediate transverse ribs are incorporated. This is not always desirable. 2.5.2.1(5)

The model adopted in this example uses gross concrete section properties of the T shape in sagging regions and a rectangular section, based on the rib width, in the hogging region.

EC2 Figure 2.3 has been used initially to define the extent of the hogging. This method can clearly be refined.

4.1.3.1.4 Effective span

2.5.2.2.2

$l_{eff} = l_n + a_1 + a_2$ Eqn 2.15

Assume 300 mm wide supporting beams

$l_n = 5700$ mm

a_1 at edge beam $= a_i$ taken as $(\frac{1}{2})\ t = 150$ mm Figure 2.4(a)

a_2 at central beam $= a_i = (\frac{1}{2})\ t = 150$ mm Figure 2.4(b)

$l_{eff} = 6000$ mm

For ratio of adjacent spans between 1 and 1.5

$l_o = 0.85 l_1 = 0.85 \times 6000 = 5100$ mm 2.5.2.2.1(4) Figure 2.3

4.1.3.1.5 Effective width of flanges

2.5.2.2.1

Effective flange width is assumed constant across the span for continuous beams in buildings. 2.5.2.2.1(2)

For a symmetrical T beam 2.5.2.2.1(3)

$b_{eff} = b_w + (\frac{1}{5})\ l_o \leq b$ Eqn 2.13

$= 125 + (\frac{1}{5}) \times 5100 \leq 600$ mm

Therefore

$b_{eff} = 600$ mm

4.1.3.1.6 Loading

$G_k = 3.6 + 1.0 = 4.6$ kN/m^2

$Q_k = 5.0$ kN/m^2

$\gamma_G = 1.35$ Table 2.2 2.3.2.3P(2)

$\gamma_Q = 1.5$ Table 2.2

Maximum ultimate load $= 1.35 \times 4.6 + 1.5 \times 5.0 = 13.7$ kN/m^2	
Minimum ultimate load $= 1.35 \times 4.6 = 6.2$ kN/m^2	

4.1.3.1.7 Flexural design

Design for ultimate limit state using linear elastic method, choosing not to redistribute moments. 2.5.3.2.2

Consider the following load combinations: 2.5.1.2

(a) Alternate spans carrying the design variable and permanent load ($\gamma_Q Q_k + \gamma_G G_k$), other spans carrying only the design permanent load, $\gamma_G G_k$.

(b) Any two adjacent spans carrying the design variable and permanent load ($\gamma_Q Q_k + \gamma_G G_k$). All other spans carrying only the design permanent load, $\gamma_G G_k$.

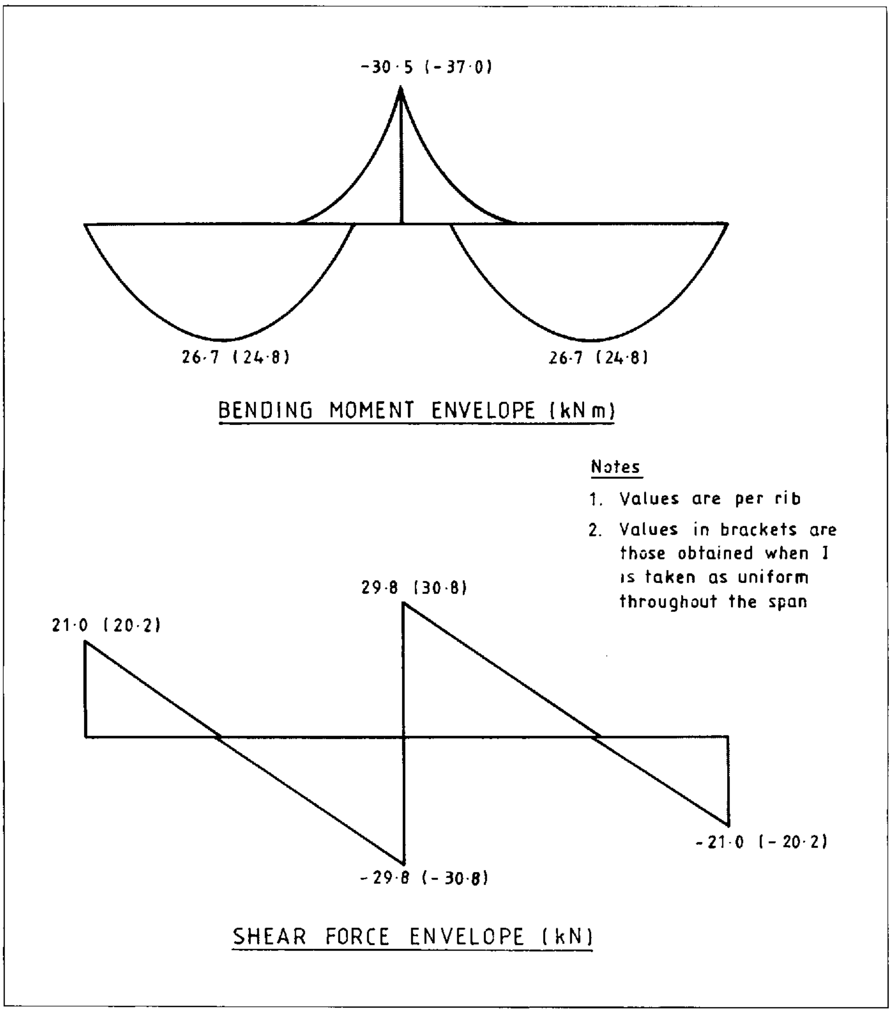


Figure 4.8 Results of analysis

The following results are taken from the analysis (see Figure 4.8).

M_{span} = 26.7 kNm/rib

M_{sup} = − 30.5 kNm/rib

$F_{Sd,sup}$ = 59.6 kN/rib

Support moment can be reduced by an amount ΔM_{Sd} 2.5.3.3(4)

where

ΔM_{Sd} = 59.6 × 0.3/8 = 2.2 kNm/rib Eqn 2.16

Therefore

M_{sup} = − 28.3 kNm/rib

$$d = 275 - 25 - 10 - \frac{16}{2} = 232 \text{ mm}$$

b = 600 mm (span), 125 mm (support)

$$\frac{M_{span}}{bd^2 f_{ck}} = 0.033$$

$$\frac{x}{d} = 0.075 < 0.45 \quad \text{OK}$$ 2.5.3.4.2(5)

Neutral axis in flange (x = 17.4 < 100 mm) . . . . OK

$$\frac{A_s f_{yk}}{bdf_{ck}} = 0.039$$

$$A_s = 295 \text{ mm}^2\text{/rib}$$

Use 2T16 (403 mm²/rib) bottom in span

$$\frac{M_{sup}}{bd^2 f_{ck}} = 0.168 > \mu_{lim} = 0.167 \text{ (Section 13, Table 13.2)}$$

Therefore

$$\frac{x}{d} > 0.45$$

This section may be analyzed to take account of the varying width of the compression zone, as shown in Figure 4.9.

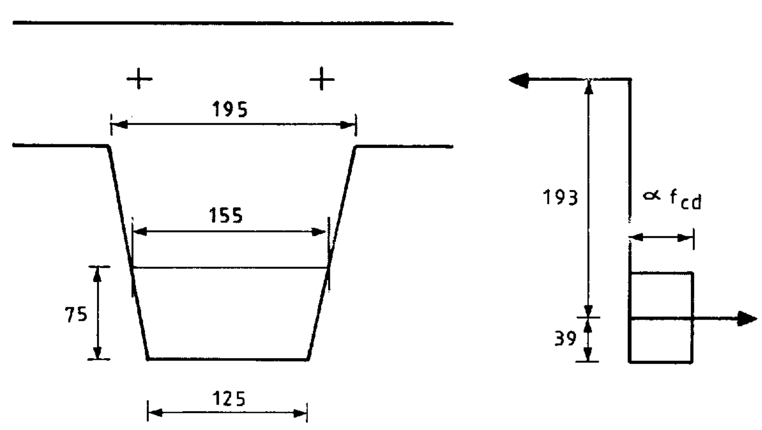


Figure 4.9 Analysis of section

Consider $x \leq 0.45d$ = 94 mm as a trial value

Using the rectangular stress block diagram with α = 0.85 gives

NAD Table 3
4.2.1.3.3(12)
Figure 4.4

$$\alpha f_{cd} = 0.85 \times \frac{25}{1.5} = 14.2 \text{ N/mm}^2$$

$$0.8x = 75 \text{ mm}$$

$$b_{av} = 140 \text{ mm}$$

$z = d - 39 = 193$ mm

$F_c = 0.8x(\alpha f_{cd})b_{av} = 75 \times \frac{14.2}{10^3} \times 140 = 149.1$ kN

$M_c = 149.1 \times \frac{(232 - 39)}{10^3} = 28.8 > 28.3$ kNm .... OK

$A_s = \frac{28.3 \times 10^6}{400 \times 193} = 367$ mm^2/rib

Use 4T12 (452 mm^2/rib) top at interior support

Minimum longitudinal reinforcement with $b_t = 160$ mm — 5.4.2.1.1(1)

$A_s \not< 0.6\, b_t d/f_{yk} \not< 0.0015\, b_t d = 56$ mm^2/rib $< A_{s,prov}$ ... OK

Maximum longitudinal reinforcement — 5.4.2.1.1(2)

$A_s = 0.04A_c = 3450$ mm^2 $> A_{s,prov}$ .... OK

4.1.3.1.8 Shear in rib — 4.3.2

$V_{Sd} = 29.8$ kN/rib at interior support

Shear resistance with no axial load

$V_{Rd1} = \tau_{Rd}\, k(1.2 + 40\rho_l)\, b_w d$ — 4.3.2.3, Eqn 4.18

$\tau_{Rd} = 0.3$ N/mm^2 — Table 4.8

$k = 1.6 - d = 1.368 \not< 1$

Based on top reinforcement: — Figure 4.12

$A_{sl} = 452$ mm^2/rib

$b_w = 125$ mm

$\rho_l = \frac{A_{sl}}{b_w d} = 0.0155 \not> 0.02$

Giving

$V_{Rd1} = 21.6$ kN/rib $< V_{Sd}$

Therefore shear reinforcement must be provided. — 4.3.2.2(3)

Use the standard design method for shear: — 4.3.2.2(7), 4.3.2.4.3

$V_{Rd3} \geq V_{Sd}$

$V_{Rd3} = V_{cd} + V_{wd}$ — Eqn 4.22

where

$V_{cd} = V_{Rd1} = 21.6$ kN/rib 4.3.2.4.3(1)

Therefore

$$V_{wd} = \frac{A_{sw}}{s} \times 0.9df_{ywd} \geq 29.8 - 21.6 = 8.2 \text{ kN/rib}$$ Eqn 4.23

Check maximum longitudinal spacing of links 5.4.2.2(7)

$$V_{Rd2} = (\tfrac{1}{2})\, \nu f_{cd} b_w \times 0.9d\,(1 + \cot\alpha)$$ Eqn 4.25

For vertical stirrups, $\cot\alpha = 0$

$$\nu = 0.7 - \frac{f_{ck}}{200} = 0.575 \geq 0.5$$ Eqn 4.21

$$V_{Rd2} = 0.5 \times 0.575 \times 16.7 \times 125 \times 0.9 \times 232 \times 10^{-3} = 125 \text{ kN}$$

$$(\tfrac{1}{5})\, V_{Rd2} < V_{Sd} \leq (\tfrac{2}{3})\, V_{Rd2}$$

Therefore

$$s_{max} = 0.6d = 139 \not> 300 \text{ mm}$$ Eqn 5.18

Try mild steel links at 125 mm crs.

$\rho_{w,min} = 0.0022$ Table 5.5

$A_{sw} = 0.0022 b_w s = 35 \text{ mm}^2$

Use R6 links @ 125 mm crs. (A_{sw} = 57 mm^2)

$$f_{ywd} = \frac{250}{1.15} = 217 \text{ N/mm}^2$$

$$V_{wd} = \frac{57}{125} \times 0.9 \times 232 \times \frac{217}{10^3} = 20.7 > 8.2 \text{ kN/rib} \ldots \text{ OK}$$

Link spacing may be increased where

$$V_{Sd} \leq (\tfrac{1}{5})\, V_{Rd2} = 25 \text{ kN/rib}$$

$$s_{max} = 0.8d \not> 300 = 185 \text{ mm}$$ Eqn 5.17

Use R6 links @ 175 mm crs. apart from region within 0.6 m of interior support

$V_{wd} = 14.7 > 3.4$ kN/rib . . . . OK

4.1.3.1.9 Shear between web and flanges — 4.3.2.5

$$v_{Sd} = \frac{\Delta F_d}{a_v}$$ Eqn 4.33

$a_v = (\tfrac{1}{2})\,l_o = 2550$ mm — Figure 4.14

Maximum longitudinal force in the flanges

$$F_c = \alpha f_{cd}(0.8x)b$$

$$\frac{x}{d} = 0.075 \text{ at mid-span}$$

$$F_c = 14.2 \times 0.8 \times 0.075 \times 232 \times \frac{600}{10^3} = 122 \text{ kN}$$

Force to one side of web

$$\Delta F_d = 122 \times \frac{600 - 195}{2 \times 600} = 41.2 \text{ kN}$$

Therefore

$$v_{Sd} = \frac{41.2}{2.55} = 16.2 \text{ kN/m}$$

$v_{Rd2} = 0.2 f_{cd} h_f = 0.2 \times 16.7 \times 100 = 334 \text{ kN/m} > v_{Sd}$ .. OK — Eqn 4.36

Eqn 4.34

$$v_{Rd3} = 2.5\tau_{Rd} h_f + \frac{A_{sf} f_{yd}}{s_f}$$ Eqn 4.37

With $A_{sf} = 0$

$v_{Rd3} = 2.5 \times 0.3 \times 100 = 75 \text{ kN/m} > v_{Sd}$ .... OK — Eqn 4.35

No shear reinforcement required

4.1.3.1.10 Topping reinforcement

No special guidance is given in EC2 regarding the design of the flange spanning between ribs. The *Handbook to BS 8110*[13] gives the following guidance.

3.6.1.5 Thickness of topping used to contribute to structural strength
Although a nominal reinforcement of 0.12% is suggested in the topping *(3.6.6.2)*, it is not insisted upon, and the topping is therefore expected to transfer load to the adjacent ribs without the assistance of reinforcement. The mode of transfer involves arching action and this is the reason for the insistence that the depth be at least one-tenth of the clear distance between the ribs.

Minimum flange depths are the same in EC2 and BS 8110 and the above is therefore equally applicable. Provide minimum reinforcement transversely and where top bars in rib, which have been spread over width of flange, are curtailed. — 2.5.2.1(5)

$A_{sf} \nless 0.6 b_t d_f / f_{yk} \nless 0.0015 b_t d_f$ — Eqn 5.14

$d_f < h_f = 100$ mm

Therefore, conservatively

$A_{sf} \nless 150$ mm²/m

Use T8 @ 200 mm crs. (251 mm²/m) or consider fabric

4.1.3.1.11 Deflection — 4.4.3.2

Actual span/effective depth ratio $= \dfrac{6000}{232} = 25.9$

Mid-span reinforcement ratio, $\rho = \dfrac{403}{600 \times 232} = 0.0029$

Therefore section is lightly stressed. — 4.4.3.2.(5)

Basic span/effective depth ratio (interpolating for ρ) = 39.2 — NAD Table 7

Modification factor for steel stress $= \dfrac{400 \times 403}{460 \times 295} = 1.19$

Since flange width > 3 × rib width, a 0.8 modification factor is required.

Since span $\ngtr$ 7 m, no further modification is required.

Permitted span/effective depth ratio = 39.2 × 1.19 × 0.8

= 37.3 > 25.9 . . . . OK

4.1.3.1.12 Cracking

For exposure class 1, crack width has no influence on durability and the limit of 0.3 mm could be relaxed. However, the limit of 0.3 mm is adopted for this example. — 4.4.2.1(6)

Satisfy the requirements for control of cracking without calculation. Check section at mid-span: — 4.4.2.3(2)

Minimum reinforcement, $A_s = k_c k f_{ct,eff} A_{ct} / \sigma_s$ — 4.4.2.2(3) Eqn 4.78

Note:
A_{ct} can be conservatively taken as the area below the neutral axis for the plain concrete section, ignoring the tension reinforcement, as shown in Figure 4.10.

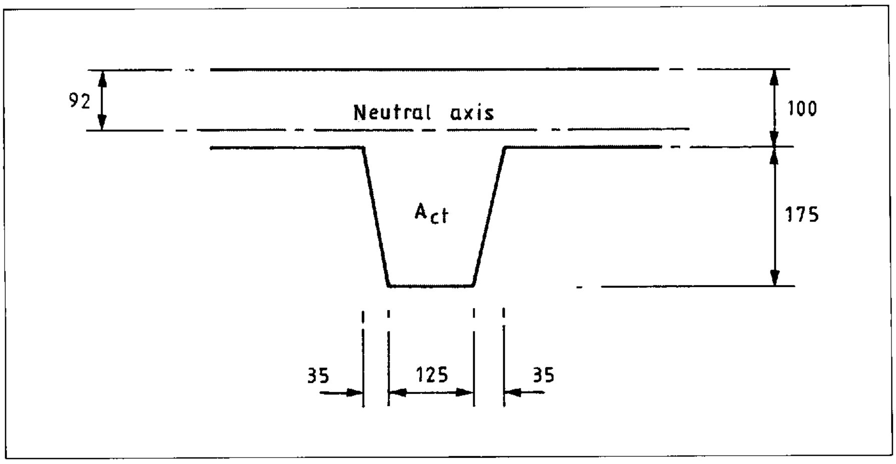


Figure 4.10 Tensile zone of plain concrete section

Depth to neutral axis = 92 mm

A_{ct} = 160 × 175 + 600 (100 − 92) = 32800 mm^2

σ_s = 100%f_{yk} = 460 N/mm^2

$f_{ct,eff}$ = recommended value 3 N/mm^2 — 4.4.2.2(3)

k_c = 0.4 for normal bending

k = 0.8

A_s = 0.4 × 0.8 × 3 × 32800/460 = 69 mm^2 < $A_{s,prov}$ . . . . . OK — Eqn 4.78

Check limit on bar size. — Table 4.11

Quasi-permanent loads = G_k + 0.3Q_k = 6.1 kN/m^2 — 4.4.2.3(3)

Ratio of quasi-permanent/ultimate loads = $\frac{6.1}{13.7}$ = 0.45 — 2.3.4, Eqn 2.9(c), NAD Table 1

Estimate of steel stress

$$0.45 \times \frac{A_{s,req}}{A_{s,prov}} \times f_{yd} = 0.45 \times \frac{295}{403} \times 400 = 132 \text{ N/mm}^2$$

Maximum bar size = 32 > 16 mm provided . . . . OK — Table 4.11

For cracks caused dominantly by loading, crack widths generally will not be excessive. — 4.4.2.3(2)

4.1.3.1.13 Detailing

Minimum clear distance between bars = ϕ ≮ 20 mm — 5.2.1(3)

Nominal clear distance in rib = 49 mm . . . . OK

Bond and anchorage lengths:	5.2.2
For $h > 250$ mm bottom reinforcement is in good bond conditions. Top reinforcement is in poor bond conditions.	5.2.2.1 Figure 5.1(c)
Therefore, ultimate bond stresses are	
Bottom reinforcement, $f_{bd} = 2.7$ N/mm^2	5.2.2.2(2) Table 5.3
Top reinforcement, $f_{bd} = 0.7 \times 2.7 = 1.89$ N/mm^2	5.2.2.2(2)
Basic anchorage length, $l_b = \dfrac{\phi f_{yd}}{4 f_{bd}}$	5.2.2.3 Eqn 5.3
For top reinforcement, $l_b = \dfrac{\phi \times 400}{4 \times 1.89} = 53\phi$	
For bottom reinforcement, $l_b = \dfrac{\phi \times 400}{4 \times 2.7} = 37\phi$	
Anchorage of bottom reinforcement at end support.	5.4.2.1.4
Treat as a solid slab and retain not less than half of the mid-span reinforcement.	5.4.3.2.2(1)

Use 2T12 L bars bottom at end support

Anchorage force for this reinforcement with zero design axial load	
$F_s = V_{Sd} \times \dfrac{a_l}{d}$	5.4.2.1.4(2) Eqn 5.15
where	
$V_{Sd} = 21$ kN/rib	
For vertical shear reinforcement calculated by the standard method	5.4.2.1.3(1)
$a_l = z(1 - \cot\alpha)/2 \nless 0$	
$\alpha = 90°$ and z is taken as $0.9d$	
Although this ribbed slab falls outside the solid slab classification requirements for analysis, treat as a solid slab for detailing and take $a_l = d$.	5.4.3.2.1(1)
Therefore	
$F_s = 21$ kN/rib	
$A_{s,req} = \dfrac{21 \times 10^3}{400} = 53$ mm$^2 < A_{s,prov}$ .... OK	
Required anchorage length for bottom reinforcement at support:	5.2.3.4 5.2.3.4.1(1) Eqn 5.4
$l_{b,net} = \dfrac{\alpha_a l_b A_{s,req}}{A_{s,prov}} \nless l_{b,min}$	
$\alpha_a = 0.7$ for curved bars in tension	
$l_{b,min} = 0.3 l_b = 11.1\phi \nless 10\phi$ or 100 mm	Eqn 5.5

In calculations of $l_{b,net}$, $A_{s,req}$ should be taken $\not< A_{s,span}/4 = 101\ mm^2$ — NAD 6.5(c), 5.4.2.1.4(3)

$$A_{s,prov} = 226\ mm^2$$

$$l_{b,net} = 0.7 \times 37 \times 12 \times \frac{101}{226} = 139\ mm > l_{b,min} \ldots\ldots\ \text{OK}$$ Eqn 5.4

Minimum transverse reinforcement (for indirect support): — 5.2.3.3

$$A_{st} = A_s/4 = 226/4 = 57\ mm^2$$

Use 1T8 bar as transverse reinforcement

Minimum top reinforcement at end support: — 5.4.2.1.2(1)

$$M_{sup} = (\tfrac{1}{4})\ 26.7 = 6.7\ kNm/rib$$

$$\frac{M}{bd^2 f_{ck}} = 0.040$$

Therefore nominal reinforcement is sufficient.

Use 2T12 L bars top as link hangers

The reinforcement details are shown in Figure 4.11. — Figure 5.12

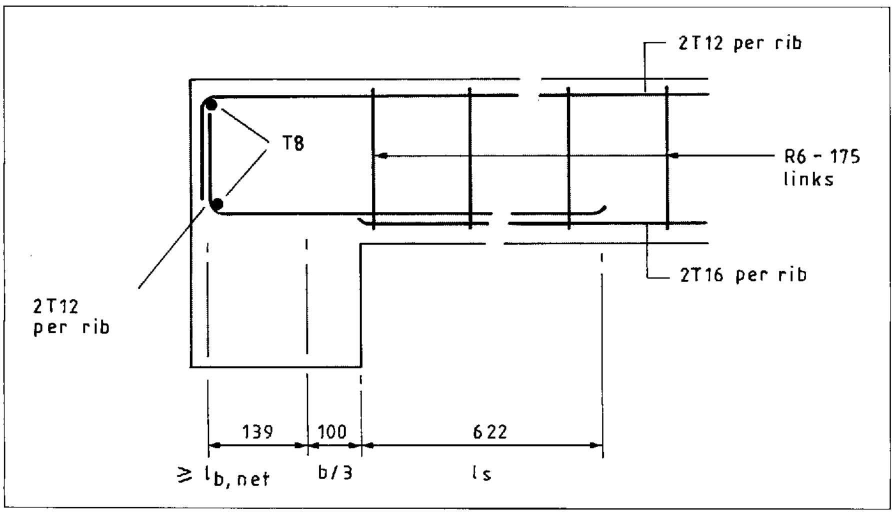


Figure 4.11 Detail at edge support

Provide full lap length, l_s, for bottom bars: — 5.2.4.1.3

$$l_s = l_{b,net}\alpha_1 \not< l_{s,min}$$ Eqn 5.7

For 100% of bars lapped and $b > 2\phi$, $\alpha_1 = 1.4$ — NAD Table 3

Hence with $\alpha_a = 1.0$ and $A_{s,req} = A_{s,prov}$ — Figure 5.6

$$l_{b,net} = l_b = 37\phi = 37 \times 12 = 444\ mm$$ Eqn 5.4

$$l_{s,min} = 0.3\ \alpha_a \alpha_1 l_b = 187\ mm \not< 15\phi \text{ or } 200\ mm$$ Eqn 5.8

Therefore

$$l_s \quad = \quad 444 \times 1.4 \quad = \quad 622 \text{ mm} \quad > \quad l_{s,min} \; \ldots\ldots\ldots\ldots\ldots\ldots \; \text{OK}$$

Transverse reinforcement at lapped splices should be provided as for a beam section. Since $\phi < 16$ mm, nominal shear links provide adequate transverse reinforcement. 5.2.4.1.2(1)

Anchorage of bottom reinforcement at interior support. 5.4.2.1.5

Treat as a solid slab and continue 50% of mid-span bars into support. 5.4.3.2.2(1)

The reinforcement details are shown in Figure 4.12. Figure 5.13(b)

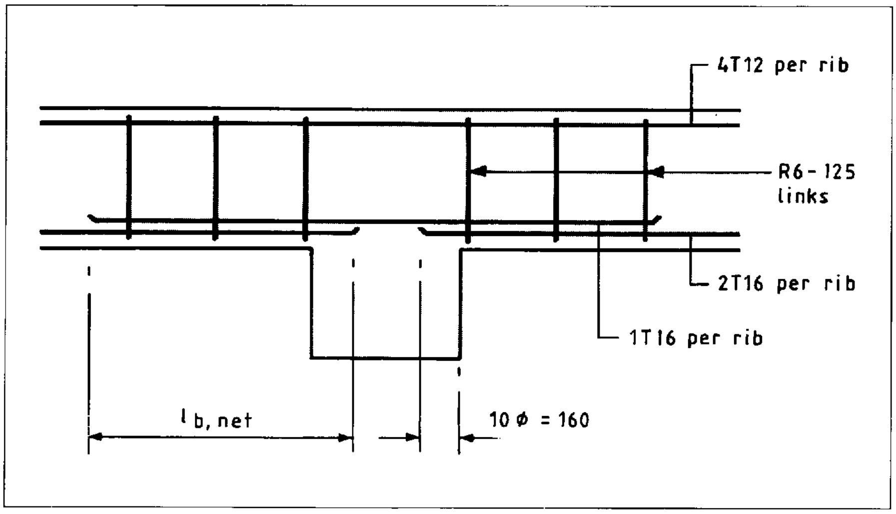


Figure 4.12 Detail at interior support

This detailing prohibits the easy use of prefabricated rib cages because of the intersection of the bottom reinforcement with the supporting beam cage. It is suggested that providing suitably lapped continuity bars through the support should obviate the need to continue the main steel into the support.

The arrangement of the reinforcement within the section including the anchorage of the links is shown in Figure 4.13. 5.2.5 NAD Tables 3 & 8 5.4.2.1.2(2) Figure 5.10

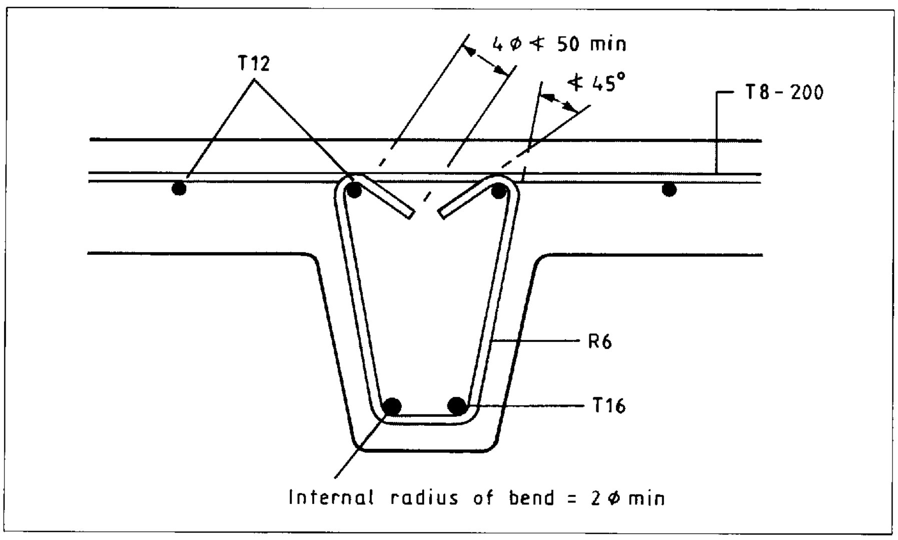


Figure 4.13 Arrangement of reinforcement

4.2 Flat slabs

4.2.1 Flat slabs in braced frames

The same frame is used in each of the following examples, but column heads are introduced in the second case.

4.2.1.1 Design example of a flat slab without column heads

Design the slab shown in Figure 4.14 to support an additional dead load of 1.0 kN/m^2 and an imposed load of 5.0 kN/m^2.

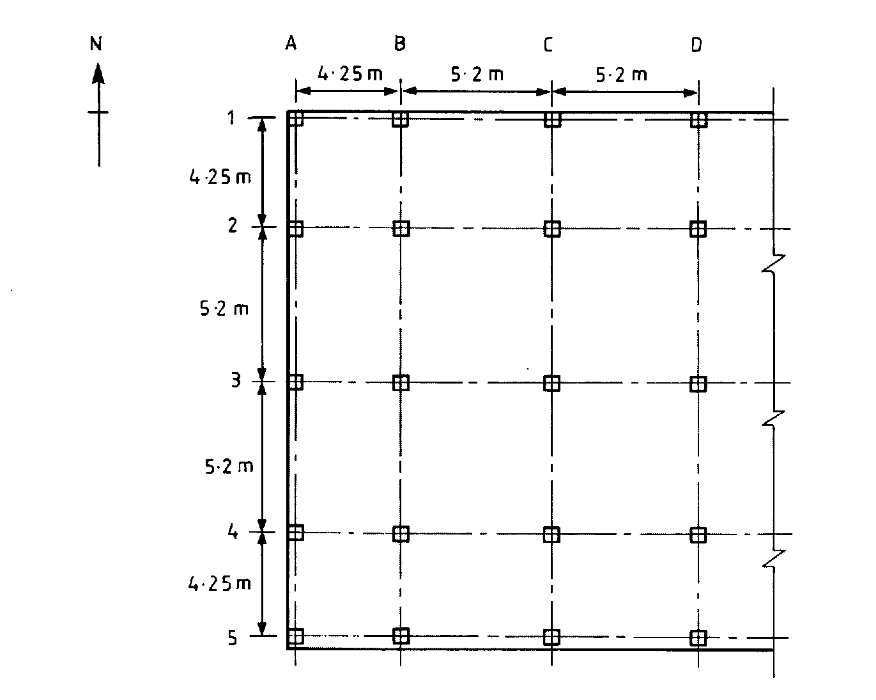


Figure 4.14 Plan of structure

The area shown is part of a larger structure which is laterally restrained in two orthogonal directions by core walls.

The slab is 225 mm thick. All columns are 300 mm square and along grid 5 there is an edge beam 450 mm deep x 300 mm wide.

4.2.1.1.1 Durability

For a dry environment, exposure class is 1. Table 4.1

Minimum concrete strength grade is C25/30. ENV 206 Table NA.1

Since a more humid environment is likely to exist at the edges of the slab, increase concrete strength grade to C30/37.

For cement content and w/c ratio, refer to ENV 206 Table 3.

Nominal cover to reinforcement = 20 mm		NAD Table 6
Nominal cover to all bars ≮ bar size		NAD 6.4(a)
≮ nominal aggregate size = 20 mm . . . . OK		4.1.3.3(5)

Use nominal cover = 20 mm

4.2.1.1.2 Materials

Type 2 deformed reinforcement, f_{yk} = 460 N/mm^2 — NAD 6.3(a)

C30/37 concrete with 20 mm maximum aggregate size

4.2.1.1.3 Load cases

It is sufficient to consider the following load cases — 2.5.1.2

(a) Alternate spans loaded with $\gamma_G G_k + \gamma_Q Q_k$ and $\gamma_G G_k$ on other spans.

(b) Any two adjacent spans carrying $\gamma_G G_k + \gamma_Q Q_k$ and all other spans carrying $\gamma_G G_k$.

$G_k = 0.225 \times 24 + 1.0 = 6.4$ kN/m^2

$\gamma_G G_k = 1.35 \times 6.4 = 8.7$ kN/m^2 — Table 2.2

$\gamma_G G_k + \gamma_Q Q_k = 8.7 + 1.5 \times 5.0 = 16.2$ kN/m^2 — Eqn 2.8(a)

4.2.1.1.4 Analysis

Analyses are carried out using idealizations of both the geometry and the behaviour of the structure. The idealization selected shall be appropriate to the problem being considered. — 2.5.1.1.P(3) and P(4)

No guidance is given in EC2 on the selection of analysis models for flat slabs, or on the division of panels into middle and column strips and the distribution of analysis moments between these strips. This is left to the assessment of individual engineers. The requirements set down in BS 8110 for the above points are taken as a means of complying with EC2 Clause 2.5.1.1P(3).

EC2 allows analysis of beams and slabs as continuous over pinned supports. It then permits a reduction in the support moment given by — 2.5.3.3(3) 2.5.3.3(4)

$$\Delta M_{Sd} = F_{Sd.sup} b_{sup}/8$$

The analysis in this example includes framing into columns. Thus the reduction ΔM_{Sd} is not taken.

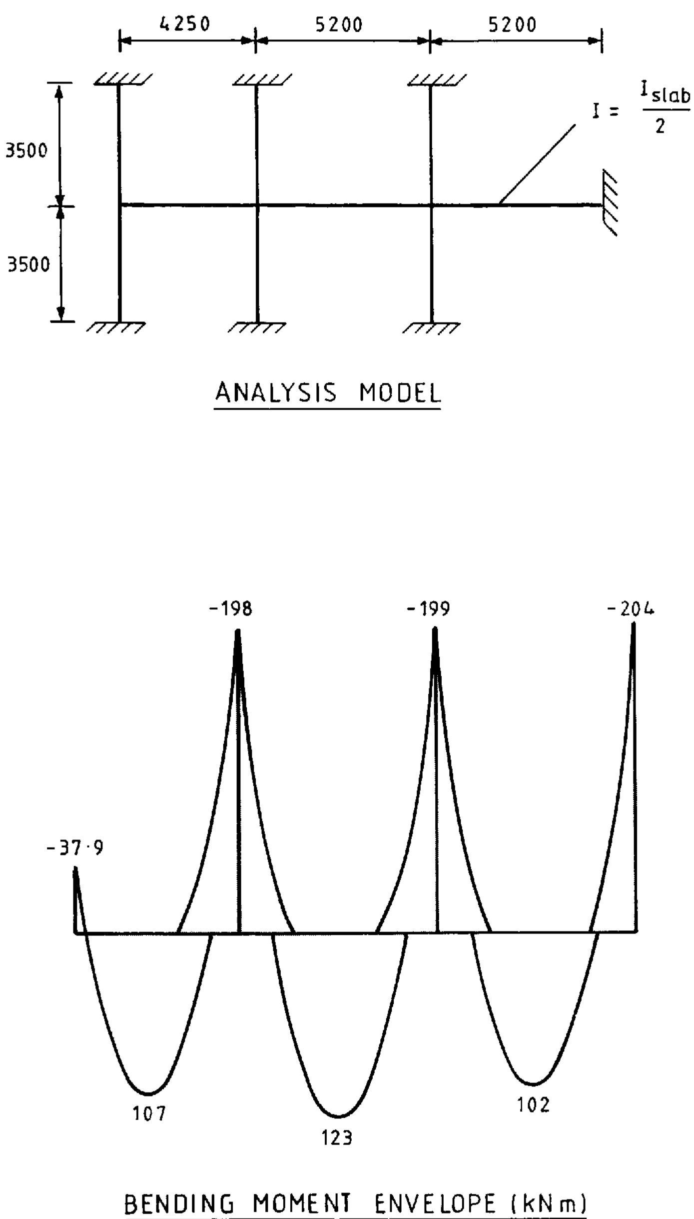
Consider two frames from Figure 4.14 as typical:

(i) Grid 3/A–D subframe
(ii) Grid B.

Analysis results for the frames described above are given in Figure 4.15. The results for each frame are practically identical as the analysis for Grid B has an increased loaded width (5.2 m), since this is the first internal support for frames in the orthogonal direction.

Member stiffnesses have been based on a plain concrete section in this analysis.

Column moments and reactions are given in Table 4.1.



Figures 4.15 Analysis of frame

Table 4.1 Column moments and reactions

Support	Max. reaction (kN)	Σ column moments (kNm)	Max. Σ column moments (kNm)
End	156.4	37.9	37.9
1st interior	444.7	6.8	21.4

4.2.1.1.5 Flexural design – Panel A–B/1–2

EC2 does not specifically address the problem of edge column moment transfer and the provisions of BS 8110 are adopted here. **BS 8110 3.7.4.2**

$M_{t,max} = 0.15 b_e d^2 f_{cu}$

Column A/2 moment transfer

Assuming 20 mm cover and 20 mm bars in the top **NAD 4.1.3.3(5)**

$d_1 = 225 - 20 - 10 = 195$ mm

$d_2 = 195 - 20 = 175$ mm

$b_e = 300 + 300$ (say) $= 600$ mm

$f_{cu} = 37$ N/mm^2

$M_{t,max} = 0.15 \times 600 \times 175^2 \times 37 \times 10^{-6} = 102$ kNm

This ought to be compared with an analysis for a loading of $1.4G_k + 1.6Q_k$, which would give approximately 5% higher edge moments than the EC2 analysis results above.

$M_{t,max} = 102 > 1.05 \times 37.9 = 39.8$ kNm . . . . OK

Design reinforcement to sustain edge moment on 600 mm width.

Using $\gamma_c = 1.5$, $\alpha = 0.85$ and $\gamma_s = 1.15$ **Table 2.3**

Referring to Section **13**, Table 13.1:

$$\frac{M_{Sd}}{bd^2 f_{ck}} = \frac{37.9 \times 10^6}{600 \times 175^2 \times 30} = 0.069$$

$$\frac{A_s f_{yk}}{bdf_{ck}} = 0.085$$

$$A_s = \frac{0.085 \times 600 \times 175 \times 30}{460} = 582 \text{ mm}^2 = 970 \text{ mm}^2/\text{m}$$

$\frac{x}{d} = 0.163 < 0.45$ (zero redistribution) . . . . OK **2.5.3.4.2(5)**

Use T16 @ 150 mm crs. (1340 mm^2/m) top at edge column

Place over width = 900 mm (see Figure 4.16)

Note:
This approach gives more reinforcement than is necessary.

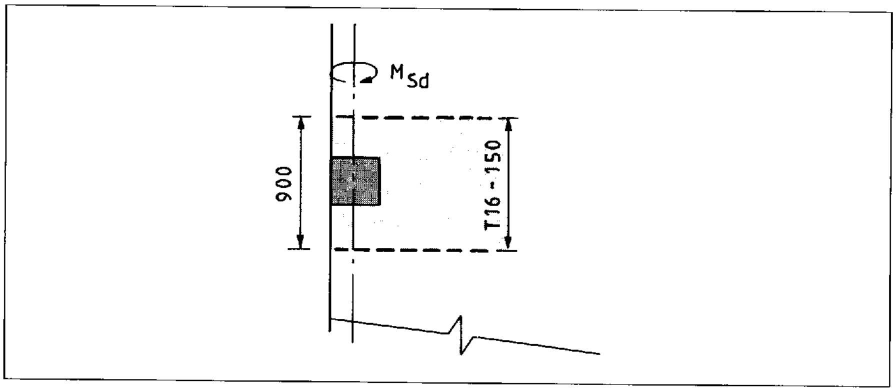


Figure 4.16 Edge column moment transfer

Check above moment against minimum value required for punching shear. 4.3.4.5.3

$m_{Sd} \geq \eta V_{Sd}$ Eqn 4.59

For moments about axis parallel to slab edge Table 4.9

$\eta = \pm\ 0.125$ per m

$V_{Sd} = 156.4$ kN

Therefore

$m_{Sd} = \pm\ 0.125 \times 156.4 = \pm\ 19.6$ kNm/m

Edge moment $= \dfrac{37.9}{0.6} = 63.2 > 19.6$ kNm/m . . . . OK

Design for m_{Sd} above in region outside edge column moment transfer zone.

$$\frac{m_{Sd}}{bd^2 f_{ck}} = \frac{19.6 \times 10^6}{1000 \times 175^2 \times 30} = 0.021$$

Minimum steel sufficient $= \dfrac{0.6 b_t d}{f_{yk}} \not< 0.0015 b_t d$ 5.4.2.1.1

$= 0.0015 \times 1000 \times 175 = 263\ mm^2/m$

Use T12 at 300 mm crs. (373 mm^2/m) top and bottom (minimum)

Maximum spacing $= 3h \not> 500 = 500 > 300$ mm . . . . OK NAD Table 3 5.4.3.2.1(4)

Column A/1 moment transfer

Assume the design forces for the frame on grid 1 are directly related to those for grid 3 in proportion to their loaded widths.

$$\text{Load ratio} = \frac{(4.25/2)}{5.2} = 0.41$$

The ratio of the edge column distribution factors for the frames is 2.0.

$$M_{Sd} = 37.9 \times 0.41 \times 2.0 = 31.1 \text{ kNm}$$

Using design approach as for column A/2:

$$b_e = 300 + \frac{300}{2} \text{ (say)} = 450 \text{ mm}$$

$$M_{t,max} = 0.15 \times 450 \times 175^2 \times 37 \times 10^{-6} = 76 \text{ kNm}$$

$$> 1.05 \times 31.1 = 32.7 \text{ kNm}$$ .... OK

Design reinforcement to sustain edge moment on 450 mm width

$$\frac{M_{Sd}}{bd^2f_{ck}} = \frac{31.1 \times 10^6}{450 \times 175^2 \times 30} = 0.075$$

$$\frac{A_sf_{yk}}{bdf_{ck}} = 0.093$$

$$A_s = \frac{0.093 \times 450 \times 175 \times 30}{460} = 478 \text{ mm}^2 = 1062 \text{ mm}^2\text{/m}$$

Use T16 @ 150 mm crs. (1340 mm²/m) top for a width of 600 mm

Check above moment against minimum value required for punching shear. 4.3.4.5.3

$$m_{Sd} \geq \eta V_{Sd} \text{ where } \eta = \pm 0.5 \text{ per m for corner columns}$$ Eqn 4.59

$$= \pm 0.5 \times (0.41 \times 156.4) \text{ say } = \pm 32.1 \text{ kNm/m}$$ Table 4.9

Edge moment = 31.1/0.45 = 69.1 kNm/m > 32.1 .... OK

In region of slab critical for punching shear:

$$\frac{M_{Sd}}{bd^2f_{ck}} = \frac{32.1 \times 10^6}{1000 \times 175^2 \times 30} = 0.035$$

$$\frac{A_sf_{yk}}{bdf_{ck}} = 0.042$$

$$A_s = \frac{0.042 \times 1000 \times 175 \times 30}{460} = 480 \text{ mm}^2\text{/m}$$

Use T16 @ 300 mm crs. (670 mm^2/m) top and bottom outside 600 mm wide moment transfer zone and over area determined in punching calculation

The division of panels into column and middle strips is shown in Figure 4.17. BS 8110 Figure 3.12

Although BS 8110 indicates a 2.36 m wide column strip at column B2, a 2.6 m width has been used in the following calculations. This is considered reasonable as a loaded width of 5.2 m has been taken in the analysis for grid B and grid 2.

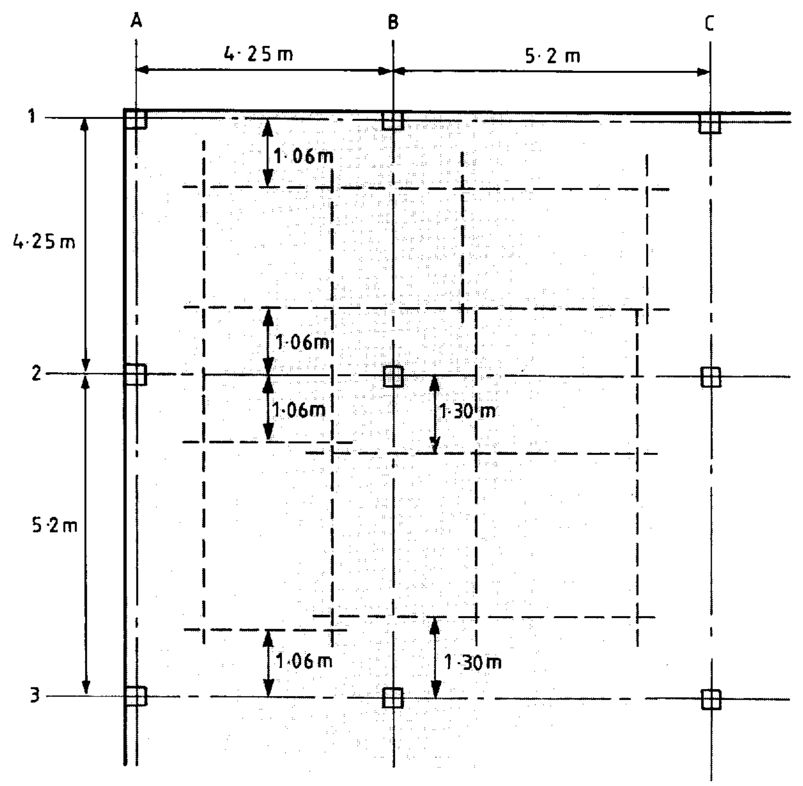


Figure 4.17 Assumed strip widths (arrangement symmetrical about diagonal A/1–C/3)

Column B/2 support moments

Analysis moment = 198 kNm in both directions

Column strip $M_{Sd,cs}$ = 0.75 × 198 = 149 kNm BS 8110 Table 3.20

b = 1300 × 2 = 2600 mm

$$\frac{M_{Sd,cs}}{bd^2 f_{ck}} = \frac{149 \times 10^6}{2600 \times 175^2 \times 30} = 0.062$$

$$\frac{A_s f_{yk}}{bdf_{ck}} = 0.076$$

$$A_s = \frac{0.076 \times 2600 \times 175 \times 30}{460} = 2255 \text{ mm}^2$$

Use 13T16 (2613 mm^2) top in column strip. Provide 9T16 @ 150 mm crs. in central 1.3 m and 2T16 @ 300 mm crs. on either side

BS 8110 3.7.3.1

Check whether minimum moment required for punching shear has been met. **4.3.4.5.3**

With $\eta = -0.125$ **Table 4.9**

$$M_{Sd} = \eta V_{Sd} = -0.125 \times 444.7 = -55.6 \text{ kNm}$$ **Eqn 4.59**

This is to be carried over a width of $0.3l$. Since V_{Sd} includes for a loaded width of 5.2 m, it is assumed that the larger panel width may be used.

$$0.3l = 0.3 \times 5.2 = 1.56 \text{ m}$$

By inspection reinforcement (9T16 in central 1.3 m) is sufficient. ..... OK

Middle strip (using average panel width)

$$\frac{M_{Sd,ms}}{bd^2 f_{ck}} = \frac{0.25 \times 198 \times 10^6}{(4725 - 2600) \times 175^2 \times 30} = 0.026$$

$$\frac{x}{d} = 0.059 < 0.45$$ .... OK **2.5.3.4.2(5)**

$$\frac{A_s f_{yk}}{bdf_{ck}} = 0.031$$

$$\frac{A_s}{b} = \frac{0.031 \times 1000 \times 175 \times 30}{460} = 354 \text{ mm}^2/\text{m}$$

Use T16 @ 300 mm crs. (377 mm^2/m) top in middle strip

It is noted that EC2 Clause 2.5.3.3(5) would allow the use of the moment at the face of the support (subject to limits in EC2 Clause 2.5.3.4.2(7)), but this is considered more appropriate to beams or solid slabs and the peak moment over the support has been used in the above design.

Span moments

No special provisions are required in EC2. Hence the design basis of BS 8110 is adopted for the division of moments. The same pattern of reinforcement will be provided in all panels.

The column strip moments are given in Table 4.2 where

$$M_{Sd,cs} = 0.55\, M_{Sd}$$

Table 4.2 Column strip span moments

Span	Total moment M_{Sd} (kNm)	$M_{Sd,cs}$ (kNm)	b (m)	$\frac{M_{Sd,cs}}{b}$ (kN)
End	107	58.9	2.12	27.8
1st. interior	123	67.7	2.36	28.6

Using the greater value:

$$\left(\frac{M_{Sd,cs}}{b}\right)\frac{1}{d^2 f_{ck}} = \frac{28.6 \times 10^3}{175^2 \times 30} = 0.031$$

$$\frac{A_s f_{yk}}{bdf_{ck}} = 0.037 \qquad \frac{x}{d} = 0.071 < 0.45 \ \ldots\ldots\ldots\ldots\ldots \ \text{OK}$$ 2.5.3.4.2(5)

$$\frac{A_s}{b} = \frac{0.037 \times 175 \times 30 \times 10^3}{460} = 422 \text{ mm}^2\text{/m}$$

Use T12 @ 250 mm crs. (452 mm²/m) bottom in column strips

Using the middle strip moment for the first interior span

$$b = 4.725 - 2.36 = 2.365 \text{ m (average panel width)}$$

$$\frac{M_{Sd,ms}}{bd^2 f_{ck}} = \frac{0.45 \times 123 \times 10^6}{2365 \times 175^2 \times 30} = 0.026$$

$$\frac{A_s f_{yk}}{bdf_{ck}} = 0.031 \qquad \frac{x}{d} = 0.059 < 0.45 \ \ldots\ldots\ldots\ldots\ldots \ \text{OK}$$ 2.5.3.4.2(5)

$$\frac{A_s}{b} = \frac{0.031 \times 175 \times 30 \times 10^3}{460} = 354 \text{ mm}^2\text{/m}$$

Use T12 @ 300 mm crs. (377 mm²/m) bottom in middle strips

Minimum longitudinal reinforcement, using $d_{max} = 195$ mm

$$= \frac{0.6 b_t d}{f_{yk}} \not< 0.0015 b_t d$$

$$= 0.0015 \times 1000 \times 195 = 293 \text{ mm}^2\text{/m} \ \ldots\ldots\ldots \ \text{OK}$$

4.2.1.1.6 Punching 4.3.4

Column B/2 (300 mm × 300 mm internal column)

Critical perimeter located at 1.5d from face of column. 4.3.4.1(3) Figure 4.16

d = 185 mm (average)

For a rectangular column/wall check geometry 4.3.4.2.1(1)

Perimeter = 4 × 300 = 1200 mm $\not>$ 11d = 2035 mm .. OK

$\frac{\text{length}}{\text{breadth}}$ = 1 $\not>$ 2 .... OK

Hence

u = $2\pi \times 1.5 \times 185 + 1200$ = 2944 mm Figure 4.18

V_{Sd} = 444.7 kN

Note: No reduction in this value has been taken.

The applied shear per unit length: 4.3.4.3(4)

$v_{Sd} = \frac{V_{Sd}\beta}{u}$ where β internal column = 1.15 Eqn 4.50 Figure 4.21

$v_{Sd} = \frac{444.7 \times 10^3 \times 1.15}{2944} = 174$ N/mm

Shear resistance without links 4.3.4.5.1

$v_{Rd1} = \tau_{Rd}k(1.2 + 40\rho_l)d$ Eqn 4.56

τ_{Rd} = 0.34 N/mm^2 Table 4.8

$k = 1.6 - d = 1.6 - 0.185 = 1.415 \geq 1.0$

ρ_l = reinforcement ratio within zone 1.5d from column face (T16 @ 150 mm crs. top each way gives 1340 mm^2/m)

$\rho_l = \sqrt{\rho_{lx} \times \rho_{ly}} \not> 0.015$

$= \frac{1340}{1000 \times 185} = 0.0072$

Note:
The amount of tensile reinforcement in two perpendicular directions > 0.5%. 4.3.4.1(9)

Assume $\rho_{lx} + \rho_{ly}$ = 2 (0.0072) > 0.005 .... OK

Therefore

$v_{Rd1} = 0.34 \times 1.415 \times (1.2 + 40 \times 0.0072) \times 185 = 133$ N/mm

$v_{Sd} = 174$ N/mm $> v_{Rd1}$

Therefore shear reinforcement required such that $v_{Rd3} \geq v_{Sd}$ — 4.3.4.3(3)

Slab depth $\geq$ 200 mm . . . . OK — 4.3.4.5.2(5)

Check that applied shear does not exceed the maximum section capacity

$v_{Rd2} = 2.0\, v_{Rd1} = 2.0 \times 133 = 266 > 174$ N/mm . . . . . OK — NAD Table 3

Shear stress around column perimeter $= \dfrac{444.7 \times 10^3}{1200 \times 185} = 2.0$ N/mm^2 — 4.3.4.5.2(1)

$\leq 0.9\sqrt{f_{ck}} = 4.9$ N/mm^2 . . . . OK — NAD 6.4(d)

Design shear reinforcement using EC2 Eqn 4.58 since

$v_{Sd}/v_{Rd1} = 174/133 \leq 1.6$ — NAD 6.4(d)

$$v_{Rd3} = v_{Rd1} + \frac{\Sigma A_{sw} f_{yd}}{u}$$ — Eqn 4.58

Using type 2 deformed high yield bars as links

$$f_{yd} = \frac{f_{yk}}{\gamma_s} = \frac{460}{1.15} = 400 \text{ N/mm}^2$$ — Table 2.3

Therefore

$$\Sigma A_{sw} \geq (174 - 133)\frac{2944}{400} = 302 \text{ mm}^2$$

Minimum reinforcement ratio = 100% × value given in EC2 Table 5.5. — NAD Table 3

$\rho_{w,min} = 0.0012$ by interpolation — 5.4.3.3(2) Table 5.5

$$\rho_w = \frac{\Sigma A_{sw}}{\text{area within critical perimeter} - \text{column area}}$$ — 4.3.4.5.2(4)

Denominator =

$$(300 + 3 \times 185)^2 - (1.5 \times 185)^2(4 - \pi) - 300^2 = 575000 \text{ mm}^2$$

Thus $\Sigma A_{sw,min} \geq 0.0012 \times 575000 = 690$ mm^2

Maximum spacing of links is determined by the ratio v_{Sd}/v_{Rd2} where it is assumed that v_{Rd2} is calculated in accordance with EC2 4.3.2.4.3(4). — 5.4.3.3(4) 5.4.2.2(7)

v_{Sd} = 174 N/mm

$v_{Rd2} = (\frac{1}{2})\, \nu f_{cd} b_w \times 0.9d$ — Eqn 4.25

$\nu = 0.7 - \frac{f_{ck}}{200} = 0.55$ — Eqn 4.21

$f_{cd} = \frac{30}{1.5} = 20$ N/mm^2 — 2.3.3.2 Table 2.3

Therefore

$v_{Rd2} = (\frac{1}{2}) \times 0.55 \times 20 \times 0.9 \times 185 = 916$ N/mm

$v_{Sd}/v_{Rd2} = 174/916 = 0.19 \leq 0.2$

$s_{max} = 0.8d \not> 300$ mm — Eqn 5.17

Longitudinal spacing $\not> 0.75d$ = 138 mm — NAD 6.5(f)
Transverse spacing $\not> d$ — 5.4.2.2(9)

Placing shear links on 100 mm grid in 700 mm square gives 48 links with 44 inside the critical perimeter. — 4.3.4.5.2(2)

By inspection the minimum preferred bar size will govern and mild steel links could be used.

f_{yk} = 250 N/mm^2

$\Sigma A_{sw} \geq 0.0022 \times 575000 = 1265$ mm^2 — Table 5.5

Use 44 R8 links (2220 mm^2)

Where necessary the punching shear resistance outside the shear reinforced area should be checked by considering further critical perimeters. — 4.3.4.5.2(3)

Check where

$v_{Sd} = v_{Rd1} = 133$ N/mm

Hence

$$u = \frac{V_{Sd}\beta}{v_{Rd1}} = \frac{444.7 \times 10^3 \times 1.15}{133} = 3845 \text{ mm}$$

Therefore distance from column face

$= (3845 - 1200)/2\pi = 420 \text{ mm} = 2.27d$

This would be approximately at the next critical perimeter taken to be at a distance $0.75d$ beyond the previous one. No further shear reinforcement required. — BS 8110 Figure 3.17

The tensile reinforcement (T16 @ 150 mm crs.) should extend for a full anchorage length beyond the perimeter at 420 mm from the column face.

Column A/1 (300 mm × 300 mm corner column)

Critical perimeter located at 1.5*d* from face of column (see Figure 4.18). 4.3.4.1(3)

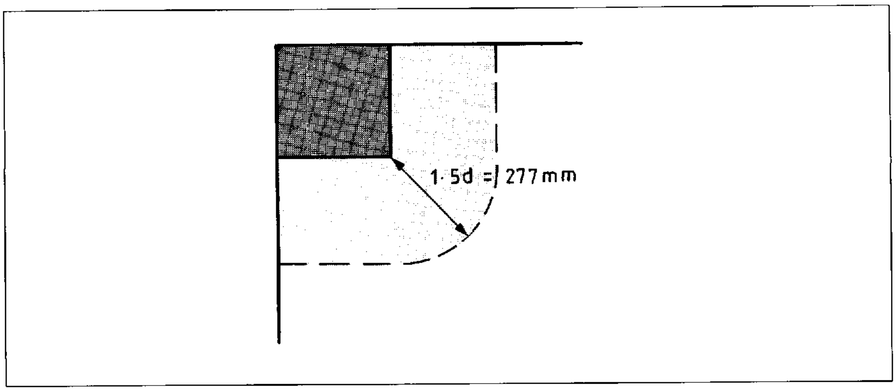


Figure 4.18 Critical perimeter at corner column

$u = 600 + 277\pi/2 = 1035$ mm

$V_{Sd} = 0.41 \times 156.4 = 64.1$ kN

Applied shear per unit length, with $\beta = 1.5$ 4.3.4.3(4) Figure 4.21

$$v_{Sd} = \frac{V_{Sd}\beta}{u} = \frac{64.1 \times 10^3 \times 1.5}{1035} = 93 \text{ N/mm}$$ Eqn 4.50

Reinforcement within zone 1.5*d* from column face is T16 @ 150 mm crs. top each way (see Figure 4.19).

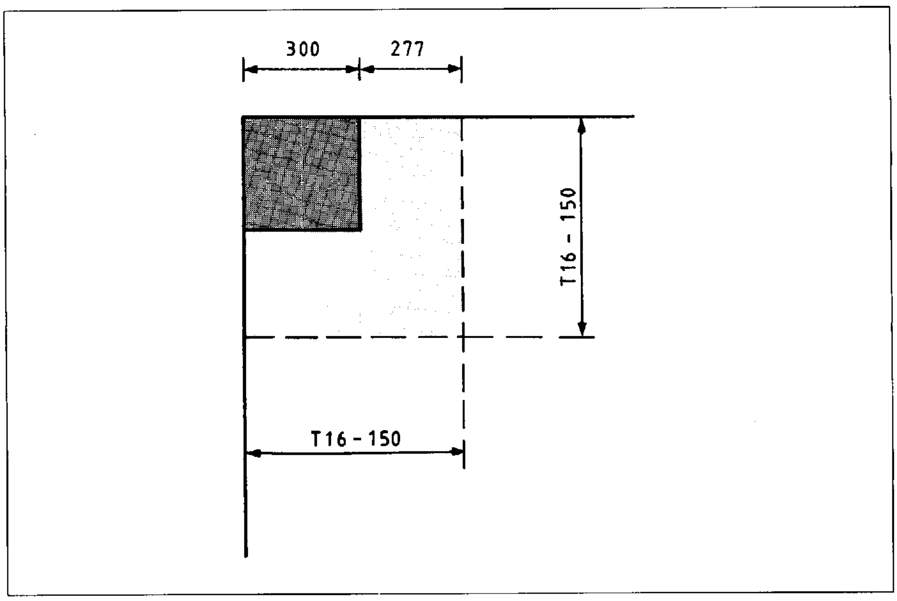


Figure 4.19 Corner column detail

$v_{Rd1} = 133$ N/mm (as for column B/2)

$v_{Rd1} > v_{Sd}$

Therefore no shear reinforcement required

4.3.4.3(2)

Column A/2 (300 mm × 300 mm edge column)

Critical perimeter located at 1.5*d* from face of column.

$$u = 900 + 277\pi = 1770 \text{ mm}$$

$$V_{Sd} = 156.4 \text{ kN}$$

Applied shear per unit length, with $\beta = 1.4$ — **Figure 4.21**

$$v_{Sd} = \frac{V_{Sd}\beta}{u} = \frac{156.4 \times 10^3 \times 1.4}{1770} = 124 \text{ N/mm}$$ **Eqn 4.50**

$$v_{Rd1} = 133 \text{ N/mm (as for column B/2)}$$

$$v_{Rd1} > v_{Sd}$$

Therefore no shear reinforcement required

4.3.4.3(2)

4.2.1.1.7 Deflection

Control by limiting span/effective depth ratio using NAD Table 7. — **4.4.3.2**

For flat slabs the check should be carried out on the basis of the longer span. — **4.4.3.2(5)(d)**

For span < 8.5 m, no amendment to basic span/effective depth ratio is required. — **4.4.3.2(3)**

Note 2 to NAD Table 7 states that modifications to the tabulated values for nominally reinforced concrete should not be carried out to take into account service stresses in the steel (refer to EC2: Clause 4.4.3.2(4)). However it is assumed that correction ought to be carried out for $0.15\% \leq \rho < 0.5\%$ but that the resulting values should not exceed those tabulated in the NAD for nominally reinforced concrete.

NAD Table 7 gives basic span/effective depth ratios which are assumed to be based on $f_{yk} = 400 \text{ N/mm}^2$. — **4.4.3.2(4)**

when

$$f_{yk} = 460 \text{ N/mm}^2 \text{ and } A_{s,req} = A_{s,prov}$$

$$\text{Modification factor} = \frac{400}{f_{yk}} \times \frac{A_{s,req}}{A_{s,prov}} = 0.87$$

Basic span/effective depth ratios for flat slabs are — **NAD Table 7 & 6.4(e)**

lightly stressed ($\rho = 0.5\%$) = 30
nominally reinforced ($\rho = 0.15\%$) = 41

Span reinforcement is typically T12 @ 250 mm crs. (452 mm²/m)

$$\frac{100A_s}{bd_{min}} = \frac{100 \times 452}{1000 \times 175} = 0.26\%$$

By interpolation (ρ = 0.26%), basic span/effective depth ratio = 37.5

$$\frac{\text{max. span}}{d_{min}} = \frac{5200}{175} = 29.7 < 37.5 \times 0.87 = 32.6 \text{ .... OK}$$

4.2.1.1.8 Crack control

Use method without direct calculation. 4.4.2.3

Estimate service stress, σ_s, under quasi-permanent loads as follows: 4.4.2.3(3)

$$G_k + \psi_2 Q_k = G_k + 0.3Q_k = 6.4 + 0.3 \times 5 = 7.9 \text{ kN/m}^2$$ 2.3.4 Eqn 2.9(c)

Ratio of quasi-permanent to ultimate design loads = 7.9/16.2 = 0.49 NAD Table 1

Therefore

$$\sigma_s = 0.49 \times f_{yd} \times \frac{A_{s,req}}{A_{s,prov}} < 200 \times \frac{A_{s,req}}{A_{s,prov}}$$

Limit bar size using EC2 Table 4.11 or bar spacing using EC2 Table 4.12. The relevant limits are shown in Table 4.3.

Table 4.3 Crack control limits

	$A_{s,req}/A_{s,prov}$		
	1.0	≤0.8	
Steel stress (N/mm^2)	200	160	
Bar size (mm)	25	32	Table 4.11
Bar spacing (mm)	250	300	Table 4.12

Maximum bar size used is less than 25 mm throughout. .... OK

Check minimum reinforcement requirement 4.4.2.3(2)

$A_s \geq k_c k f_{ct,eff} A_{ct}/\sigma_s$ Eqn 4.78

$$A_{ct} = \frac{A_c}{2}$$

σ_s = 100% × f_{yk} = 460 N/mm^2

$f_{ct,eff}$ = minimum value suggested, 3 N/mm^2

k_c = 0.4, k = 0.8

Therefore

$$A_s \geq 0.4 \times 0.8 \times 3 \times \frac{A_c}{2 \times 460} = 0.001\,A_c$$

$< 0.0015\,b_t d$ (minimum flexural steel) .... OK

4.2.1.1.9 Detailing

Consider combined requirements for flexure/shear and for punching for top steel over supports.

Column B/2

For flexure/shear bars should extend for a distance $d + l_{b,net} \not< 2d$ beyond the point at which they are no longer needed ($a_l = d =$ shift in moment diagram). 5.4.2.1.3 (1)&(2) Figure 5.11 5.4.3.2.1(1)

$$l_b = \frac{\phi}{4} \times \frac{f_{yd}}{f_{bd}} \quad \text{where } f_{yd} = 400 \text{ N/mm}^2$$ Eqn 5.3

For $h \leq 250$ mm bond conditions are good and Figure 5.1

$$f_{bd} = 3.0 \text{ N/mm}^2$$ Table 5.3

Therefore

$$l_b = \frac{\phi}{4} \times \frac{400}{3} = 33.3\phi$$

$$l_{b,net} = \alpha_a l_b \frac{A_{s,req}}{A_{s,prov}}$$ 5.2.3.4.1 Eqn 5.4

For straight bars $\alpha_a = 1.0$ and if $\dfrac{A_{s,req}}{A_{s,prov}} = 1.0$

$$l_{b,net} = l_b = 534 \text{ mm for T16 bars, say 550 mm.}$$

Curtail alternate bars as shown in Figure 4.20.

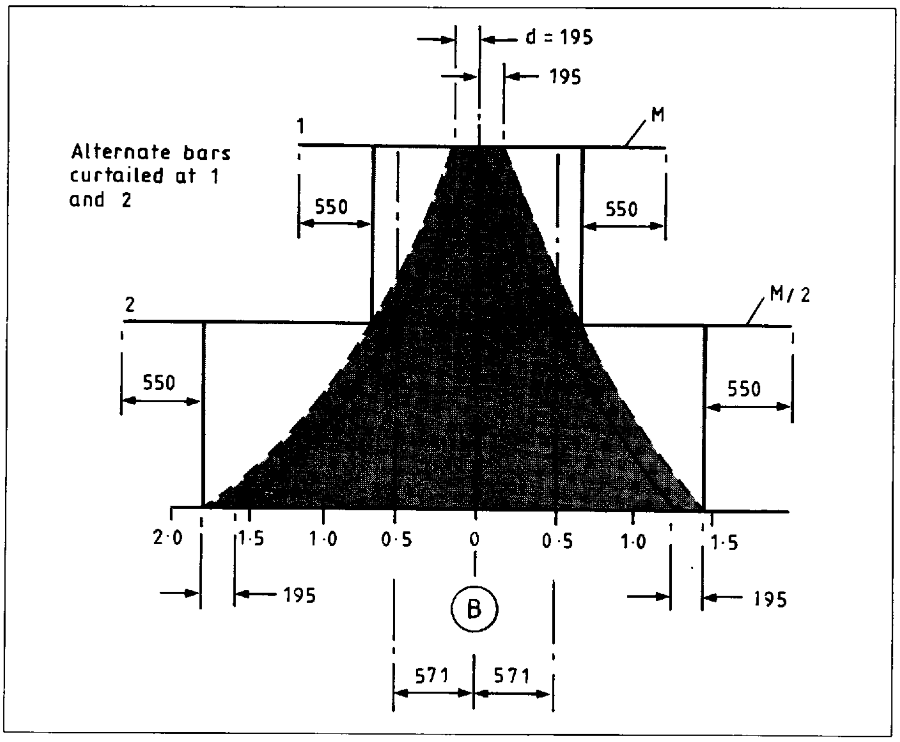


Figure 4.20 Curtailment diagram

Check that bars are anchored past relevant critical punching perimeter.

Earlier calculation required column strip reinforcement to extend beyond a perimeter 420 mm from column face i.e. 570 mm from grid. It is assumed sufficient to provide an anchorage $l_{b,net}$ beyond this perimeter. Inspection of Figure 4.20 shows that this is satisfied.

4.2.1.2 Design example of a flat slab with column heads

The previous example will be used with column heads introduced at the internal columns to avoid the need for shear reinforcement.

The rest of the design is unaffected by the change.

4.2.1.2.1 Punching at column B/2 (300 mm × 300 mm internal column)

In the previous example it was found that $v_{Sd} = v_{Rd1}$ at 420 mm from the column face where $u = 3845$ mm.

Provide a column head such that $l_H = 1.5h_H$ (see Figure 4.21). Figure 4.22

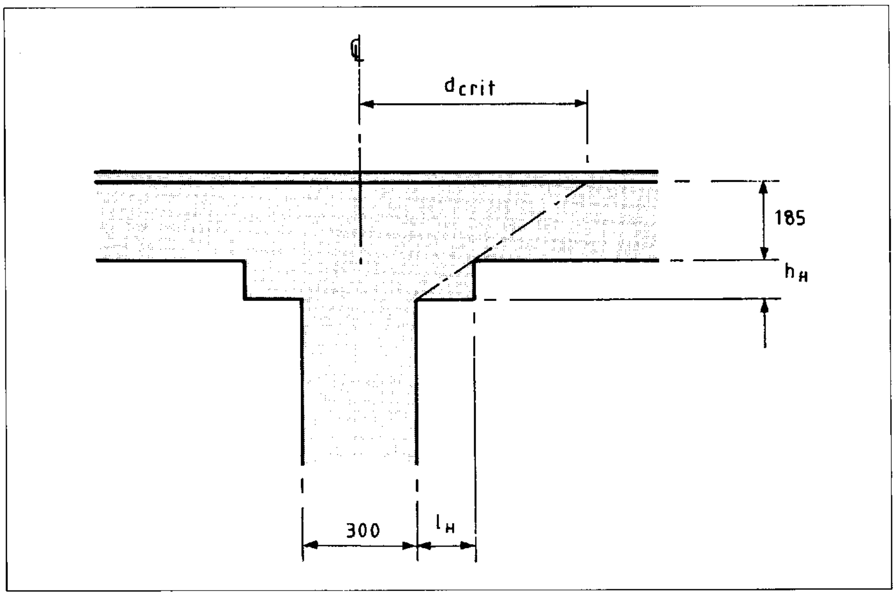


Figure 4.21 Slab with column head

For a circular column head, assume that EC2 Equation 4.51 applies to the case where $l_H = 1.5h_H$. 4.3.4.4(1)

Note:
It is suggested that EC2 Equation 4.55 should read $d_{crit} = 1.5d_H + 0.5l_c$, which reduces to the same as Equation 4.51 when $l_H = 1.5h_H$.

Assume an effective column diameter, $l_c = 300$ mm

To avoid shear reinforcement:

$$2\pi d_{crit} \geq 3845 \text{ mm}$$

$$d_{crit} \geq 612 \text{ mm}$$

$$l_H \geq 612 - 1.5d - 0.5l_c = 612 - 1.5(185) - 150 = 185 \text{ mm}$$ Eqn 4.51

$$h_H \geq \frac{185}{1.5} = 123 \text{ mm say } 125 \text{ mm}$$

$$l_c + 2l_H = 670 \text{ mm}$$

Circular column head 125 mm below slab and 670 mm diameter is sufficient to avoid shear reinforcement

If a square column head is preferred, $l_1 = l_2 = l_c + 2l_H$

$$d_{crit} = 1.5d + 0.56\sqrt{(l_1\, l_2)} \leq 1.5d + 0.69l_1$$ Eqn 4.52

$$= 1.5d + 0.56l_c + 1.12l_H$$

To avoid shear reinforcement

$$d_{crit} \geq 612 \text{ mm}$$

$$l_H \geq \frac{1}{1.12}(612 - 1.5d - 0.56l_c)$$

$$= \frac{1}{1.12}(612 - 1.5 \times 185 - 0.56 \times 300) = 149 \text{ mm}$$

$$h_H \geq \frac{149}{1.5} = 100 \text{ mm}$$

$$l_c + 2l_H = 600 \text{ mm}$$

Square column head 100 mm below slab and 600 mm wide is sufficient to avoid shear reinforcement

4.2.2 Flat slabs in laterally loaded frames

In the following example, the structure used in Section 4.2.1 is considered to be unbraced in the North–South direction.

4.2.2.1 Design example of an unbraced flat slab frame

This example considers only the analysis of the frame on grid B, consisting of three upper storeys plus a lightweight roof structure, as shown in Figures 4.22 and 4.23.

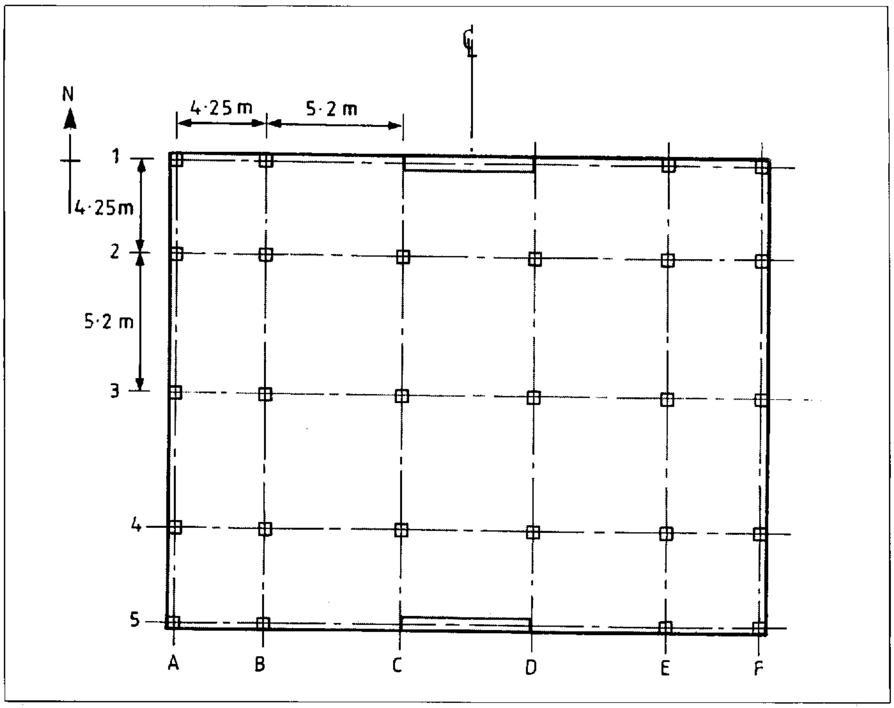


Figure 4.22 Plan of structure

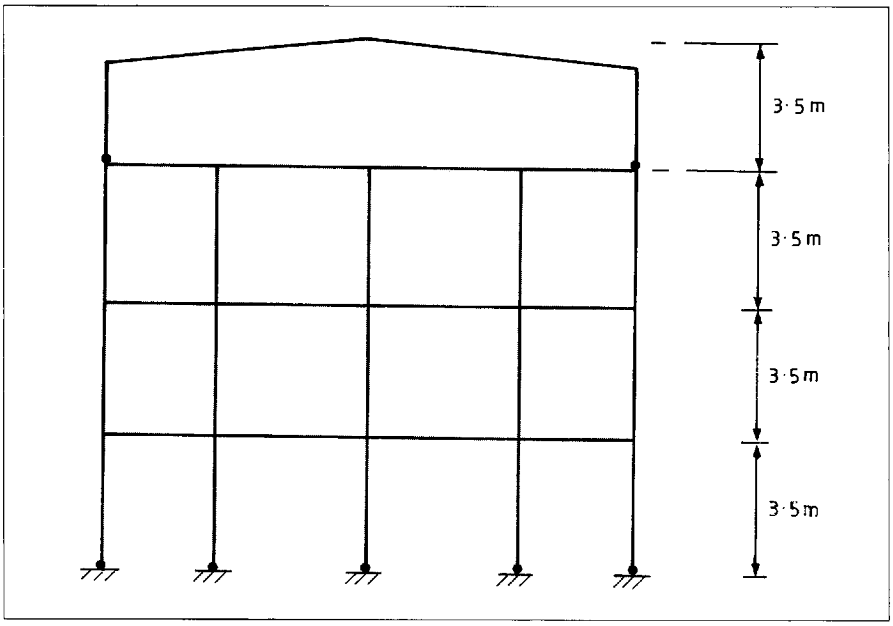


Figure 4.23 Frame on grid B

4.2.2.1.1 Design loads

Office floors

Dead load = 6.4 kN/m^2
Imposed load = 5.0 kN/m^2

Roof imparts load to columns B/1 and B/5

Dead load = 20 kN
Imposed load = 30 kN

Assume characteristic wind load = 1.0 kN/m^2

This is 90% of the value obtained from CP3: Chapter V: Part 2[11]. NAD 4(c)

Note:
The distribution of horizontal load between each frame is determined by their relative stiffness.

4.2.2.1.2 Frame classification

Determine whether sway frame or non-sway frame. **4.3.5.3.3 (1)&(3)**

Check slenderness ratio of columns in the frame.

A calculation is required for those columns that resist more than 0.7 of the mean axial load, $N_{Sd,m}$, at any level. Service loads are used throughout (i.e. γ_F = 1.0). **A.3.2** **A.3.2(3)**

It is also assumed that these are vertical loads without any lateral loads applied. **Figure A3.4**

$$N_{Sd,m} = \frac{\gamma_F F_v}{n}$$

F_v = Σ all vertical loads at given level (under service condition) **A.3.2(1)**

n = Number of columns

Consider a simple analytical model of the top floor to determine columns concerned as shown in Figure 4.24.

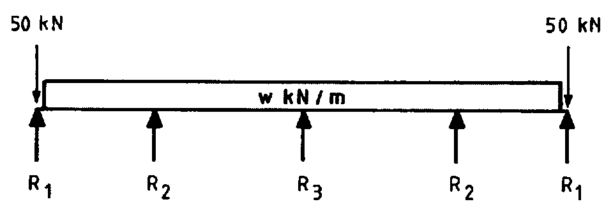


Figure 4.24 Load arrangement at third floor

w = 11.4 kN/m^2 × loaded width

= 11.4 × 5.2 (determined in Section **4.2.1.1**) = 59.3 kN/m

R_1 = 3 × 59.3 × 4.25/8 + 50 = 145 kN

R_2 = 5 × 59.3 × 4.25/8 + (59.3 × 5.2/2) = 312 kN

R_3 = 59.3 × 5.2 = 308 kN

$$N_{Sd,m} = \frac{100 + (18.9 \times 59.3)}{5} = 244 \text{ kN}$$

$0.7N_{sd,m}$ = 171 kN > R_1 = 145 kN

Therefore slenderness of internal columns only needs to be checked.

Clearly, this will normally be the case for multi-bay frames unless the edge columns carry large cladding loads.

λ = l_o/i where **4.3.5.3.5(2)**

$$i = \sqrt{\frac{I_{col}}{A}} = \frac{300}{\sqrt{12}} = 86.6 \text{ mm}$$

For a horizontally loaded flat slab frame determine the stiffnesses of the frame and thus the effective lengths of the columns using half the slab stiffness. Consider the centre column from foundation to first floor.

$$k_A = \frac{\Sigma I_{col}/l_{col}}{\Sigma \alpha I_b/l_{eff}} \text{ ; } E_{cm} \text{ assumed constant}$$ 4.3.5.3.5(1) Eqn 4.60

$$I_{col} = \frac{300^4}{12} = 0.675 \times 10^9 \text{ mm}^4$$

$$I_b = \frac{4725 \times 225^3}{2 \times 12} = 2.24 \times 10^9 \text{ mm}^4$$

$$l_{col} = 3500 \text{ mm}$$

$$l_{eff} = 5200 \text{ mm}$$ 2.5.2.2.2

$$\alpha = 1.0$$

Therefore

$$k_A = \frac{2(0.675 \times 10^9/3500)}{2(2.24 \times 10^9/5200)} = 0.5$$

$$k_B = \infty \text{ (pinned at foundation)}$$

Assuming that EC2 Figure 4.27(b) is appropriate to determine β

$$l_o = \beta l_{col} = 2.15 \times 3500 = 7500 \text{ mm}$$ Figure 4.27

Hence

$$\lambda = 7500/86.6 = 87$$

For non-sway frames A.3.2(3)

$$\lambda \leq \lambda_{lim} = \frac{15}{\sqrt{\nu_u}} \not< 25$$ 4.3.5.3.5(2)

$$\nu_u = \frac{N_{Sd}}{A_c f_{cd}}$$

Ultimate design load for centre column, ignoring self-weight of column.

$$N_{Sd} = 3 \times 16.2 \times 5.2^2 = 1314 \text{ kN}$$

$$f_{cd} = \frac{f_{ck}}{\gamma_c} = \frac{30}{1.5} = 20 \text{ N/mm}^2$$ Eqn 4.4 Table 2.3

Therefore

$$\nu_u = \frac{1314 \times 10^3}{300^2 \times 20} = 0.73$$

$$\lambda_{lim} = \frac{15}{\sqrt{\nu_u}} = 17.6 \not< 25$$ 4.3.5.3.5(2)

Since $\lambda > 25$ the structure is classified as a sway frame

The analysis and design would need to follow the requirements of EC2 Clause A.3.5 to take into account the sway effects.

EC2 Clause 2.5.3.4.2(4) does not generally allow redistribution in sway frames.

The method above is included to demonstrate its complexity. However, note the omission of guidance in EC2 Clause A.3.2(3) on which nomogram to use in EC2 Figure 4.27.

As an alternative means of determining the frame classification, it is suggested that an analysis as detailed in BS 5950[14] is used to demonstrate that the EC2 requirements are met for non-sway frames. **4.3.5.3.3(3) BS 5950: Part 1 5.1.3**

Assuming in the above example that the column sizes are increased such that a non-sway frame results, the following load cases need to be considered for design.

These same load cases would also be applicable to sway frames where amplified horizontal loads are introduced to take account of the sway induced forces, complying with EC2 Clause A.3.1(7) (b).

4.2.2.1.3 Load cases and combinations **2.5.1.2**

With the rigorous approach the design values are given by **2.3.2.2 P(2)**

$$\Sigma\gamma_G G_k + \gamma_{Q,1} Q_{k,1} + \sum_{i>1} \gamma_{Q,i} \psi_{o,i} Q_{k,i}$$ **Eqn 2.7(a)**

where

$Q_{k,1}$ = primary variable load, $Q_{k,2}$ = secondary variable load

ψ_o = 0.7 generally **NAD Table 1**

The γ_F values are given in EC2 Table 2.2.

Load cases with two variable actions (imposed and wind) are:

(a) Imposed load as primary load
$1.35G_k + 1.5Q_k + 1.05W_k$

(b) Wind load as primary load
$1.35G_k + 1.05Q_k + 1.5W_k$

In addition, load cases with only one variable action are:

(c) Dead load plus wind
$1.0G_k$ (favourable) $+ 1.5W_k$
$1.35G_k$ (unfavourable) $+ 1.5W_k$

(d) Dead load plus imposed
$1.35G_k + 1.5Q_k$

For non-sensitive structures it is sufficient to consider the load cases (a) and (b) above without patterning the imposed loads. **NAD 6.2(e) 2.5.1.2P(1)**

The NAD allows the use of EC2 Equation 2.8(b) to give a single imposed and wind load case:

$1.35G_k + 1.35Q_k$ (all spans) $+ 1.35W_k$

> Final load combinations for the example given here
>
> (i) $1.35G_k + 1.5Q_k$ (as Section **4.2.1.1.3**)
> (ii) $1.0G_k + 1.5W_k$ (single load case)
> (iii) $1.35G_k + 1.5W_k$ (single load case)
> (iv) $1.35G_k + 1.35Q_k + 1.35W_k$ (single load case)

4.2.2.1.4 Imperfections

2.5.1.3(4)

Consider the structure to be inclined at angle

$$\nu = \frac{1}{100\sqrt{l}} \geq 0.005 \text{ radians}$$

Eqn 2.10
NAD
Table 3

l = frame height = 10.5 m

$$\alpha_n = \sqrt{\frac{1}{2}\left(1 + \frac{1}{n}\right)} \quad \text{where n = number of columns = 5}$$

Eqn 2.11

$$= 0.78$$

$$\nu_{red} = \alpha_n \nu = 0.78 \times 0.005 = 0.0039 \text{ radians}$$

Take account of imperfections using equivalent horizontal force at each floor. 2.5.1.3(6)

$$\Delta H_j = \Sigma V_j \nu_{red}$$

ΣV_j = total load on frame on floor j

Using $1.35G_k + 1.5Q_k$ on each span gives

$$\Sigma V_j = (18.9 \times 5.2) \times 16.2 = 1592 \text{ kN}$$

Therefore

$$\Delta H_j = 1592 \times 0.0039 = 6.2 \text{ kN per floor}$$

Assuming the frame by virtue of its relative stiffness picks up 4.725 m width of wind load:

$$W_k = (4.725 \times 3.5) \times 1.0 = 16.5 \text{ kN per floor}$$

> Therefore the effects of imperfections are smaller than the effects of design horizontal loads and their influence may be ignored in load combinations (ii) to (iv).

2.5.1.3(8)

4.2.2.1.5 Design

The design of the slab will be as described in Section **4.2.1.1**.

5 COLUMNS

5.1 Introduction

The design of column sections from first principles using the strain compatibility method is covered.

Examples of slender column design are also presented to extend the single example given in Section **2**.

5.2 Capacity check of a section by strain compatibility

5.2.1 Introduction

Two examples are considered:

1. Where the neutral axis at ultimate limit state lies within the section; and
2. Where the neutral axis at ultimate limit state lies outside the section.

The first of these is very simple while the algebra necessary for the second is more complex. For convenience, the same section will be used for both examples. This is shown in Figure 5.1.

Assume

f_{yk} = 460 N/mm^2 and f_{ck} = 30 N/mm^2

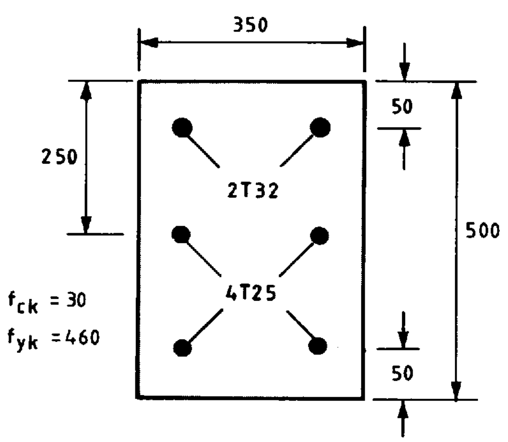


Figure 5.1 Column section

5.2.2 Example 1

Calculate the moment that the section can sustain when combined with an axial load of 2750 kN.

5.2.2.1 Basic method

If the neutral axis is within the section, the compressive force generated by the concrete at ultimate limit state is given by 4.2.1.3.3 Figure 4.2

$$N_{Rd,c} = 0.459 f_{ck} bx$$

and the moment by

$$M_{Rd,c} = N_{Rd,c}(h/2 - 0.416x)$$

The strain at the more compressed face is taken as 0.0035

The procedure adopted is as follows:

(1) Assume a value for x

(2) Calculate $N_{Rd,c}$

(3) Calculate the strain at each steel level

(4) Calculate force generated by reinforcement ($N_{Rd,s}$)

(5) $N_{Rd} = N_{Rd,c} + N_{Rd,s}$

(6) If N_{Rd} is not close enough to 2750 kN, modify the value of x and return to step (2)

(7) If N_{Rd} is approximately 2750 kN, calculate $M_{Rd,c}$ and $M_{Rd,s}$

(8) $M_{Rd} = M_{Rd,c} + M_{Rd,s}$

The design yield strain for the reinforcement

$$= \frac{460}{1.15 \times 200000} = 0.002$$

5.2.2.2 First iteration

Assumed value for x is 250 mm

$$N_{Rd,c} = 0.459 \times 30 \times 350 \times 250/1000 = 1205 \text{ kN}$$

$$\epsilon_{s,top} = \frac{0.0035}{250} \times 200 = 0.0028$$

Strain > 0.002; therefore $f_s = 400 \text{ N/mm}^2$

$$N_{Rd,s1} = 2 \times 804 \times 400/1000 = 643 \text{ kN}$$

$$\epsilon_{s,mid} = 0 \text{ and } N_{Rd,s2} = 0$$

$$\epsilon_{s,bot} = -\epsilon_{s,top}; \text{ therefore } f_s = -400 \text{ N/mm}^2$$

$$N_{Rd,s3} = -2 \times 491 \times 400/1000 = -393 \text{ kN}$$

Hence

$$N_{Rd} = 1205 + 643 - 393 = 1455 \text{ kN}$$

This is considerably less than 2750 kN, hence x must be increased.

$$\text{Try new value for } x = \frac{250 \times 2750}{1455} = 473 \text{ mm}$$

5.2.2.3 Second iteration

$$N_{Rd.c} = 0.459 \times 30 \times 350 \times 473/1000 = 2289 \text{ kN}$$

$$N_{Rd.s1} = 643 \text{ kN as before}$$

$$\epsilon_{s.mid} = \frac{0.0035}{473}(473 - 250) = 0.00165$$

$$f_{s.mid} = 0.00165 \times 200000 = 330 \text{ N/mm}^2$$

$$N_{Rd.s2} = 330 \times 2 \times 491/1000 = 324 \text{ kN}$$

$$\epsilon_{s.bot} = \frac{0.0035}{473}(473 - 450) = 0.00017$$

$$f_s = 0.00017 \times 200000 = 34 \text{ N/mm}^2$$

$$N_{Rd.s3} = 34 \times 2 \times 491/1000 = 33 \text{ kN}$$

Hence

$$N_{Rd} = 2289 + 643 + 324 + 33 = 3289 \text{ kN}$$

This is too large, hence x should be reduced. Linear interpolation gives

$$x = 250 + \left(\frac{2750 - 1455}{3289 - 1455}\right)(473 - 250) = 407 \text{ mm}$$

5.2.2.4 Third iteration

$$N_{Rd.c} = 0.459 \times 30 \times 350 \times 407/1000 = 1961 \text{ kN}$$

$$N_{Rd.s1} = 643 \text{ kN as before}$$

$$\epsilon_{s.mid} = \frac{0.0035}{407}(407 - 250) = 0.00135$$

$$f_s = 270 \text{ N/mm}^2 \text{ and } N_{Rd.s2} = 265 \text{ kN}$$

$$\epsilon_{s.bot} = \frac{0.0035}{407}(407 - 450) = -0.00037$$

$$f_s = -74 \text{ N/mm}^2 \text{ and } N_{Rd.s3} = -73 \text{ kN}$$

Hence

$$N_{Rd} = 1961 + 643 + 265 - 73 = 2796 \text{ kN}$$

This is within 2% of the given axial load of 2750 kN .... OK

5.2.2.5 Moment

$M_{Rd,c} = 1961 \times (250 - 0.416 \times 407)/1000 = 158.2$ kNm

$M_{Rd,s1} = 643 \times 0.2 = 128.6$ kNm

$M_{Rd,s2} = 0$

$M_{Rd,s3} = 73 \times 0.2 = 14.6$ kNm

$M_{Rd} = 158.2 + 128.6 + 14.6 = 301.4$ kNm

5.2.3 Example 2

Calculate the moment and axial force that can be sustained by the section where the neutral axis depth is 600 mm.

Note:
The example has been given in this way so that repeated iterations are not necessary. These would not provide any new information to the reader.

5.2.3.1 Basic method

When the neutral axis is outside the section the ultimate compressive strain is less than 0.0035 and is given by:

$$\epsilon_u = \frac{0.002x}{x - 3\,h/7} = \frac{0.002 \times 600}{600 - 3 \times 500/7} = 0.0031$$

4.3.1.2(1) (viii) & Figure 4.11

The conditions in the section are shown in Figure 5.2.

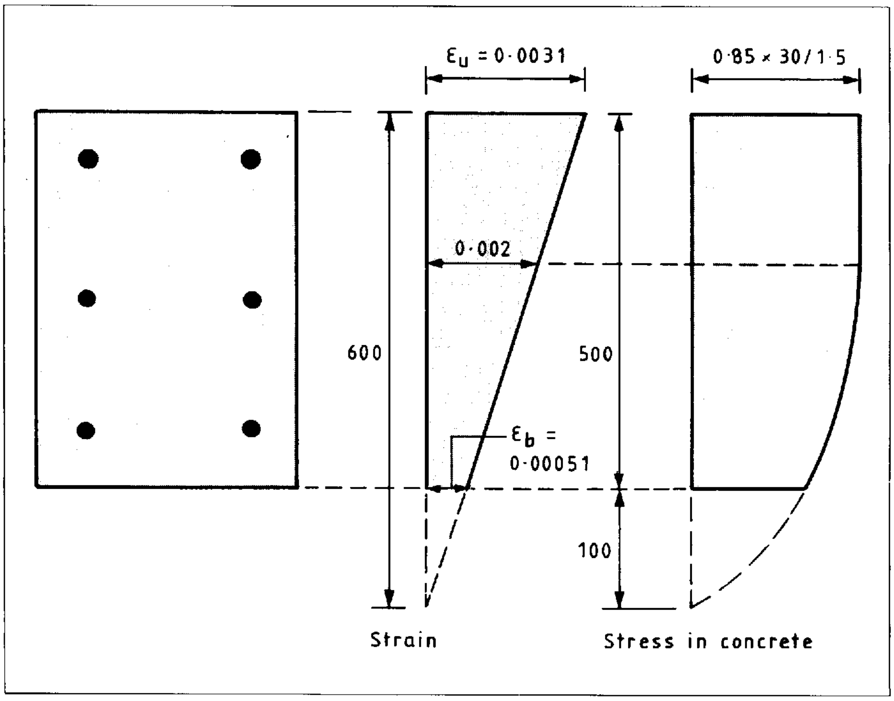


Figure 5.2 Conditions in section for Example 2

The technique adopted for the calculation of $N_{Rd,c}$ and $M_{Rd,c}$ is to calculate the effect of the stress block on a depth of 600 mm and then deduct the influence of the part lying outside the section.

5.2.3.2 Concrete forces and moments

The equations for the full stress block are:

$$N'_{Rd,c} = 0.5667(1 - \beta/3)bxf_{ck}$$

$$M'_{Rd,c} = cN'_{Rd,c}$$

where

$$c = h/2 - \frac{x(\beta^2 - 4\beta + 6)}{12 - 4\beta} \text{ ; and}$$

$$\beta = 0.002/\epsilon_u$$

Note:
It will be found that, if $\epsilon_u = 0.0035$, these equations give the values used in the first example.

The equations for the force and moment produced by the part of the stress block lying outside the section are

$$\Delta N_{Rd,c} = 0.5667\alpha(1 - \alpha/3)(x - h)bf_{ck}$$

$$\Delta M_{Rd,c} = \Delta N_{Rd,c}c'$$

where

$$c' = -\left[x - h/2 - \frac{(x - h)(8 - 3\alpha)}{12 - 4\alpha}\right]$$

$$\alpha = \epsilon_b/0.002$$

$$\epsilon_b = \text{strain at bottom of section}$$

From the strain diagram, $\epsilon_b = 0.00051$

Hence

$$\alpha = 0.255 \text{ and } \beta = 0.645$$

$$N'_{Rd,c} = 0.5667(1 - 0.645/3) \times 350 \times 600 \times 30/1000 = 2802 \text{ kN}$$

$$c = 250 - \frac{600(0.645^2 - 4 \times 0.645 + 6)}{12 - 4 \times 0.645} = 5.67 \text{ mm}$$

Hence

$$M'_{Rd,c} = 5.67 \times 2802/1000 = 15.9 \text{ kNm}$$

$$\Delta N_{Rd,c} = 0.5667 \times 0.255(1-0.255/3)(600-500)350 \times 30/1000 = 139 \text{ kN}$$

$$c' = -\left[600 - 250 - \frac{(600 - 500)(8 - 3 \times 0.255)}{12 - 4 \times 0.255}\right] = -284 \text{ mm}$$

$\Delta M_{Rd,c} = -139 \times 284/1000 = -39.4$ kNm

Hence

$N_{Rd,c} = 2802 - 139 = 2663$ kN

$M_{Rd,c} = 15.9 + 39.4 = 55.3$ kNm

5.2.3.3 Steel forces and moments

Strain in upper layer of bars $= \dfrac{0.0031}{600} \times 550 = 0.0028$

This is > 0.002; hence $f_s = 400$ N/mm^2

$N_{Rd,s1} = 643$ kN

$M_{Rd,s1} = 643 \times 0.2 = 128$ kNm

Strain in middle layer of bars $= \dfrac{0.0031}{600} \times 350 = 0.00181$

Hence

$f_s = 362$ N/mm^2

$N_{Rd,s2} = 355$ kN, $M_{Rd,s2} = 0$

Strain in bottom layer of bars $= \dfrac{0.0031}{600} \times 150 = 0.000775$

Hence

$f_s = 155$ N/mm^2

$N_{Rd,s3} = 152$ kN

$M_{Rd,s3} = -30.4$ kNm

$N_{Rd} = 2663 + 643 + 355 + 152 = 3813$ kN

$M_{Rd} = 55.3 + 128 - 30.4 = 153$ kNm

5.3 Biaxial bending capacity of a section

5.3.1 General

To carry out a rigorous check of a section for biaxial bending by hand is very tedious but possible if the simplified rectangular stress block is used. It is not suggested that the example given here is a normal design procedure for common use but it could be employed in special circumstances. There would be no difficulty in developing an interactive computer program to carry out design, in this way, by trial and error.

5.3.2 Problem

Demonstrate that the section shown in Figure 5.3 can carry ultimate design moments of 540 and 320 kNm about the two principal axes in combination with an axial load of 3000 kN. The characteristic strength of the reinforcement is 460 N/mm^2 and the concrete strength is 30 N/mm^2.

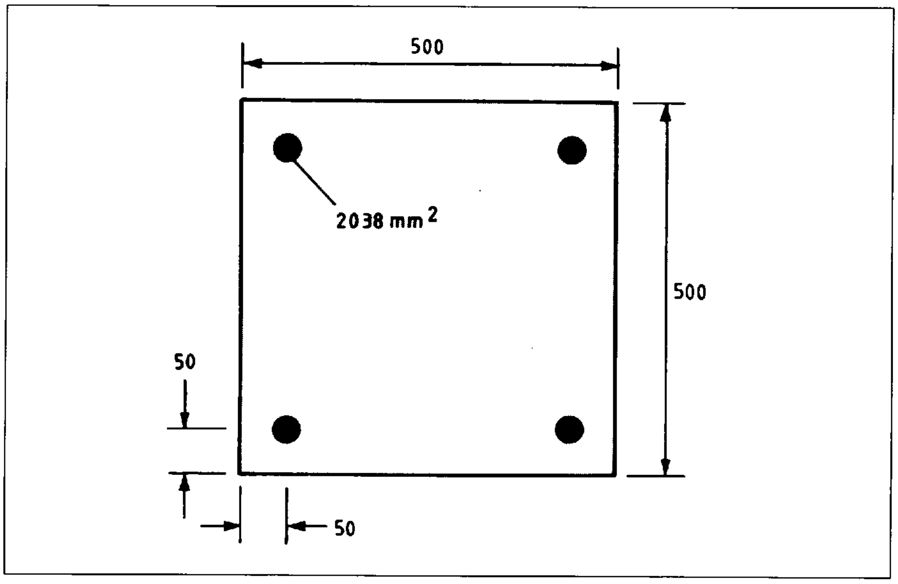


Figure 5.3 Column section

5.3.3 Basic method

4.2.1.3.3(12)

The conditions in the section are shown in Figure 5.4.

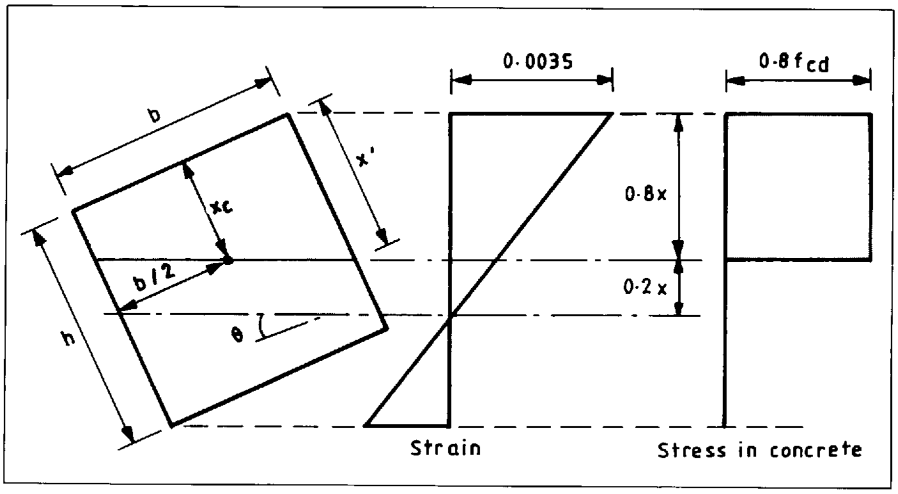


Figure 5.4 Conditions in section

Note:
It is assumed that EC2[1] Section 4.2.1.3.3(12) implies that α should be taken as 0.8 for biaxial bending but the NAD[1] would allow 0.85.

NAD Table 3

It can be seen from the diagram that the axial force provided by the concrete is given by

$$N_c = 0.8bx_c f_{cd}$$

The moments about the centroid of the concrete section are given by

$$M_{cx} = N_c\bar{x}$$

where

$$\bar{x} = \frac{h}{2} - \frac{1}{2x_c}\left[\left(x_c - \frac{b}{2}\tan\Theta\right)^2 + b\tan\Theta\left(x_c - \frac{b\tan\Theta}{6}\right)\right]$$

$$M_{cy} = \frac{0.8f_{cd}b^3\tan\Theta}{12}$$

These equations are valid where $x' < h$. When $x' > h$, rather simpler equations can be derived.

The location of the reinforcement is shown in Figure 5.5.

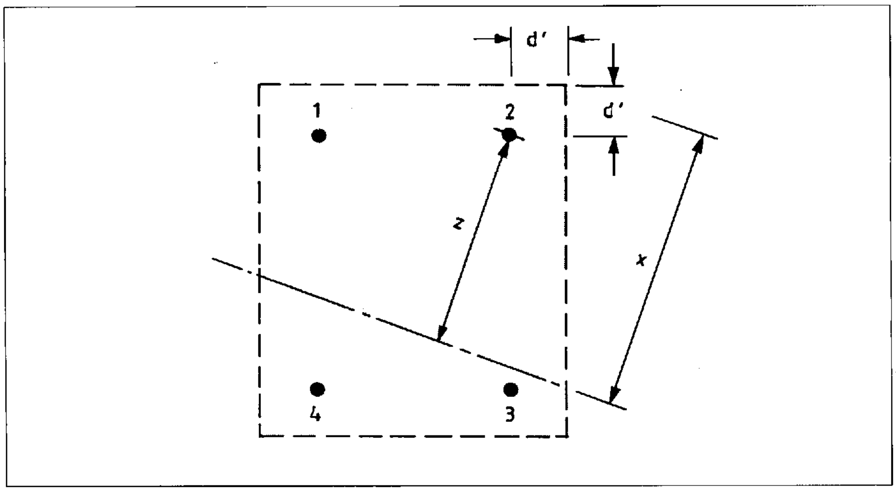


Figure 5.5 Location of reinforcement

The stress in a bar is given by

$$f_s = \left(\frac{200000 \times 0.0035}{x}\right)z = \frac{700z}{x} \leq f_{yd}$$

where

$$z = \left[\frac{x_c}{0.8} \pm (b/2 - d')\tan\Theta - d_b\right]\cos\Theta$$

d_b = depth from top face of section to bar considered. This will be d' for top bars and $h - d'$ for bottom bars.

The force in each bar is f_sA_s and the moments are obtained by multiplying the forces by the distance of the bars from the centroid of the concrete section. Dimensions to the right or upwards are taken as positive.

The total moments and forces carried by the section are the sum of the steel and concrete contributions.

The correct values of x and Θ have to be found by iteration.

5.3.4 Initial data

$$f_{cd} = \frac{f_{ck}}{1.5} = \frac{30}{1.5} = 20\ \text{N/mm}^2$$

Stress over upper 0.8 of the depth of the compression zone

$$= 0.8f_{cd} = 16 \text{ N/mm}^2$$

$$f_{yd} = \frac{f_{yk}}{1.15} = 400 \text{ N/mm}^2$$

As a first estimate of Θ, assume that the neutral axis is perpendicular to the direction of principal bending. This gives

$$\Theta = \tan^{-1}\left(\frac{320}{540}\right) = \tan^{-1}0.59$$

Try Θ = 30° which gives

$$\tan\Theta = 0.58 \text{ and } \cos\Theta = 0.87$$

The limiting value of x_c is where $x' = h$

Hence

$$x_{c,max} = h - (b/2)\tan\Theta$$

$$= 500 - 250 \times 0.58 = 355 \text{ mm}$$

This gives

$$N_c = \frac{355 \times 500 \times 16}{1000} = 2840 \text{ kN}$$

The reinforcement will increase this value significantly, hence x_c will be less than 355 mm. Try x_c = 300 mm.

5.3.5 Calculation

The simplest way to carry out the calculation is by writing the equations into a spreadsheet and then adjusting the values of x_c and Θ until the correct axial load and ratio M_x/M_y is obtained. The resulting output for the final iteration is given below. It will be seen that the result is satisfactory.

Section breadth (b)		500				
Overall depth (h)		500				
Embedment (d')		50				
Steel area		8152				
Concrete strength		30		Average stress		16
Steel strength		460		Design stress		400
Estimate of angle		34.2°				
(radians)		0.5969026				
Estimate of x_c		282.5		Neutral axis depth		432.58
Tan (angle)		0.6795993		Lever arm (x)		91.72
Cos (angle)		0.8270806				
Bar no.	z	f	F	M_x	M_y	
1	138.29	223.78	456.07	91.21	−91.21	
2	363.13	400.00	815.20	163.04	163.04	
3	32.29	52.26	106.50	−21.30	21.30	
4	−192.54	−311.57	−634.97	126.99	126.99	
Steel totals			742.80	359.95	220.12	
Concrete			2260.00	207.29	113.27	
Design resistances			N_{Sd}	M_x	M_y	M_x/M_y
			3002.80	567.23	333.39	1.701432

5.4 Braced slender column

5.4.1 General

The calculation of the effective length of columns has been adequately covered in Section 2. In the following example, the effective length is assumed.

5.4.2 Problem

Calculate the reinforcement required in a 400 mm × 400 mm column subjected to a design axial load of 2500 kN combined with the first order bending moments shown in Figure 5.6.

The effective length has been calculated as 8.8 m. 4.3.5.3.5

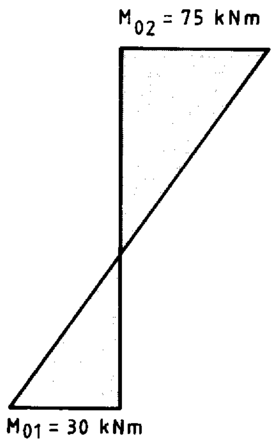


Figure 5.6 First order moments

Assume

f_{yk} = 460 N/mm^2 and f_{ck} = 30 N/mm^2

5.4.3 Slenderness ratio 4.3.5.3.2

$$\lambda = l_o/i = (l_o/h)\sqrt{12}$$

$$= (8800/400)\sqrt{12} = 76.2$$

5.4.4 Design requirements for slenderness

Minimum slenderness ratio = greater of 25 or $15/\sqrt{\nu_u}$ 4.3.5.3.5(2)

$$\nu_u = N_{Sd}/(A_c f_{cd})$$

$$= 2500 \times 10^3/(400 \times 400 \times 30/1.5) = 0.78$$

Hence $15/\sqrt{\nu_u}$ = 17.0 < 25

Therefore minimum slenderness ratio = 25

Slenderness ratio > 25, therefore column is slender

Critical slenderness ratio $\lambda_{crit} = 25(2 - e_{o1}/e_{o2})$ **4.3.5.5.3(2)**

$e_{o1} = -30 \times 10^6/(2500 \times 10^3) = -12$ mm

$e_{o2} = 75 \times 10^6/(2500 \times 10^3) = 30$ mm

Hence

$\lambda_{crit} = 25(2 + 12/30) = 60$

Slenderness ratio > λ_{crit}, therefore design is required for second order effects

5.4.5 Eccentricities

Additional eccentricity **4.3.5.4(3)**

$e_a = \nu l_o/2$

$\nu = 1/200$ **2.5.1.3(4) Eqn 2.10**

$e_a = 8800/400 = 22$ mm

Equivalent first order eccentricity is greater of **4.3.5.6.2**

$0.6e_{o2} + 0.4e_{o1} = 0.6 \times 30 - 0.4 \times 12 = 13.2$ mm ; or

$0.4e_{o2} = 0.4 \times 30 = 12.0$ mm

Hence

$e_e = 13.2$ mm

Ultimate curvature, $1/r = 2K_2\epsilon_{yd}/0.9d$ **4.3.5.6.3(5) Eqn 4.72**

Assume $d = 400 - 60 = 340$ mm

$$1/r = \frac{2 \times 460 \times K_2}{200000 \times 1.15 \times 0.9 \times 340} = 13.07K_2 \times 10^{-6} \text{ radians}$$

Second order eccentricity

$e_2 = 0.1K_1 l_o^2(1/r)$ **Eqn 4.69**

$K_1 = 1$ **Eqn 4.71**

Hence

$e_2 = 8800^2 \times 13.07 \times 10^{-7} K_2 = 101.2\,K_2$ mm

Total eccentricity **4.3.5.6.2(1) Eqn 4.65**

$e_{tot} = e_e + e_a + e_2 = 13.2 + 22 + 101.2K_2$ mm

5.4.6 Iterative calculation to establish K_2 and hence A_s

Make initial assumption of $K_2 = 1$

This gives

$e_{tot} = 136.4$ mm

$N/bhf_{ck} = 2500 \times 10^3/(400 \times 400 \times 30) = 0.52$

$M/bh^2f_{ck} = 136.4 \times 2500 \times 10^3/(400^3 \times 30) = 0.178$

$d'/h = 60/400 = 0.15$

Using chart in Section **13**, Figure 13.2(c) gives $K_2 = 0.69$

Take this modified value of K_2 to recalculate e_{tot}.

Therefore

$e_{tot} = 13.2 + 22 + 101.2 \times 0.69 = 105.0$ mm

Hence

$M/bh^2f_{ck} = 0.137$

This reduces K_2 to 0.62 and M/bh^2f_{ck} to 0.128

Try reduction of K_2 to 0.60

This gives $M/bh^2f_{ck} = 0.125$ which corresponds to $K_2 = 0.60$ in the chart.

Hence

$$\frac{A_s f_{yk}}{bhf_{ck}} = 0.38$$

$A_s = 3965$ mm^2

Use 4T32 and 2T25 (4200 mm^2)

5.5 Slender column with biaxial bending

5.5.1 General

This example has bending dominantly about one axis and is designed to illustrate the application of EC2 Section 4.3.5.6.4.

There is some ambiguity in the drafting of this Section but the interpretation below seems reasonable.

5.5.2 Problem

Design a 400 mm square column, having an effective length of 8 m in both directions, to withstand the design ultimate first order moments shown in Figures 5.7 and 5.8 combined with a design axial load of 2000 kN. The concrete strength class is C30/37 and the reinforcement has a characteristic strength of 460 N/mm^2.

$\lambda = (8000/400)\sqrt{12} = 69.3$ in both directions

Assume $d'/h = 0.15$

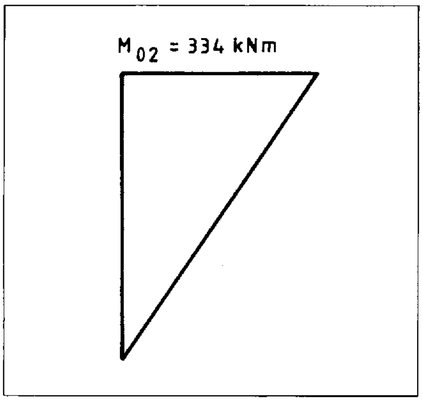


Figure 5.7 First order moments in z direction

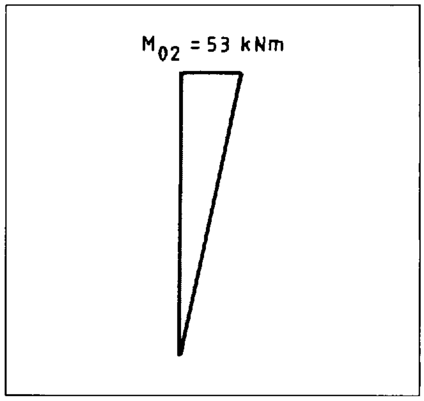


Figure 5.8 First order moments in y direction

5.5.3 Assumptions for design of section

It is assumed that e_y and e_z in EC2 Section 4.3.5.6.4 are the first order eccentricities at the critical section. They will, therefore, be effective values as defined by Eqns 4.66 and 4.67 in EC2 Section 4.3.5.6.2.

Since

$$e_{o1} = 0$$

4.3.5.6.2 Eqn 4.66

$$e_e = 0.6e_{o2}$$

Hence

$$e_z = \frac{0.6 \times 334 \times 10^3}{2000} = 100 \text{ mm}$$

$$e_y = \frac{0.6 \times 53 \times 10^3}{2000} = 16 \text{ mm}$$

$$(e_y/b)/(e_z/h) = \frac{16}{100} = 0.16 < 0.2$$

4.3.5.6.4 Eqn 4.75

Hence separate checks for the two axes are permissible

$$e_z/h = 100/400 = 0.25 > 0.2$$

4.3.5.6.4(3)

A reduced value of h, therefore, must be used in carrying out a check for bending in the y direction.

The additional eccentricity in the z direction is

$$0.5l_o/200 = 20 \text{ mm}$$

Hence

$$e_z + e_{az} = 120 \text{ mm}$$

It is assumed that the intention of EC2 Section 4.3.5.6.4(3) is that, using the reduced section, the applied load should just give zero stress at the least stressed face, i.e. as shown in Figure 5.9.

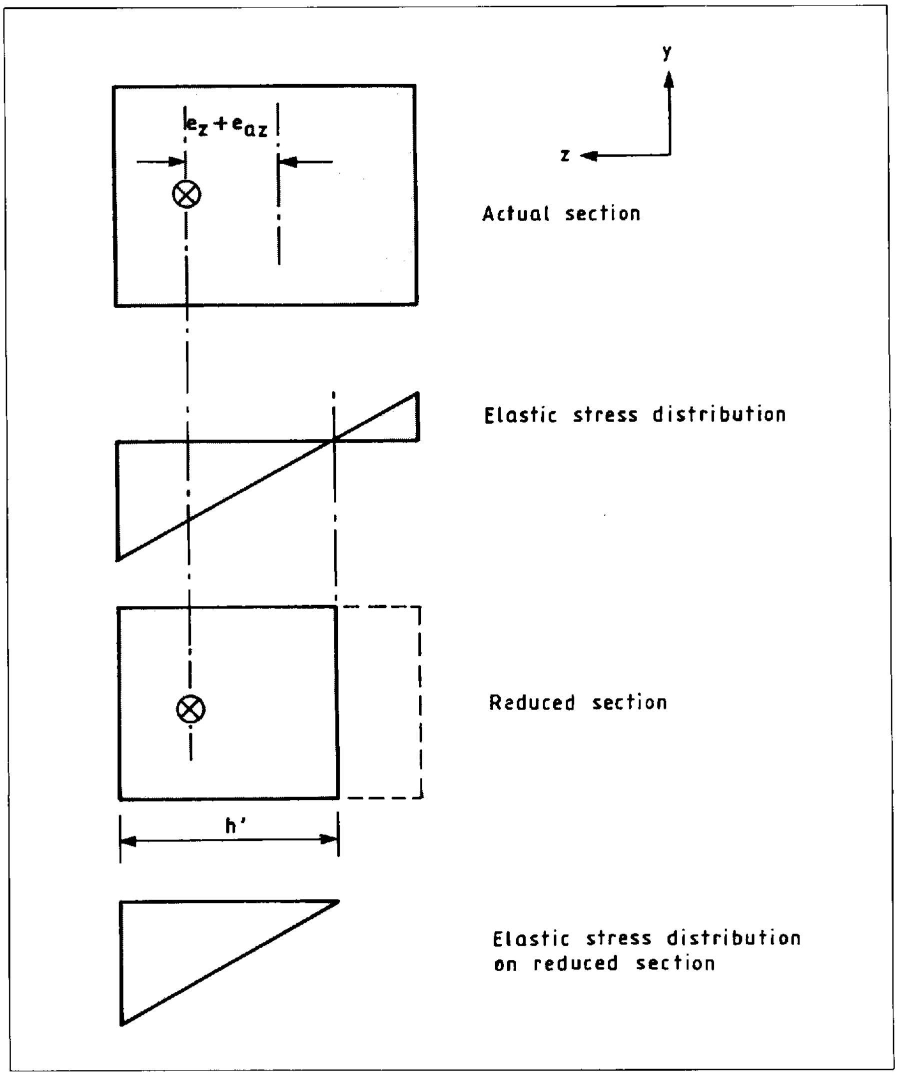


Figure 5.9 Assumption for check in the y direction

It will be seen that the point of application of the load must lie on the edge of the middle third of the reduced section.

Hence

$$h' = 3(h/2 - e_z - e_{az})$$

$$= 3(200 - 120) = 240 \text{ mm}$$

5.5.4 Check for bending in z direction

This check uses the full section dimensions

$$\frac{1}{r} = \frac{2 \times 460 \times 10^{-6} K_2}{1.15 \times 0.9 \times 340 \times 0.2} = 13.07K_2 \times 10^{-6}$$ 4.3.5.6.3 Eqn 4.72

Hence

$e_2 \;=\; 0.1 \times 8^2 \times 10^6 \times 13.07K_2 \times 10^{-6} \;=\; 83.7K_2$ mm — Eqn 4.69

(Since $\lambda > 35$, $K_1 = 1$ in EC2 Eqn 4.69)

$e_{tot} \;=\; 100 + 20 + 83.7K_2$ mm — 4.3.5.6.2 Eqn 4.65

As in the previous example, iterate using the design chart in Section **13** Figure 13.2(c) to find the appropriate value for K_2 and hence $A_s f_{yk}/bhf_{ck}$, starting with $K_2 = 1$. This procedure results in

$K_2 = 0.8$

$M/bh^2f_{ck} = 0.194$

$N/bhf_{ck} = 0.417$

Hence

$A_s f_{yk}/bhf_{ck} = 0.55$

$A_s = 0.55 \times 400^2 \times 30/460 = 5739 \text{ mm}^2$

Use 12T25 (5890 mm^2)

5.5.5 Check for bending in y direction

The assumed section is shown in Figure 5.10.

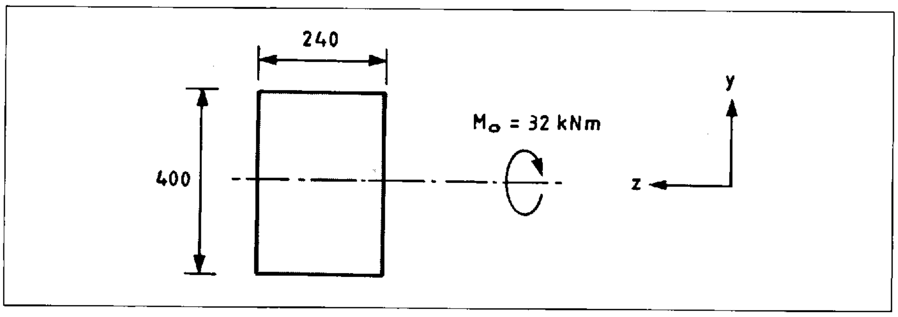


Figure 5.10 Reduced section for check in y direction

$e_{oy} = 16$ mm

$e_{ay} = e_{az} = 20$ mm

$e_{2y} = e_{2z} = 83.7K_2$ mm

Hence

$e_{tot} = 36 + 83.7K_2$ mm

$$N/bhf_{ck} = \frac{2000 \times 10^3}{240 \times 400 \times 30} = 0.694$$

$$M/bh^2f_{ck} = \frac{0.694 \times (36 + 83.7K_2)}{400} = 0.0625 + 0.145K_2$$

Using the same design chart as before, iterate to obtain K_2 and hence A_sf_{yk}/bhf_{ck}. This gives

$K_2 = 0.47$

$M/bh^2f_{ck} = 0.13$

Hence

$A_sf_{yk}/bhf_{ck} = 0.57$

$A_s = 0.57 \times 240 \times 400 \times 30/460 = 3569 \text{ mm}^2$

This is less than required for z direction bending . . . . OK

An appropriate arrangement of reinforcement is shown in Figure 5.11.

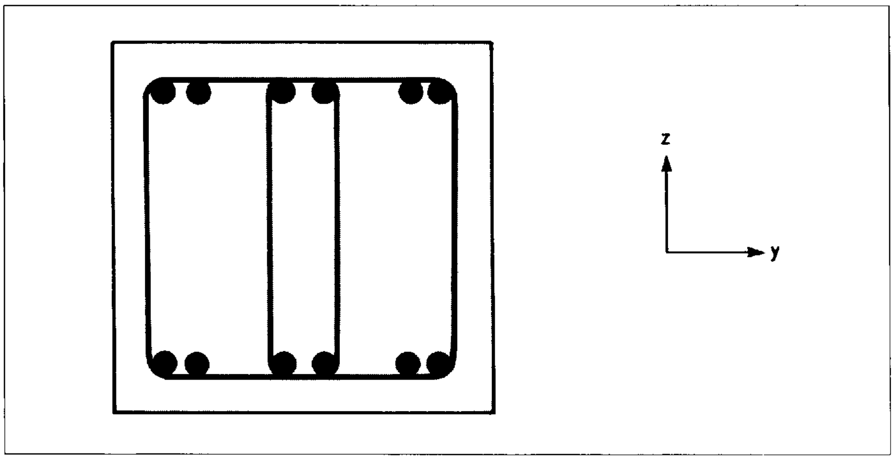


Figure 5.11 Arrangement of reinforcement

5.6 Classification of structure

5.6.1 Introduction

EC2 provides more detailed rules than BS 8110[(2)] for deciding whether or not a structure is braced or unbraced, or sway or non-sway. While it will normally be obvious by inspection how a structure should be classified (for example, with shear walls it will be braced and non-sway), there may be cases where direct calculation could give an advantage. The structure in the example following is chosen to illustrate the workings of EC2 in this area. It is entirely hypothetical and not necessarily practical.

5.6.2 Problem

Establish an appropriate design strategy for the columns in the structure shown in Figures 5.12 and 5.13. The applied vertical loads in the lowest storey are set out in Table 5.1.

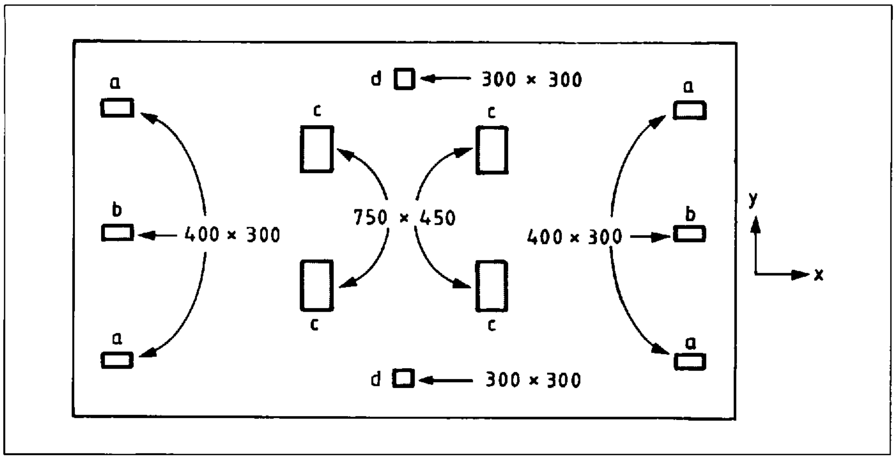


Figure 5.12 General arrangement of columns

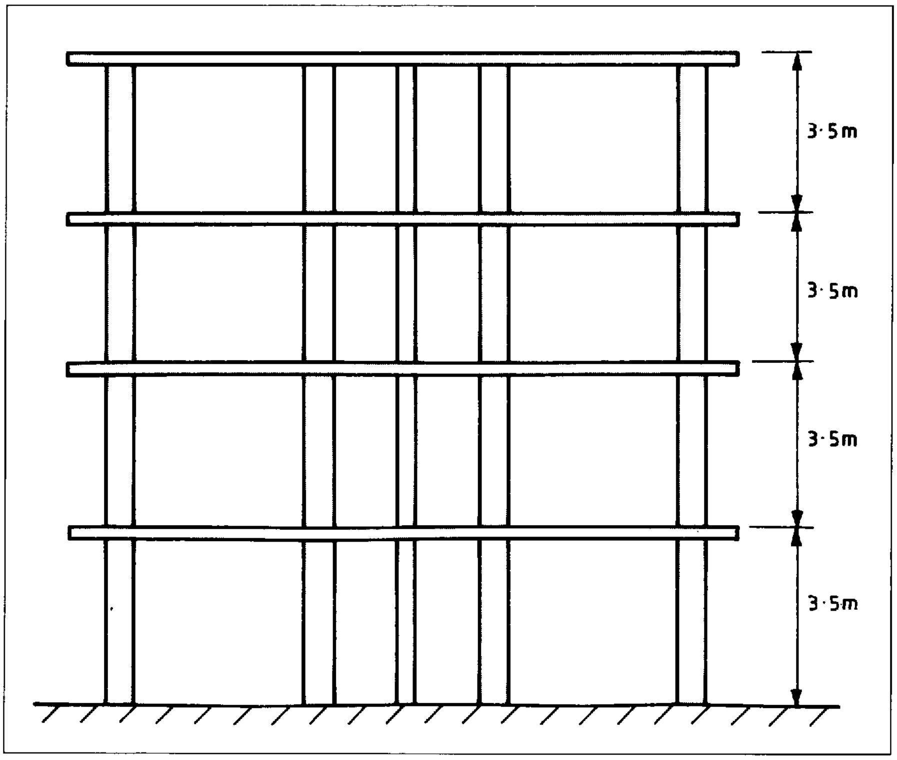


Figure 5.13 Cross-section of structure

Table 5.1 Column sizes and loads

Column type	Column dimension (mm)		2nd moment of area (mm × 10^{-6})		Service load (kN)	Ultimate load (kN)
	y	x	I_y	I_x		
a	300	400	900	1600	1900	2680
b	300	400	900	1600	2100	2960
c	750	450	15820	5695	3300	4660
d	300	300	675	675	1200	1700

5.6.3 Check if structure can be considered as braced with the 750 × 450 columns forming the bracing elements

To be considered as braced, the bracing elements must be sufficiently stiff to attract 90% of the horizontal load. Since all columns are the same length, this will be so if 4.3.5.3.2(1)

$$\frac{\Sigma I_{\text{bracing}}}{\Sigma I_{\text{tot}}} > 0.9$$

5.6.3.1 y direction

$$\frac{\Sigma I_{\text{bracing}}}{\Sigma I_{\text{tot}}} = \frac{4 \times 15820}{6 \times 900 + 4 \times 15820 + 2 \times 675} = 0.904$$

Hence the four 750 × 450 columns can be treated as bracing elements carrying the total horizontal loads and columns type a, b and d can be designed as braced in the y direction

5.6.3.2 x direction

$$\frac{\Sigma I_{\text{bracing}}}{\Sigma I_{\text{tot}}} = \frac{4 \times 5695}{6 \times 1600 + 4 \times 5695 + 2 \times 675} = 0.68$$

Structure cannot be considered as braced in x direction

5.6.4 Check if structure can be considered as non-sway

Classification of structures as sway or non-sway is covered in EC2 Appendix 3.

5.6.4.1 y direction

For braced structures of four or more storeys, the frame can be classified as non-sway if

$$h_{\text{tot}}\sqrt{F_v/E_{cm}I_c} \le 0.6$$ A3.2 Eqn A.3.2

where

h_{tot} = height of frame in metres = 4 × 3.5 = 14 m

F_v = sum of all vertical loads taking γ_f = 1

= 4 × 1900 + 2 × 2100 + 4 × 3300 + 2 × 1200 = 27400 kN

$E_{cm} I_c$ = sum of the stiffnesses of the bracing elements.

Taking E_{cm} as 32000 N/mm^2 3.1.2.5.2 Table 3.2

$E_{cm}I_c$ = 4 × 15820 × 32000 × 10^6 Nmm^2

= 2024960 × 10^9 Nmm^2

Hence

$$\sqrt{\frac{F_v}{E_{cm}I_c}} = \sqrt{\frac{27400 \times 10^3}{2024960 \times 10^9}} \text{ /mm} = 0.000116 \text{ / mm} = 0.116 \text{ / m}$$

Note:
Since the height of the building is stated to be in metres, it seems reasonable to assume that m units should be used for the other factors, though this is not stated in EC2.

Hence

$$h_{tot}\sqrt{\frac{F_v}{E_{cm}I_c}} = 14 \times 0.116 = 1.62 > 0.6$$

Therefore the bracing structure is a sway frame in the y direction

5.6.4.2 x direction

For frames without bracing elements, if $\lambda <$ greater of 25 or $15/\sqrt{\nu_u}$ for all elements carrying more than 70% of the mean axial force then the structure may be considered as non-sway. A.3.2(3)

$$\text{Mean axial force} = \frac{\text{sum of ultimate column loads}}{\text{no. of columns}}$$

$$N_{Sd,m} = \frac{4 \times 2680 + 2 \times 2960 + 4 \times 4660 + 2 \times 1700}{12}$$

$$= 38680/12 = 3223 \text{ kN}$$

$$70\%N_{Sd,m} = 2256 \text{ kN}$$

Columns type d carry less than this and are therefore ignored.

Assume effective length of 400 × 300 columns is 0.8 × 3.5 = 2.8 m (i.e. value appropriate to a non-sway condition).

$$\lambda = 24.25 < 25$$

Therefore structure is non-sway

5.6.5 Discussion

The results obtained in Sections **5.6.4.1** and **5.6.4.2** above are totally illogical as the structure has been shown to be a sway structure in the stiffer direction and non-sway in the less stiff direction.

There are two possible areas where the drafting of EC2 is ambiguous and the wrong interpretation may have been made.

(1) In Eqn A.3.2 it is specifically stated that the height should be in metres. Nothing is stated about the units for I_c, F_v and E_c. Since the output from Eqn A.3.2 is non-dimensional, the statement of the units is unnecessary unless the units for I_c, F_v and E_c are different to that for h_{tot}. Should I_c, F_v and E_c be in N and mm units while h_{tot} is in m? If this were so, then the structure would be found to be 'braced' by a large margin.

(2) In A.3.2(3) it does not state whether λ should be calculated assuming the columns to be sway or non-sway. In the calculation, the assumption was made that the λ was a non-sway value. If a sway value had been adopted, the structure would have proved to be a sway frame by a considerable margin.

Clearly, clarification is required if A.3.2 is to be of any use at all.

It is possible to take this question slightly further and make some estimate at what the answer should have been.

Considering the y direction, the ultimate curvature of the section of the 750 × 450 columns is

$$\frac{1}{r} = \frac{2 \times 460 \times K_2}{0.2 \times 10^6 \times 1.15 \times 0.9 \times 700} = 6.35K_2 \times 10^{-6}$$

Inspection of the design charts and levels of loading suggest K_2 is likely to be about 0.6. Assuming an effective length under sway conditions of twice the actual height gives a deflection of:

$$\frac{(2 \times 3.5)^2}{10} \times 6.35 \times 0.6 = 19 \text{ mm}$$

This is an overestimate of the actual deflection. It corresponds to an eccentricity of 19/750 of the section depth or 2.5%. This must be negligible, hence, in the y direction, the structure must effectively be non-sway.

5.7 Sway structures

5.7.1 Introduction

Although EC2 gives information on how to identify a sway structure, it does not give any simple approach to their design. However, Clause A.3.5.(2) states that "the simplified methods defined in 4.3.5 may be used instead of a refined analysis, provided that the safety level required is ensured". Clause A.3.5(3) amplifies this slightly, saying that "simplified methods may be used which introduce .... bending moments which take account of second order effects ...... provided the average slenderness ratio in each storey does not exceed 50 or $20/\sqrt{\nu_u}$, whichever is the greater".

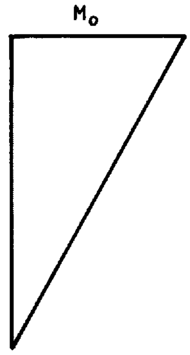
EC2 Section 4.3.5 gives the 'model column' method which is developed only for non-sway cases, so it is left to the user to find a suitable method for sway frames on the basis of the Model Column Method. BS 8110 does this, so it is suggested that the provisions of 3.8.3.7 and 3.8.3.8 of BS 8110: Part 1 are adopted, but that the eccentricities are calculated using the equations in EC2.

5.7.2 Problem

Design columns type c in the structure considered in Section **5.6.2** assuming sway in the x direction. The column loads may be taken from Table 5.1.

The design ultimate first order moments in the columns are as shown in Figure 5.14.

β has been assessed from EC2 Figure 4.27(b) as 1.6 for all columns.



Column type	M_o kNm
a	80
b	110
c	176
d	25

Figure 5.14 First order moments

5.7.3 Average slenderness ratio

The slenderness ratios are shown in Table 5.2.

Table 5.2 Slenderness ratios

Column type	No.	λ
a	4	48.5
b	2	48.5
c	4	43.1
d	2	64.7
Mean value (λ_m) = 49.4		

Since $\lambda_m < 50$, the simplified method may be used. A.3.5(3)

$$\frac{1}{r} = \frac{2K_2 \times 460}{200000 \times 1.15 \times 0.9d} = \frac{0.0044K_2}{d}$$

Hence

$$e_2 = \frac{(1.6 \times 3.5)^2}{10} \times \frac{0.0044K_2 \times 10^6}{d} = \frac{13800K_2}{d} \text{ mm}$$

$$\nu = \frac{1}{100\sqrt{14}} \geq \frac{1}{200}$$ 2.5.1.3 Eqn 2.10

This may be multiplied by α_n Eqn 2.11

Where, with 12 columns

$$\alpha_n = \sqrt{(1 + 1/12)/2} = 0.736$$

Hence

$$\nu = 0.00368$$

$$e_a = 0.00368 \times 1.6 \times 3500/2 = 10.3 \text{ mm}$$ Eqn 4.61

$$e_{tot} = e_o + e_a + 13800K_2/d = e_o + 10.3 + 13800K_2/d \text{ mm}$$ Eqn 4.65

The total eccentricities are shown in Table 5.3.

Table 5.3 Total eccentricities

Column type	d (mm)	M_o (kNm)	e_o (mm)	$\frac{N}{bhf_{ck}}$	e_{tot} (mm)
a	350	80	30	0.744	$40.3 + 39K_2$
b	350	110	37	0.822	$47.3 + 39K_2$
c	400	176	38	0.460	$48.3 + 35K_2$
d	250	25	15	0.630	$25.3 + 55K_2$

As in the previous examples, the design charts can be used iteratively to establish K_2 and hence e_2. This process gives the values shown in Table 5.4.

Table 5.4 Lateral deflections

Column type	K_2	e_2 (mm)	No. of columns
a	0.39	15.2	4
b	0.41	16.0	2
c	0.50	17.5	4
d	0.45	24.8	2
Average deflection = 17.7 mm			

All columns will be assumed to deflect by the average value. The resulting designs are shown in Table 5.5

BS 8110
3.8.3.8

Table 5.5 Summary of designs

Column type	e_{tot} (mm)	$\frac{M}{bh^2f_{ck}}$	$\frac{N}{bhf_{ck}}$	$\frac{A_sf_{yk}}{bhf_{ck}}$	A_s (mm^2)
a	58.0	0.108	0.744	0.53	4148
b	65.0	0.134	0.822	0.75	5870
c	66.0	0.067	0.460	0.10	2201
d	43.0	0.090	0.630	0.38	2230

6 WALLS

6.1 Introduction

A wall is defined as a vertical load-bearing member with a horizontal length not less than four times its thickness. 2.5.2.1(6)

The design of walls is carried out by considering vertical strips of the wall acting as columns.

6.2 Example

Design the lowest level of a 200 mm thick wall in an eight storey building supporting 250 mm thick solid slabs of 6.0 m spans on each side. The storey heights of each floor are 3.5 m, the height from foundation to the first floor being 4.5 m. The wall is fully restrained at foundation level. The building is a braced non-sway structure.

6.2.1 Design data

Design axial load (N_{Sd}) = 700 kN/m

Design moment at first floor = 5 kNm/m

Design moment at foundation = 2.5 kNm/m

Concrete strength class is C30/37. 3.1.2.4

f_{ck} = 30 N/mm^2 Table 3.1

6.2.2 Assessment of slenderness

Consider a 1.0 m vertical strip of wall acting as an isolated column.

The effective height of a column $l_o = \beta l_{col}$ 4.3.5.3.5(1)

where

l_{col} = actual height of the column between centres of restraint

β is a factor depending upon the coefficients k_A and k_B relating to the rigidity of restraint at the column ends.

$$k_A = \frac{\Sigma I_{col}/l_{col}}{\Sigma I_{slab}/l_{eff,slab}}$$

4.3.5.3.5(1) Eqn 4.60

Assuming a constant modulus of elasticity for the concrete:

$$I_{col} = \frac{1 \times 0.2^3}{12} = 6.67 \times 10^{-4}\ \text{m}^4$$

$$I_{slab} = \frac{1 \times 0.25^3}{12} = 1.3 \times 10^{-3}\ \text{m}^4$$

$$k_A = \left(\frac{6.67 \times 10^{-4}}{4.5} + \frac{6.67 \times 10^{-4}}{3.5}\right) \Big/ \left(\frac{2 \times 1.3 \times 10^{-3}}{6}\right) = 0.78$$

Base of wall is fully restrained.

Therefore 4.3.5.3.5(1) Figure 4.27(a)

k_B = 0.40 which is the minimum value to be used for k_A or k_B.

Hence

β = 0.7

l_{col} = 4500 mm

Therefore

l_o = 0.7 × 4500 = 3150 mm

The slenderness ratio $\lambda = l_o/i$ 4.3.5.3.5(2)

where

i = radius of gyration

$$= \sqrt{\frac{I}{A}} = \sqrt{\frac{1000 \times 200^3}{12 \times 1000 \times 200}} = 57.7 \text{ mm}$$

Therefore

$$\lambda = \frac{3150}{57.7} = 54.6$$

Isolated columns are considered slender where λ exceeds the greater of 25 or $15/\sqrt{\nu_u}$

where

$$\nu_u = \frac{N_{Sd}}{A_c f_{cd}}$$ 4.3.5.3.5(2)

N_{Sd} = 700 kN

A_c = 1000 × 200 = 200 × 10^3 mm²

$$f_{cd} = \frac{f_{ck}}{\gamma_c} = \frac{30}{1.5} = 20 \text{ N/mm}^2$$

Therefore

$$\nu_u = \frac{700 \times 10^3}{200 \times 10^3 \times 20} = 0.175$$

Hence

$$\frac{15}{\sqrt{\nu_u}} = \frac{15}{\sqrt{0.175}} = 35.9$$

Therefore the wall is slender

6.2.3 Design

The wall may now be designed as an isolated column in accordance with EC2[1] Clause 4.3.5.6 and as illustrated in the example in Section **5**.

Although the column or wall has been classified as slender, second order effects need not be considered if the slenderness ratio λ is less than the critical slenderness ratio λ_{crit}.

$$\lambda_{crit} = 25\,(2 - e_{o1}/e_{o2})$$ 4.3.5.5.3(2) Eqn 4.62

where

e_{o1} and e_{o2} are the first order eccentricities at the ends of the member relating to the axial load.

$$e_{o1} = \frac{M_{Sd1}}{N_{Sd}} \text{ and } e_{o2} = \frac{M_{Sd2}}{N_{Sd}}$$

M_{Sd1} and M_{Sd2} are the first order applied moments.

Therefore

$$\lambda_{crit} = 25\,(2 - M_{Sd1}/M_{Sd2})$$

where

$$M_{Sd1} \leq M_{Sd2}$$

These moments must be given their correct algebraic signs in the equation.

In this example:

$$\lambda_{crit} = 25\left[2 - \left(\frac{-2.5}{5.0}\right)\right] = 62.5 > 54.6$$

The column or wall should therefore be designed for the following minimum conditions:	4.3.5.5.3(2)
Design axial resistance (N_{Rd}) $= N_{Sd}$	Eqn 4.63
Design resistance moment (M_{Rd}) $= N_{Sd} \times \dfrac{h}{20}$	Eqn 4.64

For this example

$$M_{Rd} = 700 \times \frac{0.2}{20} = 7.0 > 5.0 \text{ kNm}$$

6.2.4 Reinforcement

The vertical reinforcement should not be less than $0.004A_c$ or greater than $0.04A_c$. 5.4.7.2(1)

Half of this reinforcement should be located at each face. 5.4.7.2(2)

The maximum spacing for the vertical bars should not exceed twice the wall thickness or 300 mm. 5.4.7.2(3)

The area of horizontal reinforcement should be at least 50% of the vertical reinforcement. The bar size should not be less than one quarter of the vertical bar size and the spacing should not exceed 300 mm. The horizontal reinforcement should be placed between the vertical reinforcement and the wall face. 5.4.7.3 (1)–(3)

Link reinforcement is required in walls where the design vertical reinforcement exceeds $0.02A_c$. 5.4.7.4(1)

In normal buildings it is unlikely that walls will be classified as slender. For practical considerations they will generally not be less than 175 mm thick and the vertical load intensity will normally be relatively low. Thus the limiting slenderness ratio given by $15/\sqrt{\nu_u}$ will be high.

In cases where the wall is slender, only slenderness about the minor axis need be considered. Even in this case it is likely that only the minimum conditions given in EC2 Clause 4.3.5.5.3(2) Eqns 4.63 and 4.64 will apply.

7 FOUNDATIONS

7.1 Ground bearing footings

7.1.1 Pad footing

Design a square pad footing for a 400 mm × 400 mm column carrying a service load of 1100 kN, 50% of this being imposed load with appropriate live load reduction. The allowable bearing pressure of the soil is 200 kN/m^2.

7.1.1.1 Base size

With 500 mm deep base, resultant bearing pressure

$$= 200 - 0.5 \times 24 = 188 \text{ kN/m}^2$$

$$\text{Area of base required} = \frac{1100}{188} = 5.85 \text{ m}^2$$

Use 2.5 m × 2.5 m × 0.5 m deep base

7.1.1.2 Durability

For components in non-aggressive soil and/or water, exposure class is 2(a). Table 4.1

Minimum concrete strength grade is C30/37. ENV 206 Table NA.1

For cement content and w/c ratio refer to ENV 206 Table 3[6].

Minimum cover to reinforcement is 30 mm. NAD Table 6

For concrete cast against blinding layer, minimum cover > 40 mm. 4.1.3.3(9)

Use 75 mm nominal cover bottom and sides

7.1.1.3 Materials

Type 2 deformed reinforcement with $f_{yk} = 460$ N/mm^2 NAD 6.3(a)

Concrete strength grade C30/37 with maximum aggregate size 20 mm

7.1.1.4 Loading

Eqn 2.8(a) Table 2.2

Ultimate column load $= 1.35G_k + 1.5Q_k = 1570$ kN

7.1.1.5 Flexural design

Critical section taken at face of column 2.5.3.3(5)

$$M_{Sd} = \frac{1570\,(2.5 - 0.4)^2}{8 \times 2.5} = 346 \text{ kNm}$$

Assuming 20 mm bars

$$d_{ave} = 500 - 75 - 20 = 405 \text{ mm}$$

Using rectangular concrete stress diagram Figure 4.4

$$f_{cd} = \frac{f_{ck}}{\gamma_c} = \frac{30}{1.5} = 20 \text{ N/mm}^2$$

Eqn 4.4 Table 2.3

$$\alpha f_{cd} = 0.85 \times 20 = 17 \text{ N/mm}^2$$

For reinforcement

$$f_{yd} = \frac{f_{yk}}{\gamma_s} = \frac{460}{1.15} = 400 \text{ N/mm}^2$$ 2.2.3.2P(1) Table 2.3

For the design of C30/37 concrete members without any redistribution of moments, neutral axis depth factor 2.5.3.4.2(5)

$$\frac{x}{d} \leq 0.45$$

Using the design tables for singly reinforced beams

$$\frac{M_{Sd}}{bd^2 f_{ck}} = \frac{346 \times 10^6}{2500 \times 405^2 \times 30} = 0.028$$

$$\frac{x}{d} = 0.063 < 0.45 \text{ .... OK}$$

$$\frac{A_s f_{yk}}{bdf_{ck}} = 0.033$$

Hence

$$A_s = 0.033 \times 2500 \times 405 \times \frac{30}{460} = 2179 \text{ mm}^2$$

Minimum longitudinal reinforcement 5.4.3.2.1 5.4.2.1.1

$$= \frac{0.6 b_t d}{460} = 0.0013 b_t d \not< 0.0015 b_t d$$

$$= 0.0015 \times 2500 \times 405 = 1519 \text{ mm}^2$$

7T20 gives 2198 $>$ 2179 mm^2 .... OK

$$\text{Bar crs.} = \frac{2500 - 2(75) - 20}{6} = 388 \text{ mm}$$

Maximum spacing $= 3h \not> 500 = 500 > 388$ mm .... OK NAD Table 3

7T20 (EW) are sufficient for flexural design. Additional checks for punching and crack control require 9T20 (EW) – refer to Sections **7.1.1.7** and **7.1.1.8**. 5.4.3.2.1(4)

Use 9T20 (EW)

7.1.1.6 Shear

Minimum shear reinforcement may be omitted in slabs having adequate provision for the transverse distribution of loads. Treating the pad as a slab, therefore, no shear reinforcement is required if $V_{Sd} \leq V_{Rd1}$. 4.3.2.1P(2) 4.3.2.2(2)

Shear force at critical section, distance d from face of column — 4.3.2.2(10)

$$V_{Sd} = \frac{1570}{2.5}\left(\frac{2.5}{2} - \frac{0.4}{2} - 0.405\right) = 405 \text{ kN}$$

Shear resistance, V_{Rd1}, with zero axial load — 4.3.2.3

$$V_{Rd1} = \tau_{Rd}k(1.2 + 40\rho_l)\, b_w d$$ Eqn 4.18

$$\tau_{Rd} = 0.34 \text{ N/mm}^2$$ Table 4.8

$$k = 1.6 - d = 1.195 \not< 1.0$$

To calculate A_{sl}, area of tension reinforcement extending $d + l_{b,net}$ beyond critical section, determine

$$l_{b,net} = \alpha_a l_b \left(\frac{A_{s,req}}{A_{s,prov}}\right) \not< l_{b,min}$$ Eqn 5.4

For curved bars with concrete side cover of at least 3ϕ — 5.2.3.4.1(1)

$$\alpha_a = 0.7$$

$$l_b = \left(\frac{\phi}{4}\right)\frac{f_{yd}}{f_{bd}}$$ Eqn 5.3

For bars in the bottom half of a pour, good bond may be assumed. Hence for $\phi \leq 32$ mm — 5.2.2.1(2)

$$f_{bd} = 3.0 \text{ N/mm}^2$$ Table 5.3

$$l_b = \frac{\phi}{4} \times \frac{400}{3} = 33.3\phi$$

For anchorage in tension

$$l_{b,min} = 0.3 \times l_b \not< 10\phi \not< 100 \text{ mm}$$ Eqn 5.5

$$= 10\phi = 200 \text{ mm}$$

Actual distance from critical section to end of bar

$$= \frac{2500}{2} - \frac{400}{2} - 405 - 75 = 570 \text{ mm}$$

$$< d + l_{b,min} = 405 + 200 = 605 \text{ mm}$$

Therefore

$A_{sl} = 0$

$V_{Rd1} = 0.34 \times 1.2 \times 1.195 \times 2500 \times 405 \times 10^{-3} = 493$ kN

$> V_{Sd} = 405$ kN

No shear reinforcement required

Check that $V_{Sd} \not> V_{Rd2}$ to avoid crushing of compression struts.

$$\upsilon = 0.7 - \frac{f_{ck}}{200} = 0.55 \not< 0.5 \text{ N/mm}^2$$ Eqn 4.20

$$V_{Rd2} = \frac{\upsilon f_{cd} b_w 0.9d}{2} = \frac{0.55 \times 20 \times 2500 \times 0.9 \times 405 \times 10^{-3}}{2}$$ Eqn 4.19

$= 5012 > 405$ kN .... OK

7.1.1.7 Punching

Length of base from face of column

$a = 1050$ mm

$$\frac{a}{h_f} = \frac{1050}{400} > 2$$ Figure 4.16

By definition the foundation should be considered as a slab.

Critical perimeter at 1.5*d* from face of column should be checked for punching. 4.3.4.1P(4) & 4.3.4.2.2

$u = 2\pi (1.5 \times 405) + 4 \times 400 = 5417$ mm

In foundations the applied shear may be reduced to allow for the soil reaction within the critical perimeter. 4.3.4.1(5)

Enclosed area

Total width $= (3 \times 405) + 400 = 1615$ mm

Corner radius $= 1.5 \times 405 = 608$ mm

Area $= 1.615^2 - (4 - \pi)\, 0.608^2 = 2.29$ m^2

$$V_{Sd} = 1570 \left(1 - \frac{2.29}{2.5^2}\right) = 995 \text{ kN}$$

The applied shear per unit length 4.3.4.3(4)

$$v_{Sd} = \frac{V_{Sd}\beta}{u}$$

$\beta = 1.0$ for pads with no eccentricity of load

Therefore

$$v_{Sd} = \frac{995 \times 10^3}{5417} = 184 \text{ N/mm}$$

The amount of tensile reinforcement in two perpendicular directions should be greater than 0.5%. This is assumed to require $\rho_{lx} + \rho_{ly} > 0.5\%$. 4.3.4.1(9)

Using 9T20 (EW), $A_s = 2830 \text{ mm}^2$ (EW)

For B1

$$d_x = 415 \text{ mm}$$

$$\frac{100A_s}{bd_x} = 0.27\%$$

For B2

$$d_y = 395 \text{ mm}$$

$$\frac{100A_s}{bd_y} = 0.28\%$$

0.27% + 0.28% = 0.55% > 0.5% . . . . OK

Punching resistance for a slab without shear reinforcement

$$v_{Rd1} = \tau_{Rd}k(1.2 + 40\rho_l)d$$ 4.3.4.5

The equation produces similar values to the shear check performed above

$$v_{Rd1} = 0.34 \times 1.195 \times 1.2 \times 405 = 197 > 184 \text{ N/mm}$$

No shear reinforcement required

Check the stress at the perimeter of the column NAD 6.4(d)

$$V_{Sd}/ud \leq 0.90\sqrt{f_{ck}} = 0.90\sqrt{30} = 4.9 \text{ N/mm}^2$$

$$d = 405 \text{ mm}$$

$$u = 4 \times 400 = 1600 \text{ mm}$$

$$\text{Stress} = \frac{1570 \times 10^3}{405 \times 1600} = 2.4 < 4.9 \text{ N/mm}^2$$ . . . . OK

7.1.1.8 Crack control

Use method without direct calculation. 4.4.2.3

Estimate service stress in reinforcement under quasi-permanent loads using the following approximation 4.4.2.3(3) 2.2.2.3P(2)

$$G_k + \psi_2 Q_k = G_k + 0.3Q_k = 550 + 0.3 \times 550 = 715 \text{ kN}$$ 2.3.4 Eqn 2.9(c) & NAD Table 1

Hence quasi-permanent load/factored load = 715/1570 = 0.46

and estimated service stress

$$= 0.46 \times f_{yd} \times \frac{A_{s,req}}{A_{s,prov}} = 0.46 \times 400 \times \frac{2179}{2830} = 142 \text{ N/mm}^2$$

Either limit bar size using EC2 Table 4.11[1] or bar spacing using EC2 Table 4.12. 4.4.2.3(2)

ϕ = 20 < 32 mm .... OK

This has been chosen to comply with Table 4.12 as well.

Using 9T20 (EW) bar spacing = 290 < 300 mm .... OK

Check minimum reinforcement requirement 4.4.2.3(2)

$A_s \geq k_c k f_{ct,eff} A_{ct}/\sigma_s$ 4.4.2.2 Eqn 4.78

For A_{ct} it is considered conservative to use $(h/2)b$

σ_s = 100% × f_{yk} = 460 N/mm²

For $f_{ct,eff}$ use minimum tensile strength suggested by EC2 – 3 N/mm²

k_c = 0.4 for bending

For k interpolate a value for h = 50 cm from values given

k = 0.5 + 0.3(80 − 50)/(80 − 30) = 0.68

Therefore

$A_{s,req}$ = 0.4 × 0.68 × 3 × 250 × 2500/460 = 1109 mm²

$A_{s,prov}$ = 2830 > 1109 mm² .... OK

7.1.1.9 Reinforcement detailing

Check that flexural reinforcement extends beyond critical section for bending for a distance ≥ $d + l_{b,net}$ 5.4.3.2.1(1) & 5.4.2.1.3

l_b = 33.3ϕ = 667 mm

Assuming straight bar without end hook

$$l_{b,net} = 1.0 \times 667 \times \frac{2179}{2830} = 514 \text{ mm}$$ Eqn 5.4

$d + l_{b,net}$ = 405 + 514 = 919 mm

Actual distance $= \frac{2500}{2} - \frac{400}{2} - 75 = 975 > 919$ mm ..... OK

The reinforcement details are shown in Figure 7.1.

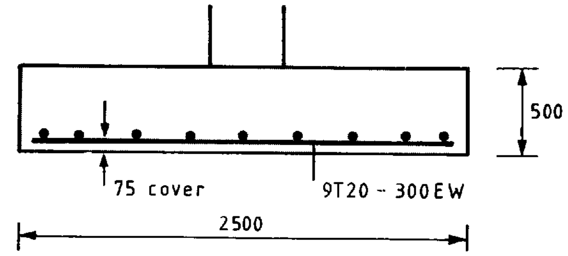


Figure 7.1 Detail of reinforcement in pad footing

7.1.2 Combined footing

Design a combined footing supporting one exterior and one interior column.

An exterior column, 600 mm × 450 mm, with service loads of 760 kN (dead) and 580 kN (imposed) and an interior column, 600 mm × 600 mm, with service loads of 1110 kN (dead) and 890 kN (imposed) are to be supported on a rectangular footing that cannot protrude beyond the outer face of the exterior column. The columns are spaced at 5.5 m centres and positioned as shown in Figure 7.2.

The allowable bearing pressure is 175 kN/m^2, and because of site constraints, the depth of the footing is limited to 750 mm.

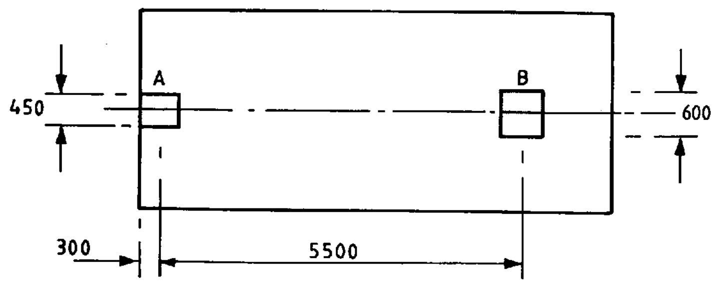


Figure 7.2 Plan of combined footing

7.1.2.1 Base size

Service loads = $G_k + Q_k$

Column A: 1340 kN and Column B: 2000 kN

Distance to centroid of loads from LH end

$$= \quad 0.3 + \frac{2000 \times 5.5}{3340} = 3.593 \text{ m}$$

For uniform distribution of load under base

Length of base = 2 × 3.593 say 7.2 m

With 750 mm deep base, resultant bearing pressure

$$= 175 - 0.75 \times 24 = 157 \text{ kN/m}^2$$

$$\text{Width of base required} = \frac{3340}{7.2 \times 157} = 2.96 \text{ say } 3.0 \text{ m}$$

Use 7.2 m × 3.0 m x 0.75 m deep base

7.1.2.2 Durability

For ground conditions other than non-aggressive soils, particular attention is needed to the provisions in ENV 206 and the National Foreword and Annex to that document for the country in which the concrete is required. In the UK it should be noted that the use of ISO 9690[15] and ENV 206 may not comply with the current British Standard, BS 8110: Part 1: 1985 Table 6.1[2] where sulphates are present.

Class 2(a) has been adopted for this design. — Table 4.1 ENV 206

Minimum concrete strength grade is C30/37. — Table NA.1

For cement content and w/c ratio refer to ENV 206 Table 3.

Minimum cover to reinforcement is 30 mm. — NAD Table 6

For concrete cast against blinding layer, minimum cover > 40 mm. — 4.1.3.3(9)

However, it is suggested that nominal cover > 40 mm is a sufficient interpretation of the above clause.

Use 75 mm nominal cover bottom and sides and 35 mm top

7.1.2.3 Materials

Type 2 deformed reinforcement with f_{yk} = 460 N/mm^2 — NAD 6.3(a)

Concrete strength grade C30/37 with maximum aggregate size 20 mm

7.1.2.4 Loading

Ultimate column loads = $1.35G_k + 1.5Q_k$
Column A: 1896 kN and Column B: 2834 kN

Eqn 2.8(a)

Table 2.2

Distance to centroid of loads from LH end

$$= 0.3 + \frac{2834 \times 5.5}{4730} = 3.595 \text{ m}$$

i.e. virtually at centre of 7.2 m long base

$$\text{Assume uniform net pressure} = \frac{4730}{7.2} = 657 \text{ kN/m} = 219 \text{ kN/m}^2$$

See Figures 7.3, 7.4 and 7.5 for loading, shear force and bending moment diagrams respectively.

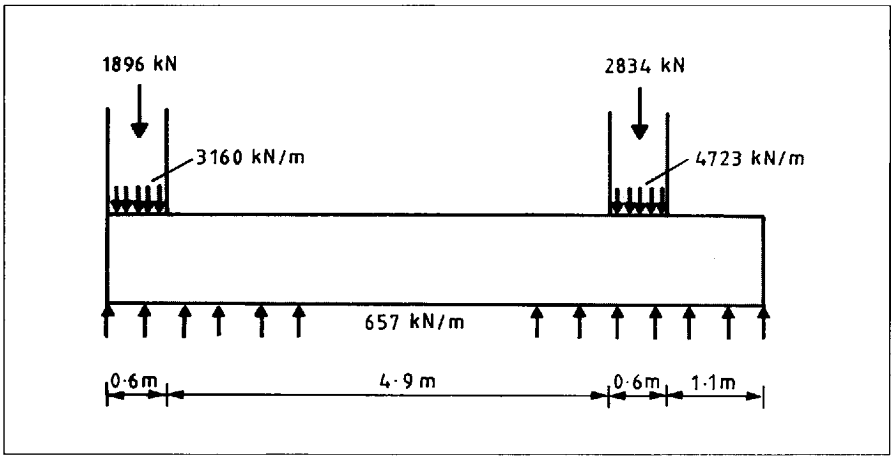


Figure 7.3 Loading diagram

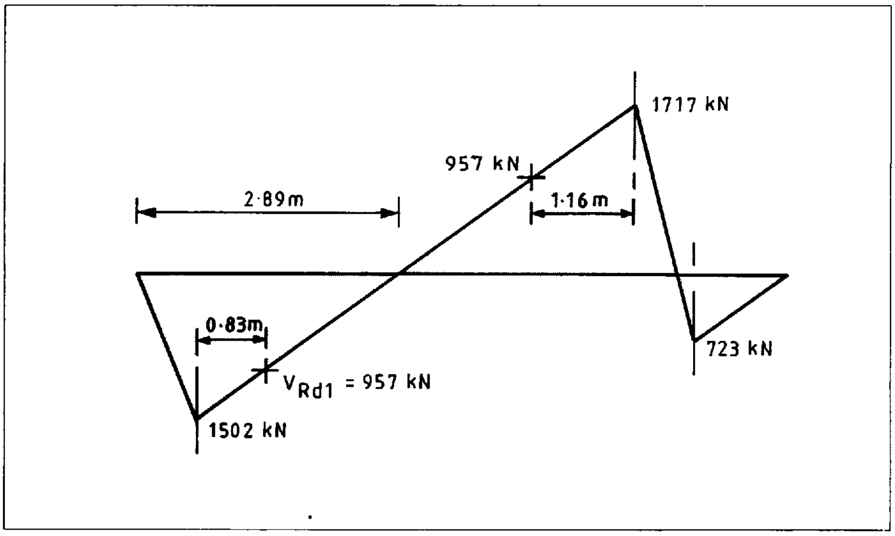


Figure 7.4 Shear force diagram

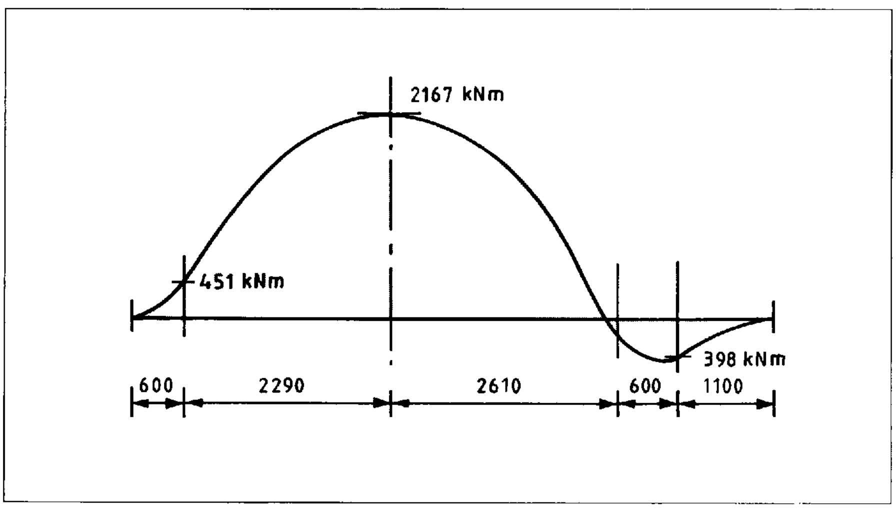


Figure 7.5 Bending moment diagram

7.1.2.5 Flexural design

7.1.2.5.1 Longitudinal direction – top steel

Mid-span

M_{Sd} = 2167 kNm

d = 750 − 35 − 20 − 32/2 = 679 say 675 mm

Using the design tables for singly reinforced beams

$$\frac{M_{Sd}}{bd^2 f_{ck}} = \frac{2167 \times 10^6}{3000 \times 675^2 \times 30} = 0.053$$

$$\frac{x}{d} = 0.123 < 0.45 \text{ limit with zero redistribution} \; \ldots\ldots\ldots \; \text{OK}$$ 2.5.3.4.2(5)

$$\frac{A_s f_{yk}}{bdf_{ck}} = 0.064$$

$$A_s = 0.064 \times 3000 \times 675 \times \frac{30}{460} = 8452 \text{ mm}^2 = 2818 \text{ mm}^2/\text{m}$$

Use 12T32 @ 250 mm crs. (3217 mm²/m)

Continue bars to RH end of base to act as hangers for links.

Particular attention is drawn to the clauses for bar sizes larger than 32 mm. These clauses are restrictive about laps and anchorages, such that designers may need to resort to groups of smaller bars instead. 5.2.6.3P(1) & P(2)

Maximum spacing = $3h \not> 500$ = 500 > 250 mm .... OK NAD Table 3 5.4.3.2.1(4)

7.1.2.5.2 Longitudinal direction – bottom steel

At column face

M_{Sd} = 398 kNm

d = 750 − 75 − 10 = 665 mm

$$\frac{M_{Sd}}{bd^2 f_{ck}} = \frac{398 \times 10^6}{3000 \times 665^2 \times 30} = 0.010$$

$$\frac{A_s f_{yk}}{bdf_{ck}} = 0.012$$

$$A_s = 0.012 \times 3000 \times 665 \times \frac{30}{460} = 1561 \text{ mm}^2 = 520 \text{ mm}^2/\text{m}$$

For minimum steel $A_{s,min}$ = $0.0015 b_t d$ = 998 mm²/m 5.4.2.1.1

Use 12T20 @ 250 mm crs. (1258 mm²/m)

7.1.2.5.3 Transverse direction – bottom steel

$$M_{Sd} = \left(1.5 - \frac{0.45}{2}\right)^2 \times \frac{219}{2} = 178 \text{ kNm/m}$$

Minimum steel governs.

Use T20 @ 250 mm crs. (1258 mm^2/m)

7.1.2.6 Shear

Critical shear section at distance d from face of column — 4.3.2.2(10)

Column B interior side

$$V_{Sd} = 1717 - 0.675 \times 657 = 1273 \text{ kN}$$

$$V_{Rd1} = \tau_{Rd}\, k(1.2 + 40\rho_l)b_w d$$

4.3.2.3
Eqn 4.18
Table 4.8

$$\tau_{Rd} = 0.34 \text{ N/mm}^2$$

$$k = 1.6 - d \not< 1.0 = 1.0$$

$$\rho_l = 0.00476$$

Ensure bars are continued sufficiently.

$$V_{Rd1} = 957 \text{ kN}$$

$$V_{Sd} > V_{Rd1}$$

Therefore shear reinforcement required.

Shear capacity with links

$$V_{Rd3} = V_{cd} + V_{wd} = V_{Rd1} + V_{wd}$$

4.3.2.4.3
Eqn 4.22

Therefore

$$V_{wd} \geq 1273 - 957 = 316 \text{ kN}$$

$$V_{wd} = \frac{A_{sw}}{s} \times 0.9 d f_{ywd}$$

Eqn 4.23

$$f_{ywd} = 400 \text{ N/mm}^2, \quad d = 675 \text{ mm}$$

$$\frac{A_{sw}}{s} \geq \frac{316 \times 10^3}{0.9 \times 675 \times 400} = 1.30 \text{ mm}^2\text{/mm}$$

Where shear reinforcement is required, the minimum amount is 100% of the EC2 Table 5.5 value. — NAD Table 3; 5.4.3.3(2)

With $f_{yk} = 460$, $\rho_{w,min} = 0.0012$ by interpolation — Table 5.5

For links

$\rho_w \quad = \quad A_{sw}/sb_w$ Eqn 5.16

Therefore

$$\left(\frac{A_{sw}}{s}\right)_{min} = 0.0012 \times 3000 = 3.6 > 1.30 \text{ mm}^2\text{/mm}$$

Therefore minimum links govern.

Determine link spacing, using EC2 Eqn 5.17–19.

$V_{Rd2} \quad = \quad \upsilon f_{cd}\, b_w(0.9\, d)/2$ Eqn 4.25

$\quad = \quad 0.55 \times 20 \times 3000 \times 0.9 \times 675 \times 10^{-3}/2 = 10020$ kN

$V_{Sd}/V_{Rd2} = 1273/10020 = 0.13 < 0.2$

Use EC2 Eqn 5.17 to determine link spacing.

$s_{max} \quad = \quad 0.8d$ (Note: 300 mm limit in Eqn 5.17 does not apply to slabs) 5.4.3.3(4)

$\not> 0.75d = 506$ mm NAD 6.5(f)

Transverse spacing of legs across section 5.4.2.2(9)

$\leq d$ or 800 mm = 675 mm

Use 12 legs T10 @ 250 mm crs. in each direction where $V_{Sd} > V_{Rd1}$

$$\frac{A_{sw}}{s} = \frac{12 \times 78.5}{250} = 3.77 > 3.6 \text{ mm}^2\text{/mm} \text{ . . . . OK}$$

Check diagonal crack control 5.4.2.2(10)

$V_{cd} \quad = \quad V_{Rd1} = 957$ kN

$V_{Sd} \quad = \quad 1273$ kN (max.)

$V_{Sd} \quad < \quad 3V_{cd}$

No further check required. 4.4.2.3(5)

Distances to where $V_{Sd} = V_{Rd1}$ from face of columns A and B

$$x_a = \frac{1502 - 957}{657} = 0.830 \text{ m}$$

$x_b \quad = \quad 1.157$ m

Check shear in areas where bottom steel is in tension and

$\rho_l \quad = \quad 0.0015$ (min. steel)

V_{Rd1} = $0.34(1.2 + 0.06)3000 \times 665 \times 10^{-3} = 854 > 723$ kN .. OK

No links required at RH end of base

In orthogonal direction, shear at d from column face

$$= \frac{219(3.0 - 0.45 - 0.6 \times 2)}{2} = 148 \text{ kN/m}$$

From above

$$V_{Rd1} = \frac{854}{3.0} = 284 > 148 \text{ kN/m}$$ .... OK

No links required in orthogonal direction

7.1.2.7 Punching

Length of one side of critical perimeter at $1.5d$ from face of column — 4.3.4.1P(4) & 4.3.4.2.2

$= 3 \times 690 + 600 = 2670$ mm

This extends almost the full width of the base = 3000 mm

Hence it is sufficient just to check line shear as above and shear around perimeter of column face, where

$$V_{Sd}/ud \leq 0.90\sqrt{f_{ck}} = 0.90 \times \sqrt{30} = 4.9 \text{ N/mm}^2$$ NAD 6.4(d)

The shear stress at the column face perimeter with d = 675 mm is less than 4.9 N/mm² in both cases (see Table 7.1). .... OK

Table 7.1 Punching shear at column face

Column	Perimeter (mm)	Load (kN)	Stress (N/mm²)
A	1650	1896	1.7
B	2400	2834	1.75

7.1.2.8 Crack control

Use method without direct calculation. — 4.4.2.3

Estimate service stress in reinforcement under quasi-permanent loads, using the following approximation: — 4.4.2.3(3)

$$G_k + \psi_2 Q_k = G_k + 0.3Q_k$$

The relevant loads are shown in Table 7.2.

Table 7.2 Column loads for cracking check

Load	Column A	Column B
$G_k + 0.3Q_k$ (kN)	934	1377
$1.35G_k + 1.5Q_k$ (kN)	1896	2834
Ratio	0.49	0.48

Estimated steel stress = $0.49 \times f_{yd} \times \dfrac{A_{s,req}}{A_{s,prov}}$

$$= 0.49 \times 400 \times \frac{8452}{12 \times 804} = 172 \text{ N/mm}^2$$

Either limit bar size using EC2 Table 4.11 or bar spacing using Table 4.12. 4.4.2.3(2)

In Table 4.11 bar size ≤ 25 mm > 32 mm used.

In Table 4.12 spacing ≤ 285 mm in pure flexure > 250 mm used. . OK

Check minimum reinforcement requirement 4.4.2.3(2) 4.4.2.2(3) Eqn 4.78

$$A_s \geq k_c k f_{ct,eff} A_{ct} / \sigma_s$$

For A_{ct} it is considered conservative to use $(h/2)b$.

$$\sigma_s = 100\% \times f_{yk} = 460 \text{ N/mm}^2$$

For $f_{ct,eff}$ use minimum tensile strength suggested in EC2, 3 N/mm^2.

$$k_c = 0.4 \text{ for bending}$$

For k interpolate a value for h = 75 cm, which gives k = 0.53.

Therefore

$$A_s \geq 0.4 \times 0.53 \times 3 \times 750 \times 3000/(2 \times 460) = 1555 \text{ mm}^2$$

12T32 gives A_s > 1555 mm^2 . . . . OK

7.1.2.9 Detailing

Check bar achorage detail at LH end.

The anchorage should be capable of resisting a tensile force 5.4.2.1.4(2)

$$F_s = V_{Sd} a_l / d$$

with

$$a_l = d$$ 5.4.3.2.1(1)

$$F_s = V_{Sd}$$

$$V_{Sd} = \text{column reaction} = 1896 \text{ kN}$$

The bond strength for poor conditions in the top of the pour 5.2.2.1 & 5.2.2.2

$$= 0.7 \times \text{Table 5.3 value}$$

$$f_{bd} = 0.7 \times 3 = 2.1 \text{ N/mm}^2$$

$$l_b = (\phi/4)(f_{yd}/f_{bd}) = 47.6\phi = 1524 \text{ mm}$$ Eqn 5.3

Continuing all T32 bars to end

$$A_{s,prov} = 9650 \text{ mm}^2$$

$$A_{s,req} = V_{Sd}/f_{yd} = 1896 \times 10^3/400 = 4740 \text{ mm}^2$$

Hence required anchorage, $(\frac{2}{3})l_{b.net}$ at a direct support — Figure 5.12

$= (\frac{2}{3})l_b \times 4740/9650 = 500 \text{ mm} > 0.3l_b$ .... OK

Anchorage up to face of column $= 600 - 75 = 525$ mm ..... OK

The anchorage may be increased to $l_{b.net}$, if preferred, by providing a bend at the end of the bar.

The requirement for transverse reinforcement along the anchorage length does not apply at a direct support. — 5.2.3.3

Secondary reinforcement ratio for top steel — 5.4.3.2.1

$\rho_2 \geq 0.2\rho_1 = 0.2 \times 0.00476 = 0.00095$

$d = 750 - 35 - 10 = 705$ mm

$A_s \geq 670 \text{ mm}^2/\text{m}$

Use T16 @ 250 mm crs. (804 mm²/m) transversely in top

Spacing $\leq$ 500 mm .... OK

The reinforcement details are shown in Figure 7.6.

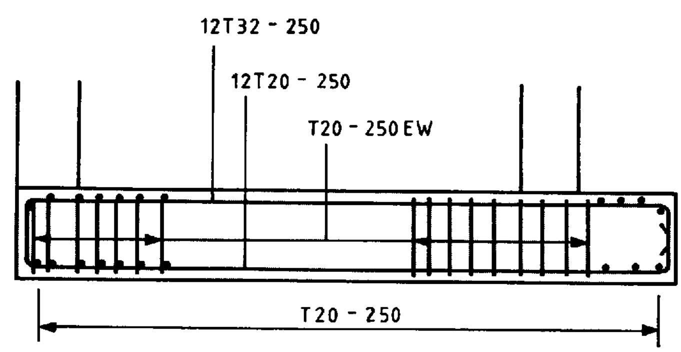


Figure 7.6 Detail of reinforcement in combined footing

7.2 Pilecap design

7.2.1 Pilecap design example using truss analogy

A four-pile group supports a 500 mm square column which carries a factored load of 2800 kN. The piles are 450 mm in diameter and spaced at 1350 mm centres.

7.2.1.1 Pilecap size

Assume a pilecap depth of 800 mm. Allow the pilecap to extend 150 mm beyond the edge of the piles, to give a base 2.1 m square as shown in Figure 7.7.

Use 2.1 m x 2.1 m x 0.8 m deep pilecap

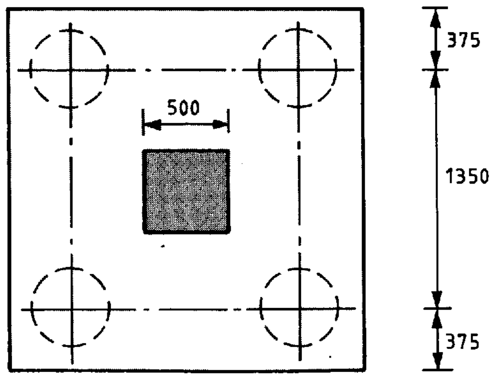


Figure 7.7 Pilecap layout

7.2.1.2 Durability

For components in non-aggressive soil and/or water, exposure class is 2(a). — Table 4.1 ENV 206

Minimum concrete strength grade is C30/37. — Table NA.1

For cement content and w/c ratio refer to ENV 206 Table 3.

Minimum cover to reinforcement is 30 mm. — NAD Table 6

Use 100 mm nominal bottom cover over piles and 50 mm sides

7.2.1.3 Materials

Type 2 deformed reinforcement with f_{yk} = 460 N/mm^2 — NAD 6.3(a)

Concrete strength grade C30/37 with maximum aggregate size 20 mm.

7.2.1.4 Element classification

A beam whose span is less than twice its overall depth is considered a deep beam. — 2.5.2.1(2)

With the effective span, l_{eff}, taken to the centre of the piles: — 2.5.2.2.2

$$\frac{l_{eff}}{h} = \frac{1350}{800} = 1.7 < 2$$

Therefore treat as deep beam for analysis.

7.2.1.5 Loading

Ultimate column load	=	2800 kN		
Pilecap (self-weight)	=	0.8 × 25	=	20 kN/m^2
Ultimate pilecap load	=	1.35 × 20	=	27 kN/m^2

Eqn 2.8(a)

7.2.1.6 Design

Deep beams under a concentrated load may be designed using a strut and tie model. — 2.5.3.7.3

Use a model with a node at the centre of the loaded area and lower nodes over the centre lines of the piles at the level of the tension reinforcement together with an effective column load to account for the pilecap weight of, for example: **BS 8110 3.11.4.1**

$$N_{Sd} = 2800 + 1.35^2 \times 27 = 2850 \text{ kN}$$

$$d_{ave} = 800 - 100 - 25 = 675 \text{ mm}$$

The total tensile force in each direction

$$F_{Sd} = \frac{N_{Sd} \times l_{eff}}{4d} = \frac{2850 \times 1350}{4 \times 675} = 1425 \text{ kN}$$

For reinforcement

$$f_{yd} = \frac{f_{yk}}{\gamma_s} = \frac{460}{1.15} = 400 \text{ N/mm}^2$$ **2.2.3.2P(1) Table 2.3**

$$A_{s,req} = \frac{1425 \times 10^3}{400} = 3563 \text{ mm}^2$$

There are no specific requirements within EC2 for the distribution of the calculated reinforcement. The provisions of BS 8110: Part 1: Clause 3.11.4.2 are adopted in this example.

With piles spaced at 3 times the diameter, the reinforcement may be uniformly distributed.

Use 8T25 at 275 mm crs. (3928 mm^2)

Maximum spacing $= 3h \not> 500 = 500 > 275$ mm .... OK **NAD Table 3**

Minimum $A_s = \dfrac{0.6 b_t d}{f_{yk}} \not< 0.0015 b_t d = 0.0015 \times 2100 \times 675 = 2127$ mm^2. . OK **5.4.3.2.1(4) 5.4.2.1.1(1)**

The reinforcement details are shown in Figure 7.8.

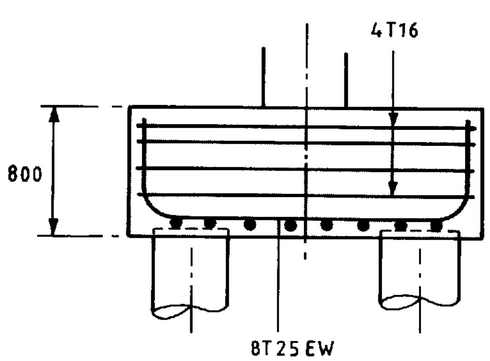


Figure 7.8 Details of pilecap reinforcement

7.2.1.7 Shear

Only in elements such as slabs may shear reinforcement be omitted where calculations justify. 4.3.2.1P(2)

Despite the classification for the pilecap given above, in line with common UK practice, it is not intended to provide shear reinforcement when $V_{Sd} \leq V_{Rd1}$. 4.3.2.2(2)

Take the critical section for shear to be located at 20% of the pile diameter into the piles, extending the full width of the pilecap. BS 8110 Figure 3.23

Distance from centre of loaded area

$$x = 1350/2 - 0.3 \times 450 = 540 \text{ mm}$$

Shear resistance

$$V_{Rd1} = \tau_{Rd}\, k(1.2 + 40\rho_l)b_w d$$ 4.3.2.3 Eqn 4.18

$$\tau_{Rd} = 0.34 \text{ N/mm}^2$$ Table 4.8

$$k = 1.6 - d \not< 1.0 = 1.0$$

$$\rho_l = \frac{3928}{2100 \times 675} = 0.00277$$

All of tension steel is to continue sufficiently past critical section; check when detailing.

$$V_{Rd1} = 0.34(1.2 + 40 \times 0.00277)\ 2100 \times 675 \times 10^{-3} = 632 \text{ kN}$$

Consider enhanced resistance close to the supports 4.3.2.2(5)

$$\beta = \frac{2.5d}{x} = \frac{2.5 \times 675}{540} = 3.125$$ 4.3.2.2(9)

$1.0 \leq \beta \leq 5.0$ . . . . OK

Shear force

$$V_{Sd} = \frac{2850}{2} = 1425 \text{ kN}$$

$$< \beta V_{Rd1} = 3.125 \times 632 = 1975 \text{ kN}$$

No shear reinforcement required

Having taken into account the increased shear strength close to the supports, it is necessary to ensure that the reinforcement is properly anchored. 4.3.2.2(11)

In this case all reinforcement will extend to centre line of pile and be anchored beyond that position. . . . . OK

7.2.1.8 Punching

Piles fall within 1.5d perimeter from column face, it is thus only necessary to check shear around column perimeter, where 4.3.4.2.2(1)

$$\text{Stress} \leq 0.9\sqrt{f_{ck}} = 0.9 \times \sqrt{30} = 4.9 \text{ N/mm}^2$$ NAD 6.4(d)

No enhancement of this value is permitted. 4.3.2.2(5)

$$\text{Stress} = \frac{2800 \times 10^3}{4 \times 500 \times 675} = 2.1 < 4.9 \text{ N/mm}^2 \text{ .... OK}$$

7.2.1.9 Crack control

Use method without direct calculation. 4.4.2.3

Estimate service stress in reinforcement under quasi-permanent loads using following method 4.4.2.3(3)

$$G_k + \psi_2 Q_k = G_k + 0.3Q_k$$

For this example the column loads, G_k = 1200 kN and Q_k = 785 kN

$$\text{Hence the quasi-permanent load/factored load} = \frac{1200 + 0.3 \times 785}{2800} = 0.51$$

Estimated steel stress

$$= 0.51 \times f_{yd} \times \frac{A_{s,req}}{A_{s,prov}} = 0.51 \times 400 \times \frac{3563}{3928} = 185 \text{ N/mm}^2$$

Either limit bar size to EC2 Table 4.11 value or bar spacing to EC2 Table 4.12 value. 4.4.2.3(2)

From Table 4.11 bar size $\leq$ 25 mm = 25 mm used .... OK

From Table 4.12 bar spacing $\leq$ 270 mm < 275 mm used

Check minimum reinforcement requirement 4.4.2.3(2) 4.4.2.2(3) Eqn 4.78

$$A_s \geq k_c k f_{ct,eff} A_{ct}/\sigma_s$$

For A_{ct} it is considered conservative to use (h/2)b.

$$\sigma_s = 100\% \times f_{yk} = 460 \text{ N/mm}^2$$

For $f_{ct,eff}$ use minimum tensile strength suggested by EC2, 3 N/mm^2.

$$k_c = 0.4 \text{ for bending}$$

$$k = 0.5 \text{ for } h \geq 80 \text{ cm}$$

Therefore

$$A_s \geq 1096 \text{ mm}^2$$

$$A_{s,prov} = 3928 \text{ mm}^2 \text{ .... OK}$$

7.2.1.10 Detailing

The reinforcement corresponding to the ties in the model should be fully anchored beyond the nodes, i.e., past the centres of piles. 5.4.5(1)

$$l_b = \left(\frac{\phi}{4}\right) \frac{f_{yd}}{f_{bd}}$$ 5.2.2.3(2)

For bars in bottom half of a pour, good bond may be assumed. 5.2.2.1

Hence

$f_{bd} = 3.0 \text{ N/mm}^2 \quad (\phi \leq 32 \text{ mm})$ Table 5.3

$$l_b = \frac{25 \times 400}{4 \times 3} = 834 \text{ mm}$$

$$l_{b,net} = \frac{\alpha_a l_b A_{s,req}}{A_{s,prov}} \geq l_{b,min}$$

Using bobbed reinforcement, $\alpha_a = 0.7$

$$l_{b,net} = 0.7 \times 834 \times \frac{3563}{3928} = 530 \text{ mm}$$

Length beyond centre of pile allowing for end cover

$= 375 - 50 = 325 < 530$ mm

Bars cannot be anchored in manner shown in EC2 Figure 5.2. Use bent-up bars with large radius bend and anchorage length

$$l_b \times \frac{A_{s,req}}{A_{s,prov}} = 756 \text{ mm}$$

Diameter of bends can be obtained from NAD Table 8[(1)]. Assume that the limits given for minimum cover in the table are equally applicable to bar centres. NAD Table 8

For T25 @ 275 mm crs., bend diameter $= 13\phi$,

bend radius $= (13/2) \times 25 = 165$ mm

The use of NAD Table 8 is conservative, as it is based on full stress in the bars at the bend. The values given appear to be consistent with BS 8110: Part 1: Clause 3.12.8.25 using $f_{cu} = 30 \text{ N/mm}^2$.

For concrete placed in the UK, it should be possible to demonstrate compliance with EC2 Clause 5.2.1.2P(1) by using the BS 8110 Clause above, with the result that smaller diameter bends may be used.

For the edge bars, which have a minimum cover $> 3\phi = 75$ mm, NAD Table 8 gives 200 mm radius bend (see Figure 7.9).

The requirement for transverse reinforcement along the anchorage length does not apply at a direct support.

Provide bars to act as horizontal links, such as 4T16 @ 150 mm crs.

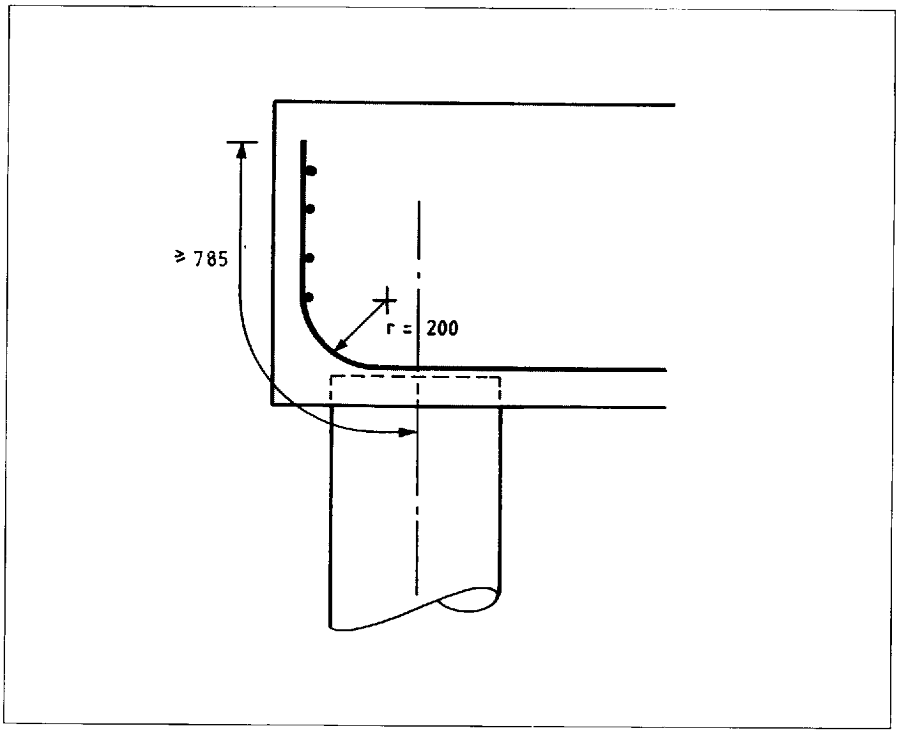


Figure 7.9 Detail of bent-up bars

7.2.2 Pilecap design example using bending theory

Take the pilecap from the preceding example but use bending theory to determine the bottom reinforcement. The shear force diagram is shown in Figure 7.10.

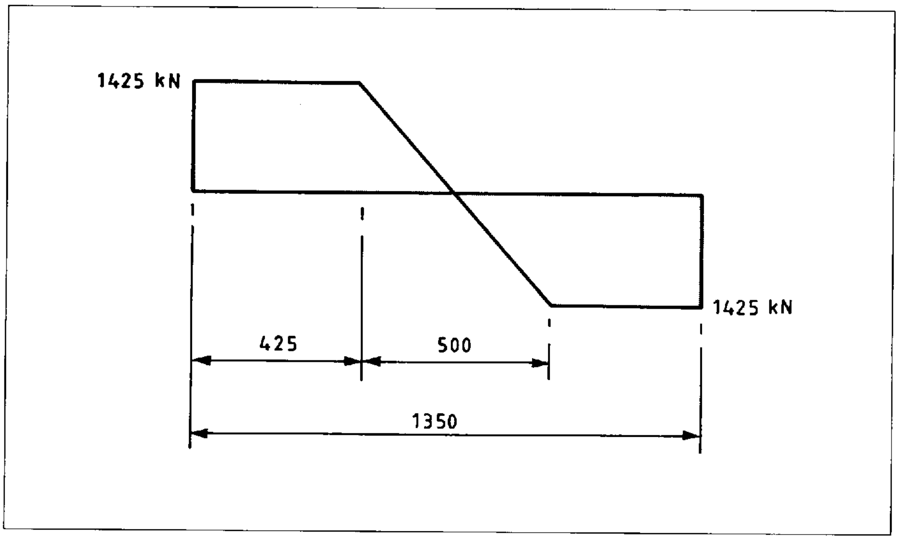


Figure 7.10 Shear force diagram

7.2.2.1 Flexural reinforcement

$$M_{Sd} = 1425\left(0.425 + \frac{0.25}{2}\right) = 784 \text{ kNm}$$

$$z = 0.975d = 658 \text{ mm}$$

$$A_s = 2979 \text{ mm}^2$$

Because of the difference in modelling, this is less reinforcement than the previous example.

7.2.2.2 Detailing

At an end support, the anchorage of bottom reinforcement needs to be capable of resisting a force: 5.4.2.1.4(2)

$$F_s = V_{Sd}a_l/d + N_{Sd}$$ Eqn 5.15

$$N_{Sd} = 0 \text{ in this case}$$

with

$$a_l = d$$ 5.4.3.2.1(1)

$$F_s = V_{Sd} = 1425 \text{ kN}$$

$$A_{s,req} = 1425 \times 10^3/400 = 3563 \text{ mm}^2$$

This is identical to the area of steel required in the previous example.

Use 8T25 as before (3928 mm^2)

Using the same detail of bobbed bars

$$l_{b,net} = 530 \text{ mm}$$

EC2 Figure 5.12(a) applies and is taken to require an anchorage length, $(2/3)l_{b,net}$ = 353 mm past the line of contact between the beam and its support.

Using a position 20% into the pile to represent the line of contact, the length available for anchorage

$$= 0.3 \times \text{pile dia.} + 375 - \text{cover}$$

$$= 0.3 \times 450 + 375 - 50 = 460 > 353 \text{ mm} \ldots\text{. OK}$$

8 SPECIAL DETAILS

8.1 Corbels

8.1.1 Introduction 2.5.3.7.2

Consider a corbel designed to carry a vertical ultimate design load of 400 kN with the line of action of the load 200 mm from the face of the support (wall, column etc), as shown in Figure 8.1.

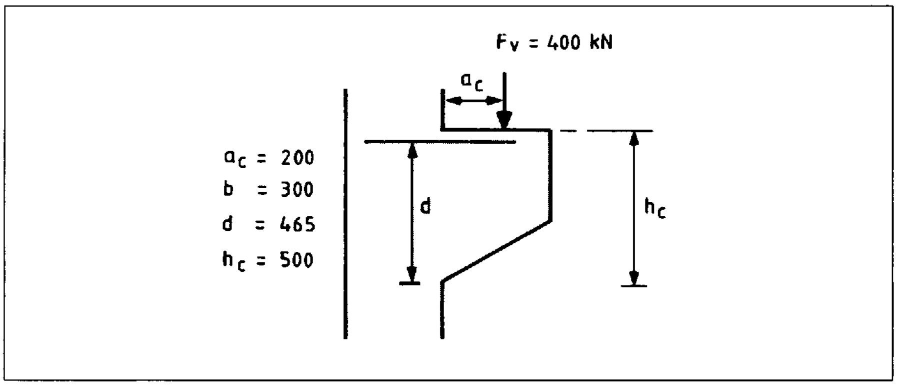


Figure 8.1 Corbel dimensions

8.1.2 Materials

f_{ck} = 30 N/mm^2 (concrete strength class C30/37)

f_{yk} = 460 N/mm^2 (characteristic yield strength)

8.1.3 Design

8.1.3.1 Check overall depth of corbel

Conservatively, the maximum shear in the corbel should not exceed V_{Rd1}. The depth of the corbel could be reduced by putting $F_v \leq V_{Rd2}$ but this would give an increased tie force and consequent detailing problems. The value of τ_{Rd} in the expression for V_{Rd1} (EC2 Eqn 4.18[1]) may be modified by the factor β defined in EC2 Clause 4.3.2.2(9). 2.5.3.7.2(5) 4.3.2.3

By inspection β will be a minimum when $x = a_c$ in EC2 Eqn 4.17. Hence V_{Rd1} will also be a minimum.

Now

$V_{Rd1} = [\beta\tau_{Rd}k(1.2 + 40\rho_l) + 0.15\sigma_{cp}]b_w d$ Eqn 4.18

$\beta = 2.5d/x$ with $1.0 \leq \beta \leq 5.0$ Eqn 4.17

$$= \frac{2.5 \times 465}{200} = 5.81 \leq 5.0$$

τ_{Rd} = 0.34 N/mm^2 Table 4.8

$k = 1.6 - d \not< 1 = 1.14$ m

ρ_l is assumed to be 0.006 (4T16)

No provision has been made to limit horizontal forces at the support; therefore a minimum horizontal force (H_c) acting at the bearing area should be assumed. This is given by 2.5.3.7.2(4)

$$H_c = 0.2F_v = \pm 80 \text{ kN}$$

$$\sigma_{cp} = \frac{N_{Sd}}{A_c} \text{ where } N_{Sd} = -80 \text{ kN}$$

Therefore

$$\sigma_{cp} = \frac{-80 \times 10^3}{465 \times 300} = -0.6 \text{ N/mm}^2$$

Hence

$$V_{Rd1} = [5 \times 0.34 \times 1.23 (1.2 + 40 \times 0.006) - 0.15 \times 0.6] \times 465 \times 300 = 407 \text{ kN}$$

Therefore

$V_{Rd1} > V_{Sd} = F_v = 400$ kN .... OK 2.5.3.7.2(5)

8.1.3.2 Determine main reinforcement requirement

Now $0.4h_c \leq a_c$, therefore a simple strut and tie model may be assumed, as shown in Figure 8.2. 2.5.3.7.2(1)

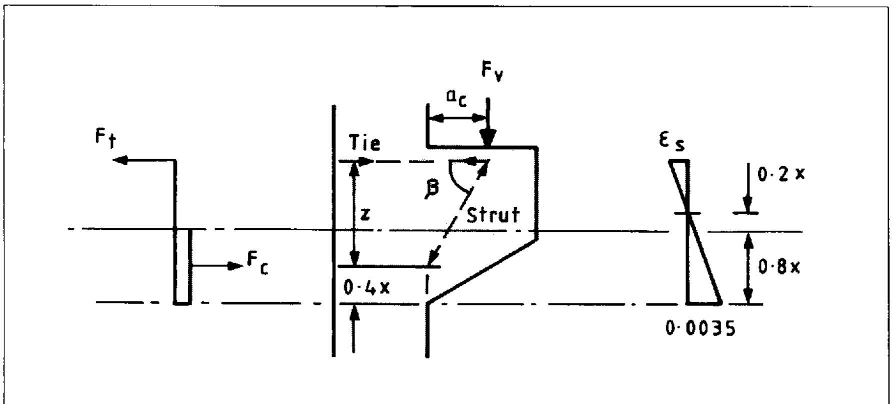


Figure 8.2 Strut and tie model

Under the vertical load

$$F_t = \frac{F_v a_c}{z} \text{ ; and}$$

$$F_c \leq \left(\frac{0.85 f_{ck}}{\gamma_c}\right) (0.8x) b \cos^2\beta$$

The determination of x will be an iterative procedure.

Choose x such that $\epsilon_s = 0.002$ and $f_{yd} = 400$ N/mm^2

Therefore

$$x = \frac{0.0035}{0.0035 + 0.002} \times 465 = 296 \text{ mm}$$

Now $z = d - 0.8x/2 = 347$ mm and $\cos\beta = 0.5$

Therefore, from above,

$$F_t = \frac{400 \times 200}{347} = 231 \text{ kN and}$$

$$F_c \leq \frac{0.85 \times 30 \times 0.8 \times 296 \times 300 \times 0.5^2}{1.5} = 301 \text{ kN}$$

For equilibrium $F_t = F_c$ and further refinement gives

$x = 235$ mm, $z = 371$ mm, $F_t = 216$ kN

In addition, EC2 Clause 2.5.3.7.2(4) requires a horizontal force of H_c to be applied at the bearing area. 2.5.3.7.2(4)

$$H_c \geq 0.2F_v = 0.2 \times 400 = 80 \text{ kN}$$

$$F_t + H_c = 296 \text{ kN}$$

$$A_{s,req} = \frac{296 \times 10^3}{460/1.15} = 740 \text{ mm}^2$$

Use 4T16 bars

8.1.3.3 Check crushing of compression strut

This has been checked directly by the calculation of F_c above. However, an indirect check may also be made.

$$V_{Rd2} = \left[\left(\tfrac{1}{2}\right) \nu f_{cd}\right] b_w 0.9d$$ Eqn 4.19

$$\nu = 0.7 - \frac{f_{ck}}{200} = 0.55 \not< 0.5$$ Eqn 4.20

Therefore

$$V_{Rd2} = \left(\tfrac{1}{2}\right) \times 0.55 \times 20 \times 300 \times 0.9 \times 465 = 690 \text{ kN}$$

Hence

$V_{Rd2} > F_v = 400$ kN .... OK

8.1.3.4 Check link reinforcement requirements 5.4.4(2)

Links are required if:

$$A_s \geq 0.4\, A_c f_{cd}/f_{yd}$$ Eqn 5.21

$$A_c = 500 \times 300 = 150 \times 10^3 \text{ mm}^2$$

$$f_{cd} = \frac{30}{1.5} = 20 \text{ N/mm}^2 \qquad f_{yd} = \frac{460}{1.15} = 400 \text{ N/mm}^2$$

Hence, links are required if

$$A_s \geq 0.4 \times 150 \times 10^3 \times 20/400 = 3000 \text{ mm}^2$$

Now

$$A_{s,prov} = 804 < 3000 \text{ mm}^2$$

Therefore links are not required

Nevertheless, in practice some links should be provided to assist in fixing the main reinforcement.

$$A_{sw} \geq 0.4A_{s,prov} = 0.4 \times 804 = 322 \text{ mm}^2$$ 5.4.4(2)

Use 4T8 links (8 legs)

8.1.3.5 Check bearing area of corbel

Allowable design ultimate bearing stress = $0.8f_{cd}$ for bearing bedded in concrete. EC2, Part 1B

Therefore area required $= \dfrac{400 \times 10^3}{0.8 \times 20} = 25000 \text{ mm}^2$

Assume transverse bearing = 250 mm

Therefore length of bearing = 100 mm

8.1.4 Detailing

The reinforcement details are shown in Figure 8.3.

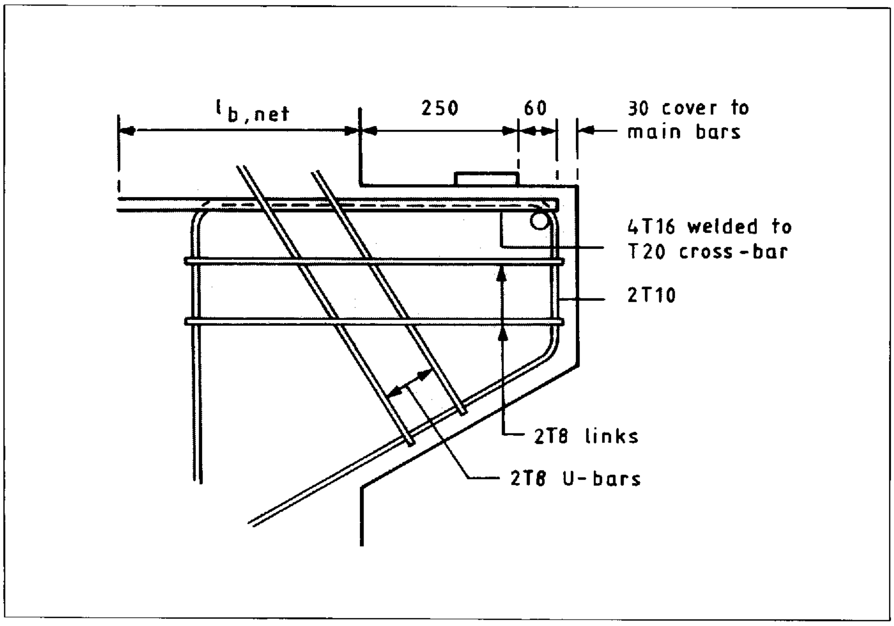


Figure 8.3 Corbel reinforcement details

8.1.4.1 Anchorage of main bars at front edge of corbel

Anchor T16 ties by means of a cross bar running horizontally and welded to the ties.

$F_t + H_c = 296$ kN

An allowable bearing stress under the cross bar can be obtained from EC2 Eqn 5.22 as 5.4.8.1

$f_{Rdu} = 3.3f_{cd}$ Eqn 5.22 (mod)

Note:
Use of this stress requires that the concrete be confined by means of links etc. In areas where the cover is small, the designer may wish to use a modified version of Eqn 50 in BS 8110[(2)].

Therefore area of bar required $= \dfrac{296 \times 10^3}{3.3 \times 20} = 4485$ mm²

For a T20 bar, length required is 225 mm.

Use T20 cross bar 240 mm long welded to T16 ties

8.1.4.2 Anchorage of main bars into support

Required anchorage length

$$l_{b.net} = \alpha_a l_b \times \frac{A_{s.req}}{A_{s.prov}} \not< l_{b.min}$$ Eqn 5.4

Now

$l_b = (\phi/4)(f_{yd}/f_{bd})$ Eqn 5.3

$f_{yd} = 400$ N/mm²

Bond conditions may be considered good as the T16 bars will be anchored into a substantial support (column or wall). 5.2.2.1(2)(b)

$f_{bd} = 3$ N/mm² Table 5.3

$l_b = (16/4) \times (400/3) = 533$ mm

Now

$A_{s.req} = 740$ mm² $A_{s.prov} = 804$ mm² $\alpha_a = 1$ 5.2.3.4.1(1)

Therefore

$$l_{b.net} = 1 \times 533 \times \frac{740}{804} = 490 \text{ mm}$$

$l_{b.min} = 0.3l_b \not< 10\phi$ or 100 mm

Provide $l_{b.net}$ = 490 mm (see Figure 8.3)

The detail at the front edge of the corbel is shown in Figure 8.4.

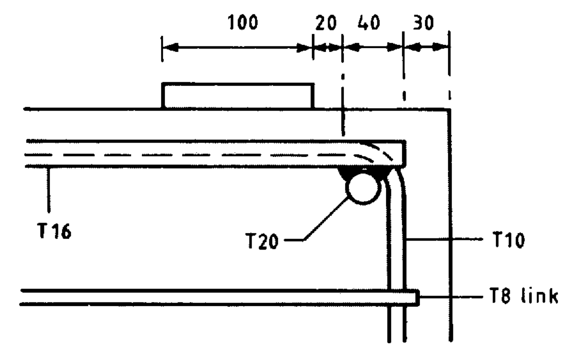


Figure 8.4 Detail at front edge of corbel

The inside face of the T20 bar is positioned not less than the cover beyond the edge of the bearing area.

This is an interpretation of BS 8110 as no guidance is given in EC2.

8.2 Nibs

8.2.1 Introduction

Consider a nib designed to carry a precast concrete floor slab imposing a vertical ultimate design load of 25 kN/m.

8.2.2 Materials

f_{ck} = 30 N/mm^2 (concrete strength class C30/37)

f_{yk} = 460 N/mm^2 (characteristic yield strength)

8.2.3 Design

Provide a 15 mm chamfer to the outside edge of the nib and assume the line of action of the load occurs at the upper edge of the chamfer.

Permissible design ultimate bearing stress = $0.6f_{cd}$ for dry bearing — EC2 Part 1B

Therefore minimum width of bearing $= \dfrac{25 \times 10^3}{0.6 \times 20 \times 1000} = 2.1$ mm

Minimum width of bearing for non-isolated member = 40 mm — BS 8110 5.2.3.2

Allowance for nib spalling = 20 mm — BS 8110 Table 5.1

Allowance for inaccuracies = 25 mm — BS 8110 5.2.4

Nominal bearing width = 40 + 20 + 25 = 85 mm

Allow an additional 25 mm for chamfer on supported member.

Width of nib projection = 85 + 25 = 110 mm

The distance of the line of action of the load from the face of the beam

$= 110 - 15 = 95$ mm

Assuming 20 mm cover to the T10 links in the beam

$a_c = 95 + 20 + 5 = 120$ mm

Check minimum depth of nib.

Assuming T8 bars, minimum internal diameter of loop is 6ϕ. NAD Table 8

Therefore minimum depth of nib $= 20 + 8 \times 8 + 20 = 104$ mm

Depth of nib = 105 mm

$M = 25 \times 0.12 = 3$ kNm/m

Effective depth (d) $= 105 - 20 - 4 = 81$ mm

$$\frac{M}{bd^2 f_{ck}} = \frac{3 \times 10^6}{1000 \times 81^2 \times 30} = 0.015$$

$$\frac{A_s f_{yk}}{bdf_{ck}} = 0.018 \text{ (Section } \mathbf{13}\text{, Table 13.1)}$$

$$A_s = \frac{0.018bdf_{ck}}{f_{yk}} = \frac{0.018 \times 1000 \times 81 \times 30}{460} = 95 \text{ mm}^2$$

Check minimum area of reinforcement 5.4.2.1.1(1)

$$A_s = 0.6\left(\frac{b_t d}{f_{yk}}\right) \not< 0.0015 b_t d$$ Eqn 5.14

$$= \frac{0.6 \times 1000 \times 81}{460} = 106 \text{ mm}^2$$

$$\not< 0.0015 \times 1000 \times 81 = 122 \text{ mm}^2$$

Check minimum area of reinforcement for crack control 4.4.2.2

$A_s = k_c k f_{ct,eff} A_{ct} / \sigma_s$ Eqn 4.78

$k_c = 0.4$ for bending

$k = 0.8$ for $h \leq 300$ mm

$f_{ct,eff} = 3.0$ N/mm²

$$A_{ct} = \frac{bh}{2} = \frac{1000 \times 105}{2} = 52.5 \times 10^3 \text{ mm}^2$$

$\sigma_s = 460$ N/mm²

Therefore

$A_s = 0.4 \times 0.8 \times 3.0 \times 52.5 \times 10^3/460 = 110$ mm²

No further check for crack control is necessary as h = 105 ≤ 200 mm. 4.4.2.3.(1)

Maximum bar spacing = $3h$ = 315 ≤ 500 mm NAD Table 3

Use T8 @ 300 mm crs. (168 mm²/m)

5.4.3.2.1(4)

The reinforcement details are shown in Figure 8.5.

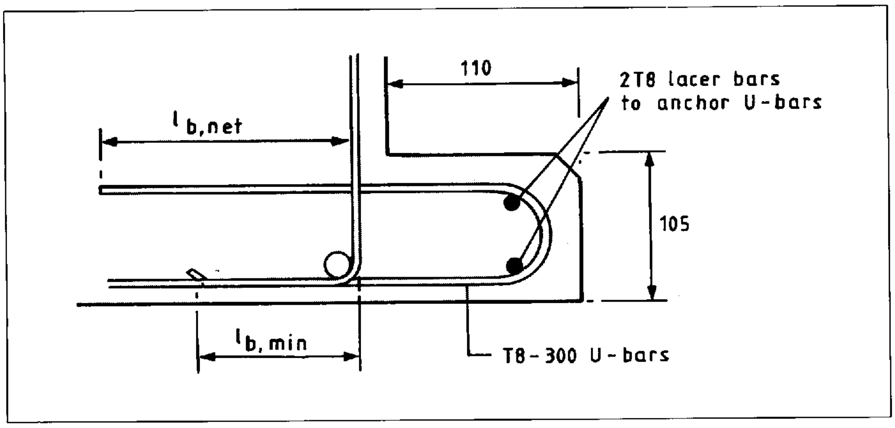


Figure 8.5 Nib reinforcement details

Check shear in nib, taking into account the proximity of the concentrated load to the support. 4.3.2.3

$$V_{Rd1} = \left[\beta\tau_{Rd}k(1.2 + 40\rho_l) + 0.15\sigma_{cp}\right]b_w d$$ Eqn 4.18

$$\beta = 2.5d/x = \frac{2.5 \times 81}{120} = 1.69$$ Eqn 4.17

$$\tau_{Rd} = 0.34 \text{ N/mm}^2$$

$$k = 1.6 - d \not< 1 = 1.52$$

$$\rho_l = \frac{A_{sl}}{b_w d} = \frac{168}{1000 \times 81} = 0.0021$$

$$\sigma_{cp} = \frac{N_{Sd}}{A_c} = 0$$

Therefore

$$V_{Rd1} = 1.69 \times 0.34 \times 1.52(1.2 + 40 \times 0.0021) \times 1000 \times 81 = 90.8 \text{ kN/m}$$

$$V_{Sd} = 25 \text{ kN/m}$$

Therefore

$V_{Rd1} > V_{Sd}$ .... OK

Check anchorage of T8 bars.

$$l_{b,net} = \alpha_a\, l_b \left(\frac{A_{s,req}}{A_{s,prov}}\right) \not< l_{b,min}$$ Eqn 5.4

Now

$l_b = (\phi/4)\ (f_{yd}/f_{bd})$ — Eqn 5.3

$f_{yd} = 400\ \text{N/mm}^2$

Bond conditions may be considered good as the bars are anchored at least 300 mm from the top of the member. — 5.2.2.1(2)(b)

$f_{bd} = 3\ \text{N/mm}^2$ — Table 5.3

$l_b = (8/4) \times (400/3) = 267\ \text{mm}$

Now $A_{s,req} = 122\ \text{mm}^2 \quad A_{s,prov} = 168\ \text{mm}^2 \quad \alpha_a = 1$

Therefore

$$l_{b,net} = 1 \times 267 \times \frac{122}{168} = 194\ \text{mm} \not< l_{b,min}$$

For bars in tension

$l_{b,min} = 0.3l_b \not< 10\phi$ or 100 mm

Therefore

$l_{b,min} = 100\ \text{mm}$

$l_{b,net} = 194$ mm (see Figure 8.5)

For bars in compression

$l_{b,min} = 0.6l_b \not< 10\phi$ or 100 mm

Therefore

$l_{b,min} = 160\ \text{mm}$

$l_{b,min} = 160$ mm (see Figure 8.5)

8.3 Simply supported ends

8.3.1 Directly supported ends

Reinforcement anchorage requirements are shown in Figure 8.6.

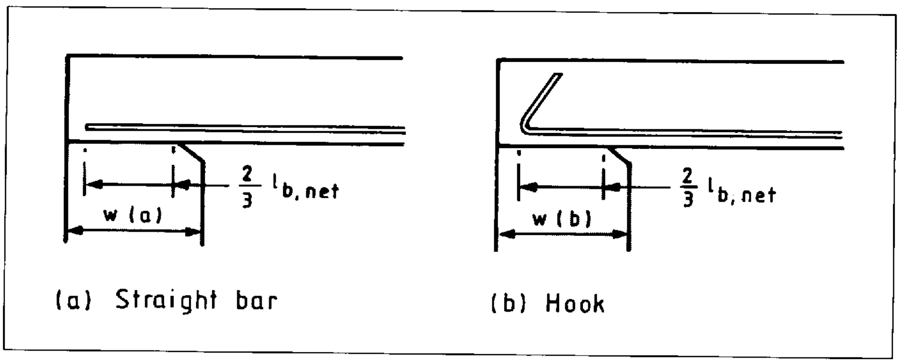


Figure 8.6 Anchorage at a direct support

Figure 8.6(a) shows anchorage of tensile reinforcement being achieved using a straight bar. It should be noted that EC2 does not permit straight anchorage or bends for smooth bars of more than 8 mm diameter. 5.4.2.1.4(3) Figure 5.12(a) 5.2.3.2(2)

Note:
The CEB–FIP Model Code[16] gives a factor of 1.0 for $l_{b,net}$ as opposed to 2/3 in EC2. Designers may wish to consider using the higher value.

Typical values for anchorage length and support width, w, can be obtained for (a) and (b) in Figure 8.6.

Assume

$$f_{ck} = 30 \text{ N/mm}^2, \quad f_{yk} = 460 \text{ N/mm}^2$$

$$\frac{A_{s,req}}{A_{s,prov}} = 1.0$$

Note: NAD 6.5
$A_{s,req}$ may be taken as one quarter of the reinforcement required at mid-span but not less than that required to resist the tensile force given by EC2 Eqn 5.15. 5.4.2.1.4(3) 5.4.2.1.4(2)

$$l_{b,net} = \alpha_a l_b \left(\frac{A_{s,req}}{A_{s,prov}}\right) \not< l_{b,min}$$ Eqn 5.4

$$l_b = (\phi/4)\ (f_{yd}/f_{bd})$$ Eqn 5.3

$$f_{bd} = 3 \text{ N/mm}^2$$ Table 5.3

$$f_{yd} = 400 \text{ N/mm}^2$$

Therefore

$$l_b = (\phi/4) \times (400/3) = 33.3\phi$$

$$l_{b,min} = 0.3 l_b \not< 10\phi \text{ or } 100 \text{ mm}$$ Eqn 5.5

α_a = 1 for straight bars; or

α_a = 0.7 for curved bars with 3ϕ transverse cover 5.2.3.4.1(1)

Therefore

$$l_{b,net}(a) = 1 \times 33.3\phi = 33.3\phi$$

$$l_{b,net}(b) = 0.7 \times 33.3\phi = 23.3\phi$$

Therefore width of support required in Figure 8.6(a), assuming 20 mm cover and 15 mm chamfer

$$w\,(a) = (\tfrac{2}{3}) \times 33.3\phi + 35 = 22.2\phi + 35 \text{ mm}$$

and width of support required in Figure 8.6(b), assuming as above

$$w\,(b) = (\tfrac{2}{3}) \times 23.3\phi + 35 = 15.5\phi + 35 \text{ mm}$$

The minimum support width is given by

$$w_{min} = (\tfrac{2}{3}) \times 10\phi + 35 = 6.7\phi + 35 \text{ mm}$$

where, in Figure 8.6(a), $A_{s,req} \leq 0.3A_{s,prov}$

and, in Figure 8.6(b), $A_{s,req} \leq 0.43A_{s,prov}$

As noted above, $\alpha_a = 0.7$ can only be used if the concrete cover perpendicular to the plane of curvature is at least 3ϕ. This is clearly difficult to achieve in beams without end diaphrams for bar sizes in excess of 12 mm.

The requirements for the various types of hooks, loops and bends are given in EC2 Figure 5.2. The minimum diameters of mandrels are given in NAD[1] Table 8. The required support widths are given in Table 8.1.

Table 8.1 Width of support (mm)

$\frac{A_{s,req}}{A_{s,prov}}$	ϕ (mm)	10	12	16	20	25	32
1.0 ≤0.3	w(a) w_{min}	257 102	302 116	391 143	479 169	590 203	746 250
1.0 ≤0.43	w(b) w_{min}	190 102	221 116	283 143	345 169	423 203	531 250

8.3.2 Indirectly supported ends

Reinforcement anchorage requirements are shown in Figure 8.7.

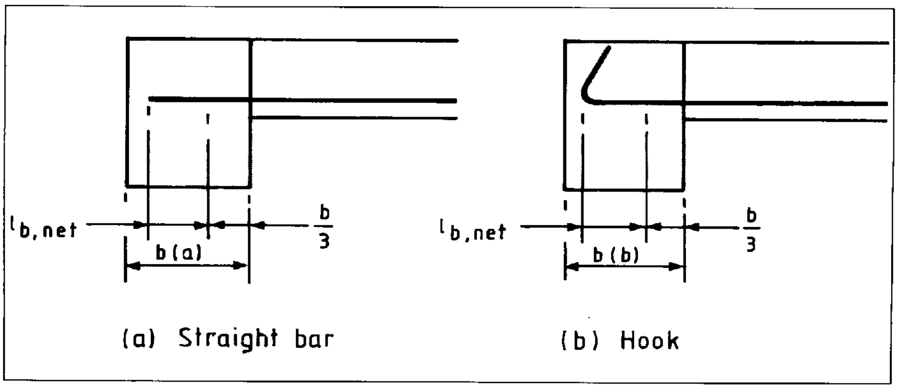


Figure 8.7 Anchorage at an indirect support

As in Section **8.3.1** above, anchorage lengths and support widths can be obtained for both straight bars and hooked bars.

The anchorage lengths are as Section **8.3.1** but the required support widths are increased.

Assuming 20 mm cover

$$b(a) = (33.3\phi + 20) \times 1.5 = 50\phi + 30 \text{ mm}$$

$$b(b) = (23.3\phi + 20) \times 1.5 = 35\phi + 30 \text{ mm}$$

The minimum support beam width is given by

$$b_{min} = (10\phi + 20) \times 1.5 = 15\phi + 30 \text{ mm}$$

where the same conditions apply as in Section **8.3.1.**

In these cases, as the beam is indirectly supported, i.e., by another beam, 3ϕ cover perpendicular to the plane of the curvature can be achieved more easily and $\alpha_a = 0.7$ can be readily used in EC2 Eqn 5.4.

The required support beam widths are given in Table 8.2.

Table 8.2 Width of support beam (mm)

$\frac{A_{s,req}}{A_{s,prov}}$	ϕ (mm)	10	12	16	20	25	32
1.0	b(a)	530	630	830	1030	1280	1630
≤0.3	b_{min}	180	210	270	330	405	510
1.0	b(b)	380	450	590	730	905	1150
≤0.43	b_{min}	180	210	270	330	405	510

8.4 Surface reinforcement

5.4.2.4

In certain circumstances it may be necessary to provide surface reinforcement located outside the links.

Surface reinforcement is provided to resist spalling from fire and where bundled bars or bar sizes greater than 32 mm are used. 5.4.2.4(3)

EC2 also refers to the use of skin reinforcement located inside the links. Skin reinforcement is provided to control cracking in the side faces of beams 1 m or more in depth. 4.4.2.3(4)

8.4.1 Design data

A beam section requiring surface reinforcement is shown in Figure 8.8.

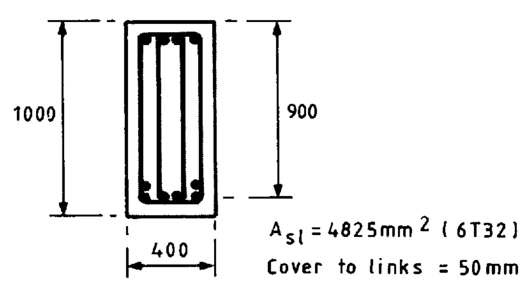


Figure 8.8 Beam section showing main reinforcement

8.4.2 Area of longitudinal surface reinforcement

$A_{s.surf} = 0.01A_{ct.ext}$ 5.4.2.4(5)

From EC2 Figure 5.15

$A_{ct.ext} = 2 \times 50 \times (1000 - 360) + 50 \times 300 = 79 \times 10^3 \text{ mm}^2$

Therefore

$A_{s.surf} = 0.01 \times 79 \times 10^3 = 790 \text{ mm}^2$

Length of $A_{ct.ext}$ internal perimeter $= 490 \times 2 + 300 = 1280$ mm

Hence

$$A_{s.surf}/\text{m} = \frac{790}{1.280} = 617 \text{ mm}^2/\text{m}$$

Use B785 fabric

This comprises 10 mm wires @ 100 mm crs. horizontally and 8 mm wires @ 200 mm crs. vertically.

Note: 5.2.3.4.3
EC2 does not directly cover the use of plain wire fabric.

Surface reinforcement may also be used as longitudinal bending reinforcement in the horizontal direction and as shear reinforcement in the vertical direction in some cases. 5.4.2.4(6)

If surface reinforcement is being used to resist shear, EC2 Clause 5.4.2.2(4) should be noted. It states that a minimum of 50% of shear reinforcement should be in the form of links. 5.4.2.2(4)

The reinforcement detail is shown in Figure 8.9.

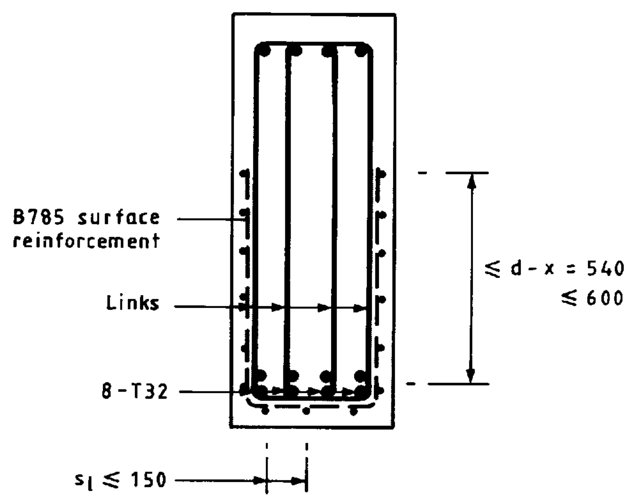


Figure 8.9 Beam section showing surface reinforcement.

9 PRESTRESSED CONCRETE

9.1 Introduction

Design of a prestressed band beam with bonded post-tensioned tendons, to support a ribbed floor slab, is set out.

This example is similar to Example 2 in the Concrete Society's *Post tensioned concrete floors: Design handbook*[17].

9.2 Design data

The floor plan and section for the structure are shown in Figure 9.1. The band beams run along the column lines in the longitudinal direction. The floor slab contains unbonded tendons, and is not designed here.

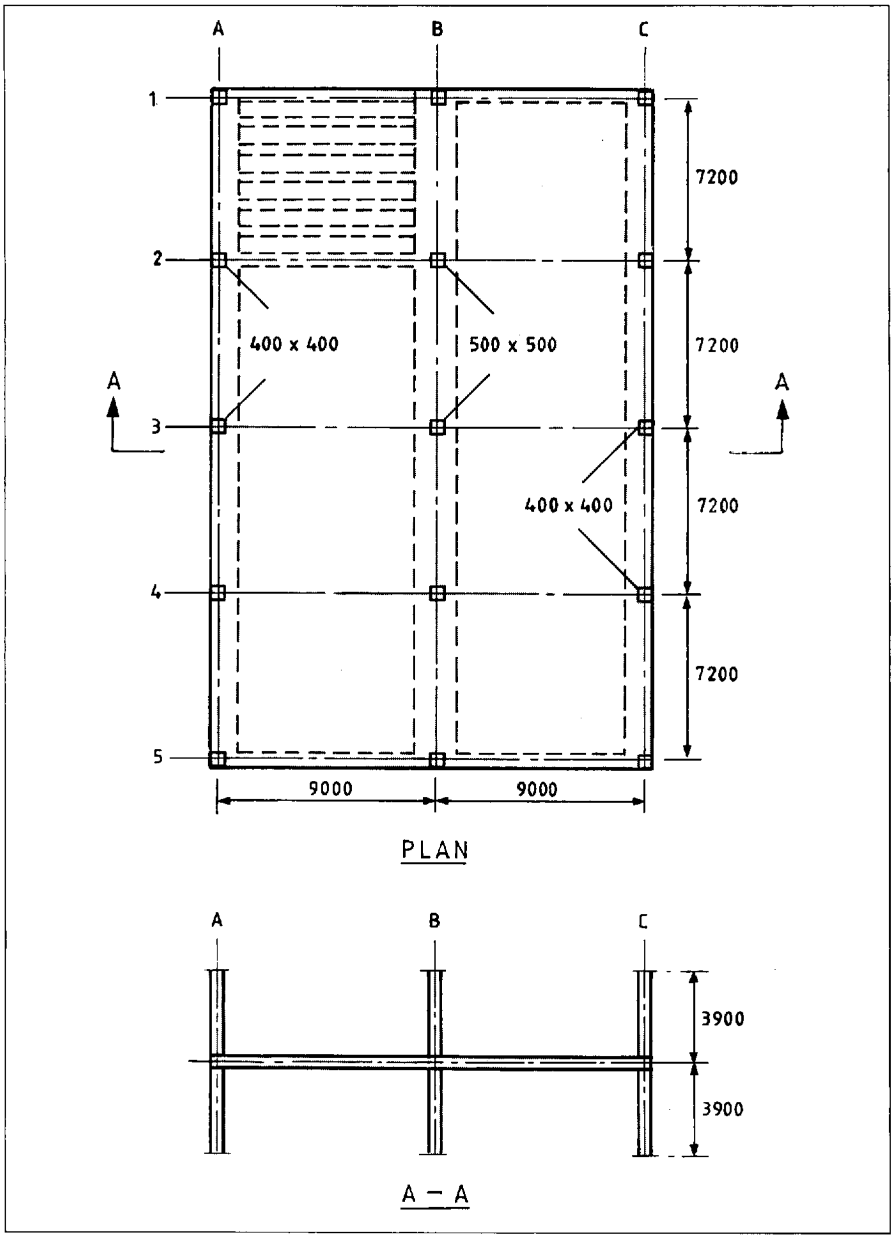


Figure 9.1 Floor plan and section

9.2.1 Beam section

The effective flange width of the beam for calculation of stiffnesses or stresses is taken as

$$b_{eff} = b_w + (\tfrac{1}{5}) l_o = 2508 \text{ mm} < b$$ 2.5.2.2.1P(2) Eqn 2.13

The beam section is shown in Figure 9.2.

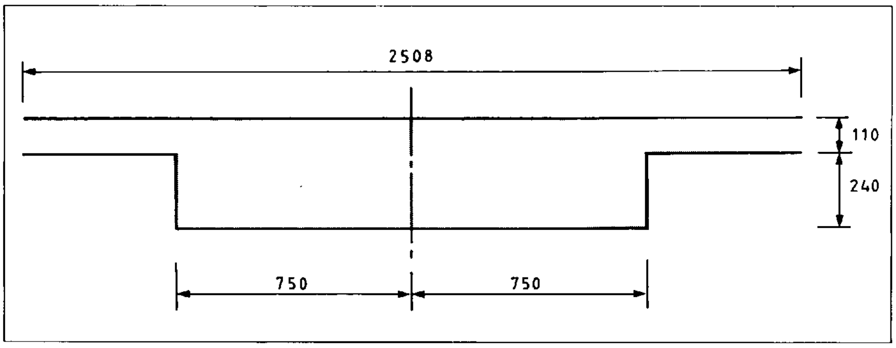


Figure 9.2 Beam section

9.2.2 Durability

For components in dry environment, exposure class is 1. Table 4.1

Minimum concrete strength grade for post-tensioned members is C25/30. 4.2.3.5.2

Minimum cover to reinforcement is 15 mm. 4.1.3.3 NAD Table 6

Use 25 mm nominal cover to reinforcement

Minimum cover to duct is given as the smaller cross-sectional dimension of the duct but not less than half the greater cross-sectional dimension. 4.1.3.3(11)

Use nominal cover to duct ≮ 50 mm

BS 8110 4.12.3.2

9.2.3 Materials

9.2.3.1 Reinforcement

Type 2 deformed bars, characteristic strength, f_{yk} = 460 N/mm^2 having high ductility NAD 6.3(a)

9.2.3.2 Prestressing steel

15.7 mm diameter superstrand, grouped in oval ducts 20 × 75 mm

Characteristic strength, f_{pk} = 1770 N/mm^2 BS 5896

A_p = 150 mm^2

E_s = 190 kN/mm^2 3.3.4.4

9.2.3.3 Concrete

In order that this example can be compared with that given in Example 2 of the *Post tensioned concrete floors: Design handbook*, a non-standard concrete strength grade has been chosen of C32/40.

f_{ck} = 32 N/mm^2

f_{ci} = 20 N/mm^2 strength at transfer

E_{cm} = $9.5 \times (32 + 8)^{1/3}$ = 32.4 say 32 kN/mm^2 3.1.2.5.2(3)

E_{ci} = $9.5 \times (20 + 8)^{1/3}$ = 28.8 say 29 kN/mm^2

9.2.4 Loading

4.2.3.5.4P(2)

Imposed loading = 5.0 kN/m^2

Effective width of slab = 10.22 m

Self-weight of slab and beam = 35.60 + 12.60 = 48.20 kN/m

9.3 Serviceability limit state

9.3.1 Tendon details

The tendon profile is shown in Figure 9.3.

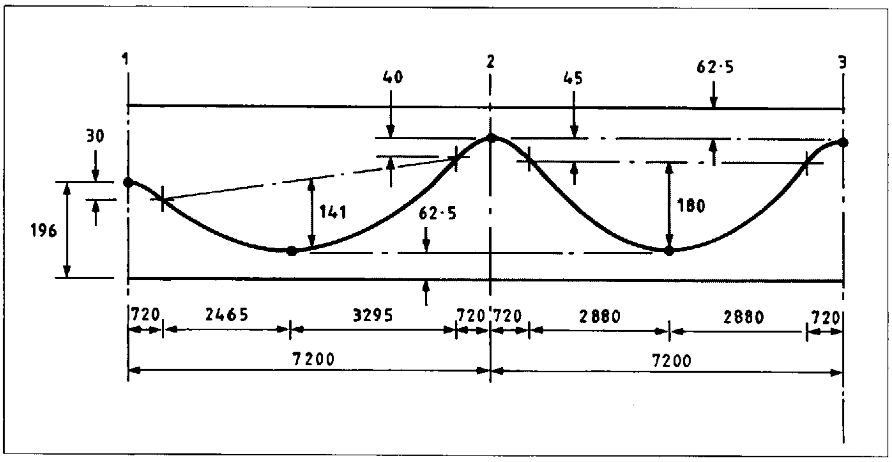


Figure 9.3 Tendon profile

Initial prestressing force is taken as

P_o = $0.75 \times f_{pk} \times A_p$ = 199.1 kN per tendon 4.2.3.5.4P(3)

Losses are assumed to be

15% of P_o at transfer
30% of P_o at service

Hence prestressing force

At transfer, $P_{m,o}$ = 169.3 kN per tendon

At service, $P_{m,\infty}$ = 139.4 kN per tendon

9.3.2 Maximum drape

Span 1–2

y = $kx(s - x)$ where s is the distance between inflexion points

Using Appendix C of *Post tensioned concrete floors: Design handbook*

$k = 1.70 \times 10^{-5}$ and $s = 5760$ mm

At mid-span

$y = 1.70 \times 10^{-5} \times 2880^2 = 141$ mm

Span 2–3 at mid-span

$y = 350 - 62.5 - 45 - 62.5 = 180$ mm

9.3.3 Prestress required

Take an equivalent balanced load equal to the self-weight.

Span 1–2

$$P_{req} = \frac{48.2 \times 5760^2}{8 \times 141 \times 1000} = 1418 \text{ kN}$$

Span 2–3

$$P_{req} = \frac{48.2 \times 5760^2}{8 \times 180 \times 1000} = 1111 \text{ kN}$$

Number of tendons required = 1418/139.4 = 10.17

Use 11 tendons throughout beam

9.3.4 Equivalent loads from prestress

The equivalent loads from the longitudinal tendons, given by $q = 8(nP_{av})a/s^2$ where $n = 11$, are calculated in Tables 9.1 and 9.2.

Table 9.1 Calculation of equivalent loads from longitudinal tendons at transfer, for the full slab width

Span	1		2	2		3
nP_{av} (kN)	1861.6	1861.6	1861.6	1861.6	1861.6	1861.6
a (mm)	30	−141	40	45	−180	45
s (mm)	1440	5760	1440	1440	5760	1440
q (kN/m)	216.9	−63.3	289.4	323.2	−80.8	323.2

Table 9.2 Calculation of equivalent loads from longitudinal tendons after all losses, for the full slab width

Span	1		2	2		3
nP_{av} (kN)	1533.4	1533.4	1533.4	1533.4	1533.4	1533.4
a (mm)	30	−141	40	45	−180	45
s (mm)	1440	5760	1440	1440	5760	1440
q (kN/m)	178.7	−52.1	238.4	266.2	−66.6	266.2

9.3.5 Load cases

For continuous beams, the following arrangements of imposed loads should be considered: 2.5.1.2(4)

(a) alternate spans loaded;

(b) any two adjacent spans loaded.

The rare, frequent and quasi-permanent load combinations should be considered where the values of ψ_1 and ψ_2 are taken from NAD Table 1[(1)]. For imposed loads in offices $\psi_1 = 0.6$ and $\psi_2 = 0.3$. 2.3.4P(2) NAD Table 1

Rare combination, $G_k + P + Q_k$ Eqn 2.9(a)

Frequent combination, $G_k + P + 0.6Q_k$ Eqn 2.9(b)

Quasi-permanent combination, $G_k + P + 0.3Q_k$ Eqn 2.9(c)

9.3.6 Maximum concrete stresses

As the beam is a T-section, the values of W_t and W_b are not equal. By calculation it can be shown that $I = 6.79 \times 10^9$ mm^4 and that the centroid of the section is at a height of 196 mm from the soffit.

$$A = 635.9 \times 10^3 \text{ mm}^2$$

Therefore

$$W_t = 44.1 \times 10^6 \text{ mm}^3$$

$$W_b = 34.6 \times 10^6 \text{ mm}^3$$

The calculation of the stresses under each load combination is not shown here. The method follows that given in the *Post tensioned concrete floors: Design handbook*. The top and bottom concrete stress for transfer conditions are given in Table 9.3 and those after all losses are given in Table 9.4.

Table 9.3 Stresses at transfer

Zone	Top stresses, f_t (N/mm^2)		Bottom stresses, f_b (N/mm^2)	
	max.	min.	max.	min.
1 (support)	3.42	—	—	1.83
1–2 (span)	3.15	2.21	3.40	2.20
2 (support)	3.81	—	—	1.34
2 (support)	4.10	—	—	0.97
2–3 (span)	3.07	1.23	4.66	2.29
3 (support)	4.45	—	—	0.53

Table 9.4 Stresses after all losses

Zone	Rare loading: Top stresses, f_t (N/mm²)		Rare loading: Bottom stresses, f_b (N/mm²)	
	max.	min.	max.	min.
1 (support)		−2.53	8.75	
1–2 (span)	5.64	1.91	3.05	−1.73
2 (support)		−4.19	10.8	
2 (support)		−3.31	9.82	
2–3 (span)	4.68	0.93	4.31	−0.50
3 (support)		−2.95	9.30	

Zone	Frequent loading: Top stresses, f_t (N/mm²)		Frequent loading: Bottom stresses, f_b (N/mm²)	
	max.	min.	max.	min.
1 (support)		−0.87	6.62	
1–2 (span)	4.37	2.19	2.69	−0.10
2 (support)		−1.96	8.02	
2 (support)		−1.17	7.00	
2–3 (span)	3.44	1.19	3.98	1.09
3 (support)		−0.77	6.48	

Zone	Quasi-permanent loading: Top stresses, f_t (N/mm²)		Quasi-permanent loading: Bottom stresses, f_b (N/mm²)	
	max.	min.	max.	min.
1 (support)		0.38	5.02	
1–2 (span)	3.41	2.19	2.69	1.13
2 (support)		−0.29	5.87	
2 (support)		0.48	4.89	
2–3 (span)	2.81	1.38	3.73	1.89
3 (support)		−0.87	4.38	

9.3.7 Allowable compressive stresses 4.4.1

To prevent longitudinal cracks the compressive stress under rare load combinations should not exceed

$0.6f_{ck} = 0.6 \times 32 = 19.2$ N/mm^2 4.4.1.1(2)

The maximum stress from Table 9.4 is 10.8 N/mm^2. . . . . OK

To control creep the compressive stress under quasi-permanent loading should not exceed

$0.45f_{ck} = 0.45 \times 32 = 14.4$ N/mm^2 4.4.1.1(3)

The maximum stress from Table 9.4 is 5.87 N/mm^2. . . . . OK

9.3.8 Limit state of cracking

No check is required at transfer since beam is totally in compression.

Design crack width for post-tensioned member under frequent load combinations

$w_k = 0.2$ mm 4.4.2.1 Table 4.10

The method adopted to determine the minimum reinforcement required is to carry out a rigorous calculation of the crack width where the flexural tensile stress under rare loads exceeds 3 N/mm^2. If the calculated crack width under frequent loads does not exceed 0.2 mm then further bonded reinforcement is not required. 4.4.1.2.(5) 4.4.1.2(7) 4.4.2.4

From Table 9.4 the stress at support 2 under the rare load combination is -4.19 N/mm^2 and hence a more detailed calculation is required. As this example is a beam, at least two longitudinal bars at the top and bottom are required to hold the links in place.

For this analysis include 2T16s in the top of the beam.

$w_k = \beta s_{rm} \epsilon_{sm}$ Eqn 4.80

$\beta = 1.7$ for load induced cracking 4.4.2.4(2)

The value of s_{rm} can be conservatively calculated as

$s_{rm} = h - x$ 4.4.2.4(8)

The value of ϵ_{sm} can be conservatively calculated as

$$\epsilon_{sm} = \frac{\sigma_s}{E_s}$$ Eqn 4.81

The values of σ_s and x, the neutral axis depth, for this example were determined from computer analysis assuming linear stress/strain relationships and no tension from the concrete.

Applied moment = -377.6 kNm (frequent load case)

$x = 213$ mm

$\sigma_s = -95.8$ N/mm^2

Hence $w_k = 1.7 \times (350 - 213) \times \dfrac{95.8}{200 \times 10^3} = 0.11 < 0.2$ mm .. OK

9.3.9 Calculation of prestress losses per tendon

$$P_{m,t} = P_o - \Delta P_c - \Delta P_\mu(x) - \Delta P_{sl} - \Delta P_t(t)$$ 2.5.4.2 Eqn 2.19

$$P_{m,o} = P_o - \Delta P_c - \Delta P_\mu(x) - \Delta P_{sl}$$ 4.2.3.5.4 Eqn 4.8

9.3.9.1 Short term losses

4.2.3.5.5 (5–8)

9.3.9.1.1 Loss due to elastic deformation

4.2.3.5.5(6)

Modular ratio $= \dfrac{E_s}{E_{ci}} = \dfrac{190}{29} = 6.55$

Maximum stress in concrete adjacent to tendons at transfer occurs at middle of span 2–3.

From Table 9.3, stress at level of tendons

$$= 4.66 - \frac{62.5}{350}(4.66 - 1.23) = 4.05 \text{ N/mm}^2$$

Average loss of force due to elastic deformation of concrete

$$\Delta P_c = 0.5 \times 4.05 \times 6.55 \times 150 \times 10^{-3} = 1.99 \text{ kN}$$

The loss, which has been conservatively based on the maximum concrete stress rather than the stress averaged along the length of the tendon, is only 1% of the jacking force and will be neglected.

9.3.9.1.2 Loss due to friction

4.2.3.5.5(8)

$$\Delta P_\mu(x) = P_o(1 - e^{-\mu(\theta + kx)})$$ Eqn 4.9

P_o = jacking force = 199.1 kN

μ = 0.19 (recommended for strand)

k = 0.0085 (from *Post tensioned concrete floors: Design handbook*)

$\theta = \Sigma\left(\dfrac{8a}{s}\right)$ for each span

x = 7.2 m for each span

Span 1–2

$$\theta = \frac{4 \times 30}{1440} + \frac{8 \times 141}{5760} + \frac{4 \times 40}{1440} = 0.392$$

$$\Delta P_2 = 199.1(1 - e^{-0.19(0.392 + 0.0085 \times 7.2)}) = 16.4 \text{ kN}$$

Therefore

$$P_2 = 199.1 - 16.4 = 182.7 \text{ kN}$$

Span 2–3

$$\Theta = \frac{8 \times 45}{1440} + \frac{8 \times 180}{5760} = 0.500$$

$$\Delta P_3 = 199.1(1 - e^{-0.19(0.892 + 0.0085 \times 14.4)}) = 34.9 \text{ kN}$$

Therefore

$$P_3 = 199.1 - 34.9 = 164.2 \text{ kN}$$

9.3.9.1.3 Loss due to wedge set (Δ_{sl} = 6 mm)

$$\Delta P_{sl} = 2p'l'$$

where

$$p' = \frac{199.1 - 164.2}{14.4} = 2.42 \text{ kN/m}$$

$$l' = \sqrt{\frac{\Delta_{sl}E_s \; A_p}{p'}} = \sqrt{\frac{0.006 \times 190 \times 150}{2.4}} = 8.4 \text{ m}$$

$$\Delta P_{sl} = 2 \times 2.42 \times 8.4 = 40.7 \text{ kN}$$

The resulting force profile is shown in Figure 9.4.

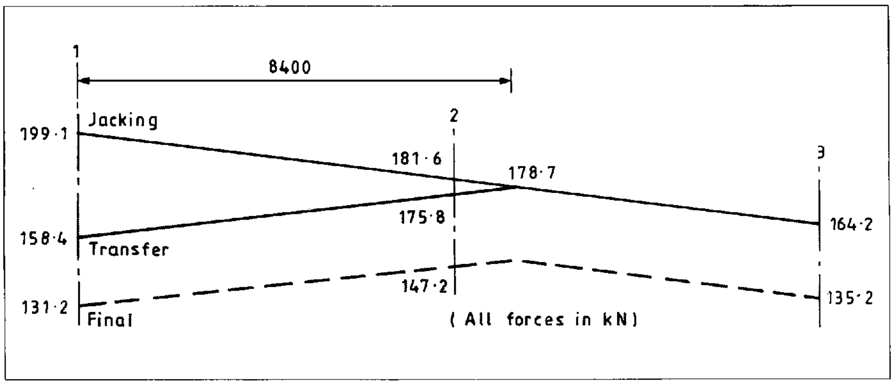


Figure 9.4 Force profiles

9.3.9.1.4 Percentage losses at transfer

At 1: $\frac{199.1 - 158.4}{199.1} \times 100 = 20.4\%$

At 2: $\frac{199.1 - 175.8}{199.1} \times 100 = 11.7\%$

At 3: $\frac{199.1 - 164.2}{199.1} \times 100 = 17.5\%$

Average loss = 16.5% (15% assumed) . . . . OK

9.3.9.2 Long term losses

9.3.9.2.1 Creep and shrinkage data 3.1.2.5.5

Notional size of section from Figure 9.2,

$$\frac{2A_c}{u} = \frac{2(2508 \times 110 + 1500 \times 240)}{2(2508 + 240)} = 230 \text{ mm}$$

For inside conditions and transfer at 7 days,

$$\phi(\infty, t_c) = 3.0$$ Table 3.3

$$\epsilon_{cs}(\infty) = 0.00058$$ Table 3.4

9.3.9.2.2 Relaxation data 4.2.3.4.1(2) Table 4.8 NAD Table 3

Long term class 2 relaxation loss for initial stress of $0.67f_{pk}$ immediately after transfer

$$\Delta\sigma_{pr} = 1.5 \times 0.02\sigma_{po} = 0.03\sigma_{po}$$

9.3.9.2.3 Loss due to creep, shrinkage and relaxation

$$\Delta\sigma_{p,c+s+r} = \frac{\epsilon_s(t,t_o)\, E_s + \Delta\sigma_{pr} + \alpha\phi(t,t_o)(\sigma_{cg} + \sigma_{cpo})}{1 + \alpha \dfrac{A_p}{A_c}\left[\left(1 + \dfrac{A_c\, z^2_{cp}}{I_c}\right)[1 + 0.8\phi(t,t_o)]\right]}$$ 4.2.3.5.5(9) Eqn 4.10

$$\alpha = \frac{E_s}{E_{cm}} = \frac{190}{32} = 5.94$$

At 1: $z_{cp} = 0$

Therefore

$$\sigma_{cg} = 0$$

$$\sigma_{cpo} = \frac{11 \times 158.4 \times 10^3}{635880} = 2.74 \text{ N/mm}^2$$

At 2: $z_{cp} = 287.5 - 196 = 91.5$ mm

$$W_{cp} = \frac{I_c}{z_{cp}} = \frac{6.79 \times 10^9}{91.5} = 74.2 \times 10^6 \text{ mm}^3$$

$$\sigma_{cg} + \sigma_{cpo} = \frac{M}{W_{cp}} + \frac{P}{A_c}$$

Using moment and force at transfer

$$\sigma_{cg} + \sigma_{cpo} = \frac{(6.17 + 30.38) \times 10^6}{74.2 \times 10^6} + \frac{11 \times 175.8 \times 10^3}{635880}$$

$$= 0.49 + 3.04 = 3.53 \text{ N/mm}^2$$

At 3: $z_{cp} = 91.5$ mm

$W_{cp} = 74.2 \times 10^6$ mm^3

$$\sigma_{cg} + \sigma_{cpo} = \frac{49.36 \times 10^6}{74.2 \times 10^6} + \frac{11 \times 164.2 \times 10^3}{635880}$$

$$= 0.67 + 2.84 = 3.51 \text{ N/mm}^2$$

Losses of prestress

At 1: $$\Delta\sigma_{p,c+s+r} = \frac{0.00058 \times 190 \times 10^3 + 31.7 + 5.94 \times 3 \times 2.74}{1 + \dfrac{5.94 \times 1650}{635880}\left[\left(1 + \dfrac{635880 \times 0^2}{6.79 \times 10^9}\right)(1 + 0.8 \times 3)\right]}$$

$$= \frac{110.2 + 31.7 + 48.8}{1 + 5.24 \times 10^{-2}(1 + 9.36 \times 10^{-5} \times 0^2)}$$

$$= \frac{190.7}{1.052} = 181.2 \text{ N/mm}^2$$

$$\Delta P_t(t) = 181.2 \times 150 \times 10^{-3} = 27.2 \text{ kN}$$

At 2: $$\Delta\sigma_{p,c+s+r} = \frac{110.2 + 35.2 + 5.94 \times 3 \times 3.53}{1 + 5.24 \times 10^{-2}(1 + 9.36 \times 10^{-5} \times 91.5^2)}$$

$$= \frac{208.3}{1.093} = 190.6 \text{ N/mm}^2$$

$$\Delta P_t(t) = 190.6 \times 150 \times 10^{-3} = 28.6 \text{ kN}$$

At 3: $$\Delta\sigma_{p,c+s+r} = \frac{110.2 + 32.8 + 5.94 \times 3 \times 3.81}{1.093}$$

$$= \frac{210.9}{1.093} = 193.0 \text{ N/mm}^2$$

$$\Delta P_t(t) = 193.0 \times 150 \times 10^{-3} = 29.0 \text{ kN}$$

Final forces at service (see Figure 9.4)

At 1: $P_t = 158.4 - 27.2 = 131.2$ kN

At 2: $P_t = 175.8 - 28.6 = 147.2$ kN

At 3: $P_t = 164.2 - 29.0 = 135.2$ kN

9.3.9.2.4 Percentage losses at service

At 1: $$\frac{199.1 - 131.2}{199.1} \times 100 = 34.1\%$$

At 2: $$\frac{199.1 - 147.2}{199.1} \times 100 = 26.1\%$$

At 3: $\dfrac{199.1 - 135.2}{199.1} \times 100 = 32.1\%$

Average loss = 30.8% (30% assumed) .... OK

9.4 Ultimate limit state

9.4.1 Calculation of applied moments

4.3.1.1P(2), P(4) & P(6)

Partial safety factors 2.3.3.1

$\gamma_G = 1.35, \quad \gamma_Q = 1.5, \quad \gamma_P = 1.0$ Table 2.2 2.5.4.4.1(2)

Load cases – as for serviceability 2.5.1.2

9.4.2 Calculation of resistance moments

4.3.1.2P(1)

The section may be analysed as shown in Figure 9.5.

Rectangular stress block for concrete in compression with 4.3.1.2(4) 4.2.1.3.3(12) Figure 4.4

$\alpha = 0.85, \quad f_{cd} = \dfrac{32}{1.5} = 21.3 \text{ N/mm}^2$

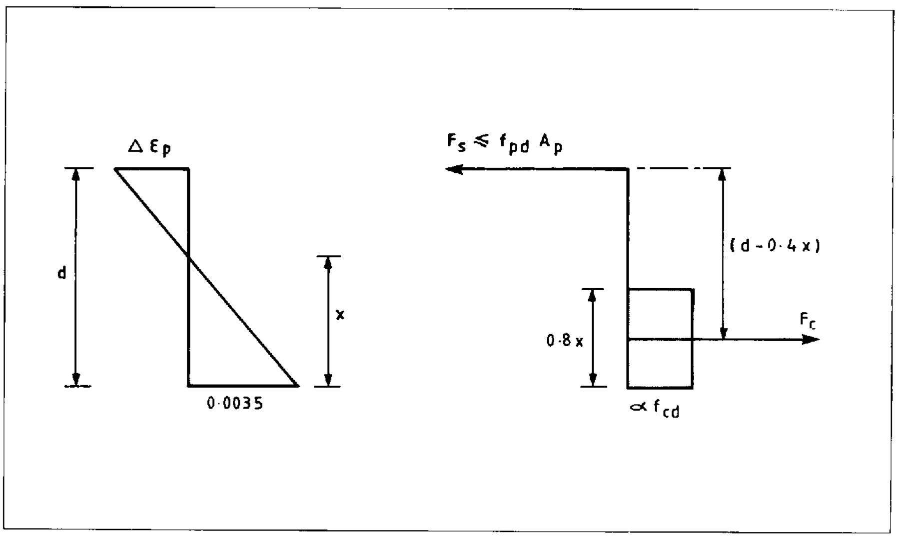


Figure 9.5 Analysis of section at ultimate limit state

Horizontal top branch to stress-strain curve for prestressing steel with 4.2.3.3.3 Figure 4.6

$$f_{pd} = 0.9\left(\frac{f_{pk}}{\gamma_s}\right) = \frac{0.9 \times 1770}{1.15} = 1385 \text{ N/mm}^2$$

For stress to reach maximum design value

$$\text{Minimum strain, } \epsilon_p = \frac{f_{pd}}{E_s} = \frac{1385}{190 \times 10^3} = 0.0073$$

$$\text{Prestrain, } \epsilon_{pm} = \frac{P_{m,\infty}}{A_p E_s} = \frac{139.4}{150 \times 190} = 0.0049$$

$$\text{Increment, } \Delta\epsilon_p = 0.0073 - 0.0049 = 0.0024$$

Maximum neutral axis/effective depth ratio

$$\frac{x}{d} = \frac{0.0035}{0.0035 + 0.0024} = 0.593$$

For values of $x \leq 0.593d$

$$F_s = f_{pd}A_p = 1385 \times 11 \times 150 \times 10^{-3} = 2285 \text{ kN}$$

$$M_{Rd} = F_s(d - 0.4x) = 2.285(d - 0.4x) \text{ kNm}$$

where

$$F_c = \alpha f_{cd} b(0.8x) = 0.85 \times 21.3 \times 0.8bx = 14.5bx$$

$F_c = F_s$ gives

$$x = \frac{2285 \times 10^3}{14.5b} = \frac{157600}{b} \text{ mm}$$

At support 1: $b = 1500$ mm, $d = 196$ mm

$$x = \frac{157600}{1500} = 105 \text{ mm}$$

$$\frac{x}{d} = \frac{105}{196} = 0.536 < 0.593 \text{ .... OK}$$

$$M_{Rd} = 2.285(196 - 0.4 \times 105) = 351.9 \text{ kNm}$$

At supports 2 and 3: $b = 1500$ mm, $d = 287.5$ mm

$$M_{Rd} = 2.285(287.5 - 0.4 \times 105) = 561.0 \text{ kNm}$$

In spans: $b = 2508$ mm, $d = 287.5$ mm

$$x = \frac{157600}{2508} = 63 \text{ mm} < h_f = 110 \text{ mm .... OK}$$

$$M_{Rd} = 2.285(287.5 - 0.4 \times 63) = 599.4 \text{ kNm}$$

9.4.3 Comparison of moments

The calculation of the moments due to the applied loads ($\gamma_G = 1.35$, $\gamma_Q = 1.5$) is not shown here. These moments are combined with the secondary moments due to prestressing ($\gamma_p = 1.0$) and compared with the resistance moments at each position. The results are summarized in Table 9.5.

Table 9.5 Moments at ultimate limit state

Zone	Secondary moments (kNm)	Moments from ultimate loads (kNm)	Applied moments (kNm)	Resistance moments, M_{Rd} (kNm)
1 (support)	122.0	−461.1	−339.1	−351.9
1−2 (span)	83.6	350.8	434.4	599.4
2 (support)	45.1	−673.9	−628.8	−561.0
2 (support)	62.6	−628.1	−565.5	−561.0
2−3 (span)	67.4	309.4	376.8	599.4
3 (support)	72.2	−604.4	−532.2	−561.0

The resistance moment is inadequate at support 2 and additional reinforcement is required.

Since

$$M = F_c(d - 0.4x) = 14.5bx(d - 0.4x),$$

$$x^2 - 2.5dx + \frac{2.5M}{14.5b} = 0$$

Hence

$$x = 1.25\left(1 - \sqrt{1 - \frac{M}{9.06bd^2}}\right)d$$

$$= 1.25\left(1 - \sqrt{1 - \frac{628.8 \times 10^6}{9.06 \times 1500 \times 287.5^2}}\right) 287.5 = 121 \text{ mm}$$

$$\frac{x}{d} = \frac{121}{287.5} = 0.421 < 0.593 \ldots\ldots\ldots\ldots \text{OK}$$

$$F_s = \frac{M}{d - 0.4x} = \frac{628.8 \times 10^3}{287.5 - 0.4 \times 121} = 2630 \text{ kN}$$

Additional area of reinforcement required

$$A_s = \frac{F_s - f_{pd}A_p}{f_{yd}} = \frac{(2630 - 2285)10^3}{400} = 863 \text{ mm}^2$$

2T16 and 2T20 gives 1030 > 863 mm^2 .... OK

Use 2T16 top and bottom throughout beam with additional 2T20 top at support 2

9.5 Minimum and maximum areas of reinforcement

5.4.2.1.1

Although it is not clear what should be assumed from EC2[1], the total area of steel has been taken as the sum of the untensioned and tensioned steel.

$$A_{s+p} = A_s + A_p$$

$$= (2 \times 201) + (11 \times 150) = 2052 \text{ mm}^2$$

9.5.1 Minimum

Minimum area of total tension reinforcement

$$\frac{0.6 b_t d}{f_{yk}} \nless 0.0015 b_t d$$

Eqn 5.14

At support, $b_t = 2508$ mm

$$\text{Minimum area} = \frac{0.6 \times 2508 \times 290}{460} = 948 \text{ mm}^2$$

$$\nless 0.00.15 \times 2508 \times 290 = 1090 \text{ mm}^2$$

Area provided $= 2052 > 1090$ mm^2 .... OK

9.5.2 Maximum

Maximum area of total tension and compression reinforcement

$= 0.04A_c = 0.04 \times 635880 = 25435 > 2052$ mm^2 .... OK

9.6 Reinforcement summary

11 tendons throughout beam
2T16s top and bottom throughout beam. Additional 2T20s top at support 2

These areas are within maximum and minimum limits.

10 SERVICEABILITY CHECKS BY CALCULATION

10.1 Deflection

Calculate the long term deflection of a 7.0 m span simply supported beam whose section is shown in Figure 10.1. The beam supports the interior floor spans of an office building.

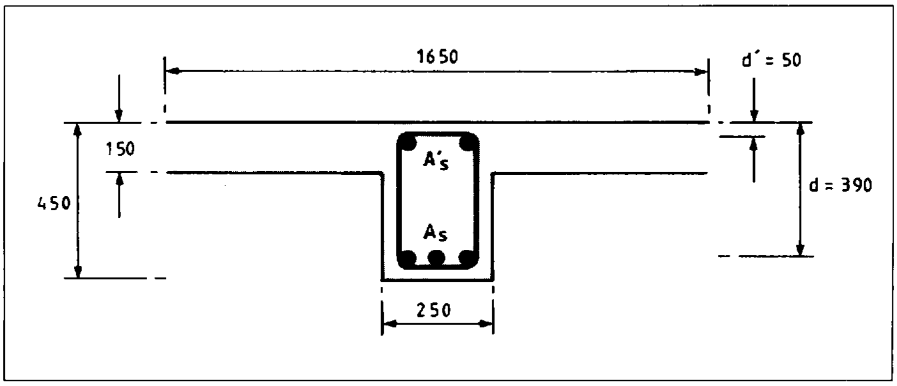


Figure 10.1 Beam section

Deflections will be calculated using the rigorous and simplified methods given in EC2[(1)], together with an alternative simplified method. The results will then be compared with the limiting span/effective depth ratios given in EC2.

10.1.1 Design data

Span = 7.0 m

G_k = 19.7 kN/m

Q_k = 19.5 kN/m

A'_s = 402 mm²

A_s = 2410 mm²

f_{ck} = 30 N/mm² (concrete strength class C30/37) 3.1.2.4 Table 3.1

10.1.2 Calculation method

The requirements for the calculation of deflections are given in Section 4.4.3 and Appendix 4 of EC2.

Two limiting conditions are assumed to exist for the deformation of concrete sections A4.3(1)

(1) Uncracked

(2) Cracked.

Members which are not expected to be loaded above the level which would cause the tensile strength of the concrete to be exceeded, anywhere in the member, will be considered to be uncracked. Members which are expected to crack will behave in a manner intermediate between the uncracked and fully cracked conditions. A4.3(2)

For members subjected dominantly to flexure, the Code gives a general equation for obtaining the intermediate value of any parameter between the limiting conditions

$$\alpha = \zeta\alpha_{II} + (1 - \zeta)\alpha_{I}$$ A4.3(2) Eqn A.4.1

where

α is the parameter being considered

α_I and α_{II} are the values of the parameter calculated for the uncracked and fully cracked conditions respectively

ζ is a distribution coeffient given by

$$\zeta = 1 - \beta_1\beta_2\left(\frac{\sigma_{sr}}{\sigma_s}\right)^2$$ A4.3(2) Eqn A.4.2

The effects of creep are catered for by the use of an effective modulus of elasticity for the concrete given by

$$E_{c,eff} = \frac{E_{cm}}{1 + \phi}$$ A4.3.(2) Eqn A.4.3

Bond and deterioration of bond under sustained or repeated loading is taken account of by coefficients β_1 and β_2 in Eqn A.4.2

Curvatures due to shrinkage may be assessed from

$$\frac{1}{r_{cs}} = \frac{\epsilon_{cs}\alpha_e S}{I}$$ A4.3(2) Eqn A.4.4

Shrinkage curvatures should be calculated for the uncracked and fully cracked conditions and the final curvature assessed by use of Eqn A.4.1.

In accordance with the Code, the rigorous method of assessing deflections is to calculate the curvatures at frequent sections along the member and calculate the deflections by numerical integration.

The simplified approach, suggested by the Code, is to calculate the deflection assuming firstly the whole member to be uncracked and secondly the whole member to be cracked. Eqn A.4.1 is used to assess the final deflection.

10.1.3 Rigorous assessment

The procedure is, at frequent intervals along the member, to calculate

(1) Moments
(2) Curvatures
(3) Deflections.

Here, calculations will be carried out at the mid-span position only, to illustrate this procedure, with values at other positions along the span being tabulated.

10.1.3.1 Calculation of moments

For buildings, it will normally be satisfactory to consider the deflections under the quasi-permanent combination of loading, assuming this load to be of long duration. A4.2(5)

The quasi-permanent combination of loading is given, for one variable action, by

$$G_k + \psi_2 Q_k$$ 2.3.4 P(2) Eqn 2.9(c)

$\psi_2 = 0.3$ — NAD Table 1

Therefore

Loading = 19.7 + (0.3 × 19.5) = 25.6 kN/m

Mid-span bending moment (M) = 25.6 × 7^2/8 = 156.8 kNm

10.1.3.2 Calculation of curvatures

In order to calculate the curvatures it is first necessary to calculate the properties of the uncracked and cracked sections and determine the moment at which cracking will occur.

10.1.3.2.1 Flexural curvature

The effective modulus of elasticity ($E_{c,eff}$) $= \dfrac{E_{cm}}{1 + \phi}$ — A4.3(2) Eqn A.4.3

For concrete strength class C30/37, E_{cm} = 32 kN/mm^2 — 3.1.2.5.2 Table 3.2

$$\frac{2A_c}{u} = \frac{2[(1650 \times 150) + (250 \times 300)]}{2(1650 + 300)} = 165 \text{ mm}$$

For internal conditions and age at loading of 7 days — 3.1.2.5.5 Table 3.3

$\phi = 3.1$

Therefore

$$E_{c,eff} = \frac{32}{1 + 3.1} = 7.8 \text{ kN/mm}^2$$

Effective modular ratio (α_e) $= \dfrac{E_s}{E_{c,eff}}$

Modulus of elasticity of reinforcement (E_s) = 200 kN/mm^2 — 3.2.4.3(1)

Therefore

$$\alpha_e = \frac{200}{7.8} = 25.64$$

$$\rho = \frac{A_s}{bd} = \frac{2410}{1650 \times 390} = 3.75 \times 10^{-3}$$

$$\rho' = \frac{A'_s}{bd} = \frac{402}{1650 \times 390} = 6.25 \times 10^{-4}$$

For the uncracked section, the depth to the neutral axis is given by

$$x = \frac{bh^2/2 - (b - b_w)(h - h_f)\left(\frac{h - h_f}{2} + h_f\right) + (\alpha_e - 1)(A'_s d' + A_s d)}{bh_f + b_w(h - h_f) + (\alpha_e - 1)(A'_s + A_s)} = 165.2 \text{ mm}$$

The second moment of the area of the uncracked section is given by

$$I_I = \frac{bh_f^3}{12} + \frac{b_w(h - h_f)^3}{12} + bh_f(x - h_f/2)^2 + b_w(h - h_f)$$

$$\left(\frac{h + h_f}{2} - x\right)^2 + (\alpha_e - 1)\, A'_s(x - d')^2 + (\alpha_e - 1)\, A_s(d - x)^2 = 7535 \times 10^6 \text{ mm}^4$$

For the cracked section the depth to the neutral axis is given by

$$\frac{x}{d} = -[\alpha_e\rho + (\alpha_e - 1)\rho'] + \sqrt{[\alpha_e\rho + (\alpha_e - 1)\rho']^2 + 2[\alpha_e\rho + (\alpha_e - 1)\rho'\frac{d'}{d}]}$$

$$x = 0.345d = 134.6 \text{ mm}$$

The second moment of area of the cracked section is given by

$$\frac{I_{II}}{bd^3} = \frac{1}{3}\left(\frac{x}{d}\right)^3 + \alpha_e\rho\left(1 - \frac{x}{d}\right)^2 + (\alpha_e - 1)\rho'\left(\frac{x}{d} - \frac{d'}{d}\right)^2$$

$$I_{II} = 0.0556bd^3 = 5448 \times 10^6 \text{ mm}^4$$

The moment which will cause cracking of the section is given by

$$M_{cr} = \frac{f_{ctm} I_I}{y_t}$$

$$y_t = h - x = 450 - 165.2 = 284.8 \text{ mm}$$

For concrete strength grade C30/37, $f_{ctm} = 2.9 \text{ N/mm}^2$ — 3.1.2.4 Table 3.1

Therefore

$$M_{cr} = \frac{2.9 \times 7535 \times 10^6 \times 10^{-6}}{284.8} = 76.7 \text{ kNm}$$

The section is considered to be cracked, since

$$M_{cr} < M = 156.8 \text{ kNm}$$

Curvature of the uncracked section is given by

$$\frac{1}{r_I} = \frac{M}{E_{c,eff} I_I} = \frac{156.8 \times 10^6}{7.8 \times 10^3 \times 7535 \times 10^6} = 2.668 \times 10^{-6} \text{ rad./mm}$$

Curvature of the cracked section is given by

$$\frac{1}{r_{II}} = \frac{M}{E_{c,eff} I_{II}} = \frac{156.8 \times 10^6}{7.8 \times 10^3 \times 5448 \times 10^6} = 3.690 \times 10^{-6} \text{ rad./mm}$$

Having obtained the values for the two limiting conditions Eqn A.4.1 is used to assess the intermediate value.

Hence

$$\frac{1}{r} = \zeta\left(\frac{1}{r_{II}}\right) + (1 - \zeta)\frac{1}{r_I}$$

A4.3(2) Eqn A.4.1

$$\zeta = 1 - \beta_1\beta_2 \left(\frac{\sigma_{sr}}{\sigma_s}\right)^2$$

For high bond bars, $\beta_1 = 1.0$

For sustained loading, $\beta_2 = 0.5$

σ_s is the stress in the tension steel calculated on the basis of a cracked section

Therefore

$$\sigma_s = \frac{\alpha_e M (d - x)}{I_{II}} = \frac{25.64 \times 156.8 \times 10^6 (390 - 134.6)}{5448 \times 10^6} = 188.5 \text{ N/mm}^2$$

σ_{sr} is the stress in the tension steel calculated on the basis of a cracked section under the loading which will just cause cracking at the section considered.

Therefore

$$\sigma_{sr} = \frac{\alpha_e M_{cr} (d - x)}{I_{II}}$$

$$= \frac{25.64 \times 76.7 \times 10^6(390 - 134.6)}{5448 \times 10^6} = 92.2 \text{ N/mm}^2$$

Therefore

$$\zeta = 1 - 0.5 \left(\frac{92.2}{188.5}\right)^2 = 0.88$$

Note:

$\frac{\sigma_{sr}}{\sigma_s}$ may be replaced by $\frac{M_{cr}}{M}$ in the above calculation

$$\frac{1}{r} = [(0.88 \times 3.69) + (1-0.88) \times 2.668] \times 10^{-6} = 3.567 \times 10^{-6} \text{ rad./mm}$$

10.1.3.2.2 Shrinkage curvature

The shrinkage curvature is given by

$$\frac{1}{r_{cs}} = \epsilon_{cs}\alpha_e \frac{S}{I}$$ A.4.3.2 Eqn A.4.4

where

ϵ_{cs} is the free shrinkage strain

For internal conditions and $2A_c/u = 165$ mm — 3.1.2.5.5 Table 3.4

$$\epsilon_{cs} = 0.60 \times 10^{-3}$$

S is the first moment of area of the reinforcement about the centroid of the section.

I is the second moment of area of the section.

S and I should be calculated for both the uncracked and fully cracked conditions.

Curvature of the uncracked section

$$S_I = A_s(d - x) - A'_s(x - d') = 495.5 \times 10^3 \text{ mm}^3$$

$$\frac{1}{r_{csI}} = \frac{0.60 \times 10^{-3} \times 25.64 \times 495.5 \times 10^3}{7535 \times 10^6} = 1.0 \times 10^{-6} \text{ rad./mm}$$

Curvature of the cracked section

$$S_{II} = A_s(d - x) - A'_s(x - d') = 581.5 \times 10^3 \text{ mm}^3$$

$$\frac{1}{r_{csII}} = \frac{0.60 \times 10^{-3} \times 25.64 \times 581.5 \times 10^3}{5448 \times 10^6} = 1.64 \times 10^{-6} \text{rad./mm}$$

Therefore

$$\frac{1}{r_{cs}} = \zeta \times \frac{1}{r_{csII}} + (1 - \zeta) \times \frac{1}{r_{csI}}$$

$$= [(0.88 \times 1.64) + (1 - 0.88) \times 1.0] \times 10^{-6} = 1.563 \times 10^{-6} \text{ rad./mm}$$

The total curvature at mid-span

$$\frac{1}{r_{tot}} = \frac{1}{r} + \frac{1}{r_{cs}} = (3.567 + 1.563) \times 10^{-6} = 5.130 \times 10^{-6} \text{rad./mm}$$

The flexural, shrinkage and total curvatures at positions x/l along the span are given in Table 10.1.

Table 10.1 Curvatures $\times 10^6$ (rad./mm)

x/l	Moment (kNm)	$\frac{1}{r_I}$	$\frac{1}{r_{II}}$	ζ	$\frac{1}{r}$	$\frac{1}{r_{cs}}$	$\frac{1}{r_{tot}}$
0	0	0	0	0	0	1.000	1.000
0.1	56.4	0.960	–	–	0.960	1.000	1.960
0.2	100.4	1.708	2.363	0.708	2.171	1.453	3.624
0.3	131.7	2.241	3.100	0.830	2.954	1.531	4.485
0.4	150.5	2.561	3.542	0.870	3.414	1.557	4.971
0.5	156.8	2.668	3.690	0.880	3.567	1.563	5.130
0.6	150.5	2.561	3.542	0.870	3.414	1.557	4.971
0.7	131.7	2.241	3.100	0.830	2.954	1.531	4.485
0.8	100.4	1.708	2.363	0.708	2.171	1.453	3.624
0.9	56.4	0.960	–	–	0.960	1.000	1.960
1.0	0	0	0	0	0	1.000	1.000

10.1.3.3 Calculation of deflections

Having calculated the total curvatures, the deflections may be calculated by numerical integration using the trapezoidal rule.

The uncorrected rotation at any point may be obtained by the first integral given by

$$\Theta_x = \Theta_{x-1} + \left(\frac{\frac{1}{r_x} + \frac{1}{r_{x-1}}}{2} \right) \times \frac{l}{n}$$

Having calculated the uncorrected rotations, the uncorrected deflections may be obtained by the second integral given by

$$a_x = a_{x-1} + \left(\frac{\Theta_x + \Theta_{x-1}}{2}\right)\frac{l}{n}$$

where the subscript x denotes the values of the parameters at the fraction of the span being considered, and the subscript x−1 denotes the values of the parameters at the preceding fraction of the span.

l is the span

n is the number of span divisions considered.

Hence the uncorrected rotation at $0.1l$

$$\Theta_{0.1l} = \Theta_o + \left(\frac{\frac{1}{r_{0.1l}} + \frac{1}{r_o}}{2}\right)\frac{l}{n}$$

$$= 0 + \left(\frac{1.96 + 1.0}{2}\right) 10^{-6} \times \frac{7000}{10} = 1.036 \times 10^{-3} \text{ rad.}$$

and the uncorrected deflection at $0.1l$

$$a_{0.1l} = a_o + \left(\frac{\Theta_{0.1l} + \Theta_o}{2}\right)\frac{l}{n}$$

$$= 0 + \left(\frac{1.036 + 0}{2}\right) 10^{-3} \times \frac{7000}{10} = 0.363 \text{ mm}$$

The uncorrected deflections may then be corrected to comply with the boundary conditions of zero deflection at both supports. This is done by subtracting from the uncorrected deflections the value of the uncorrected deflection at the right hand support multiplied by the fraction of the span at the point being considered.

The values of the uncorrected rotations, uncorrected and corrected deflections at positions x/l along the span are given in Table 10.2.

Table 10.2 Deflections (mm)

x/l	$\frac{1 \times 10^6}{r_{tot}}$	1st integral $\times 10^3$	2nd integral	Correction	Deflection
0	1.000	0	0	0	0
0.1	1.960	1.036	0.363	8.871	− 8.508
0.2	3.624	2.990	1.772	17.742	−15.970
0.3	4.485	5.828	4.858	26.613	−21.755
0.4	4.971	9.138	10.096	35.484	−25.388
0.5	5.130	12.673	17.730	44.356	−26.626
0.6	4.971	16.208	27.838	53.227	−25.388
0.7	4.485	19.518	40.342	62.098	−21.755
0.8	3.624	22.356	54.998	70.969	−15.970
0.9	1.960	24.310	71.331	79.840	− 8.508
1.0	1.000	25.346	88.711	88.711	0

Maximum deflection at mid-span
$a_{tot} = 26.6 \text{ mm} = \frac{\text{span}}{263} < \text{limit of } \frac{\text{span}}{250} = 28 \text{ mm}$

10.1.4 Simplified approach

The procedure for this approach is to

(1) Calculate the maximum bending moment and the moment causing cracking

(2) Calculate the maximum deflections for the uncracked and fully cracked conditions, and use Eqn A.4.1 to assess the final maximum deflection.

From Section **10.1.3.2.1** the maximum bending moment $M = 156.8$ kNm, and the moment causing cracking $M_{cr} = 76.7$ kNm.

The maximum deflection of the uncracked section due to flexure

$$a_{I} = \frac{5wl^4}{384E_{c,eff}I_{I}}$$

$$w = 25.6 \text{ kN/m}$$

$$l = 7.0 \text{ m}$$

$$E_{c,eff} = 7.8 \text{ kN/mm}^2$$

$$I_{I} = 7535 \times 10^6 \text{ mm}^4$$

Therefore

$$a_{I} = \frac{5 \times 25.6 \times 7^4 \times 10^{12}}{384 \times 7.8 \times 10^3 \times 7535 \times 10^6} = 13.6 \text{ mm}$$

The maximum deflection of the cracked section due to flexure

$$a_{II} = \frac{5wl^4}{384E_{c,eff}I_{II}}$$

$$I_{II} = 5448 \times 10^6 \text{ mm}^4$$

Therefore

$$a_{II} = \frac{5 \times 25.6 \times 7^4 \times 10^{12}}{384 \times 7.8 \times 10^3 \times 5448 \times 10^6} = 18.8 \text{ mm}$$

Final maximum deflection due to flexure

$$a = \zeta a_{II} + (1 - \zeta)a_{I}$$ A4.3(2) Eqn A.4.1

$$\zeta = 1 - \beta_1\beta_2\left(\frac{M_{cr}}{M}\right)^2$$

$\beta_1 = 1.0$

$\beta_2 = 0.5$

Therefore

$$\zeta = 1 - 0.5\left(\frac{76.7}{156.8}\right)^2 = 0.88$$

Therefore

$$a = (0.88 \times 18.8) + (1 - 0.88) \times 13.6 = 18.2 \text{ mm}$$

It must be appreciated that the deflection calculated above is due to flexure only. The additional deflection due to shrinkage must also be assessed. The shrinkage curvature at mid-span from Section **10.1.3.2**

$$\frac{1}{r_{cs}} = 1.563 \times 10^{-6} \text{ rad./mm}$$

$$a_{cs} = \frac{1}{8}\left(\frac{1}{r_{cs}}\right)l^2 = \frac{1.563 \times 10^{-6} \times 7^2 \times 10^6}{8} = 9.6 \text{ mm}$$

$$a_{tot} = a + a_{cs} = 18.2 + 9.6 = 27.8 \text{ mm}$$

This figure is close to the rigorously assessed value of 26.6 mm.

10.1.5 Alternative simplified approach

An alternative simplified approach, which directly takes account of shrinkage, is given in BS 8110[(2)]. BS 8110: Part 2 Section 3

The procedure here is to calculate the total curvature at one point, generally the point of maximum moment. Then, assuming the shape of the curvature diagram to be the same as the shape of the bending moment diagram, the deflection is given by

$$a = Kl^2\left(\frac{1}{r_{tot}}\right)$$

BS 8110: Part 2 3.7.2 Eqn 11

where

K is a factor dependent upon the shape of the being moment diagram.

For a simply supported beam with uniformly distributed load

$$K = 0.104$$

BS 8110: Part 2 3.7.2 Table 3.1

Total curvature at mid-span, from Section **10.1.3.2**

$$\frac{1}{r_{tot}} = 5.130 \times 10^{-6} \text{ rad./mm}$$

Therefore maximum deflection at mid-span

$$a_{tot} = 0.104 \times 7^2 \times 5.130 = 26.2 \text{ mm}$$

Again this is close to the rigorously assessed value.

10.1.6 Comparison with span/effective depth ratio

The procedure for limiting deflections by use of span/effective depth ratios is set out in EC2 Section 4.4.3.

For the example considered

$$A_{s,req} = 2392 \text{ mm}^2 \quad A_{s,prov} = 2410 \text{ mm}^2$$

$$\rho = \frac{100A_{s,prov}}{bd} = \frac{100 \times 2410}{1650 \times 390} = 0.37\%$$

Therefore the concrete is lightly stressed, $\rho \leq 0.5\%$ — 4.4.3.2(5)(c)

The NAD[1] introduces a category of nominally reinforced concrete corresponding to $\rho = 0.15\%$ — NAD 6.4(e)

Basic span/effective depth ratio for a simply supported beam, interpolating for $\rho = 0.37\%$

$$\frac{l}{d} = 28$$

NAD Table 7

For flanged beams where $b/b_w > 3.0$ the basic span/effective depth ratio should be multiplied by a factor of 0.8 — 4.4.3.2(3)

The span/effective depth ratios given in NAD Table 7 are based on a maximum service stress in the reinforcement in a cracked section of 250 N/mm^2. The tabulated values should be multiplied by the factor of $250/\sigma_s$ for other stress levels, where σ_s is the service stress at the cracked section under the frequent load combination. As a conservative assumption the Code states that the factor may be taken as

$$\frac{250}{\sigma_s} = \frac{400}{f_{yk}\left(\frac{A_{s,req}}{A_{s,prov}}\right)}$$

Therefore, for this example, allowable span/effective depth ratio

$$\frac{l}{d} = 28 \times 0.8 \left(\frac{400}{460 \times 2392/2410}\right) = 19.6$$

$\frac{l}{d}$ (allowable) $= 19.6 > \frac{l}{d}$ (actual) $= \frac{7000}{390} = 18.0$

If the span/effective depth ratio is modified using the service stress in the reinforcement as calculated in Section **10.1.3.2.1** but adjusted for the frequent load combination

$$\sigma_s = 188.5 \times 31.4/25.6 = 231 \text{ N/mm}^2$$

$\frac{l}{d}$ (allowable) $= 28 \times 0.8 \times \frac{250}{231} = 21.6 > 18.0$ (actual)

It can be seen from this example that whilst the span/effective depth ratio based on the calculated steel service stress suggests that the deflection should be well within the prescribed limits, the deflection from the rigorous and simplified analysis proves to be much nearer to the limit of span/250.

This is due to the contribution to the deflection from shrinkage, which in this example is approximately a third of the total deflection.

The values of shrinkage strain given in EC2 Table 3.4 relate to concrete having a plastic consistence of classes S2 and S3 as specified in ENV 206[6]. For concrete of class S1 and class S4 the values given in the Table should be multiplied by 0.7 and 1.2 respectively. **3.1.2.5.5(4) ENV 206 7.2.1**

Table 4 of ENV 206 categorises the class in relation to slump as given in Table 10.3.

Table 10.3 Slump classes **ENV 206 7.2.1 Table 4**

Class	Slump (mm)
S1	10 – 40
S2	50 – 90
S3	100 – 150
S4	≥ 160

Thus for classes S2 and S3 the slump may vary between 50 mm and 150 mm. It is not logical that mixes with this variation of slump, and hence w/c ratio, should have a standard value of shrinkage strain.

If the values in EC2 Table 3.4 are assumed to relate to the median slump for classes S2 and S3 of 100 mm, then the values for slumps of 40 mm to 100 mm should be multiplied by a factor between 0.7 and 1.0 and values for slumps of 100 mm to 160 mm should be multiplied by a factor between 1.0 and 1.2.

As most normal mixes will have a slump in the order of 50 mm the values of shrinkage strain for the example considered would be:

$$0.60 \times 10^{-3}\left[0.7 + \left(\frac{1 - 0.7}{60}\right) \times 10\right]$$

$$= (0.60 \times 10^{-3}) \times 0.75 = 0.45 \times 10^{-3}$$

This figure relates more closely to the value which would be given in BS 8110, for the same example, of 0.4×10^{-3}. **BS 8110: Part 2 7.4 Figure 7.2**

For the example considered, the calculated deflection due to shrinkage from the rigorous assessment would be

$$9.1 \times 0.75 = 6.8 \text{ mm}$$

and the total deflection from the rigorous assessment would be

$$a_{tot} = 26.6 - 9.1 + 6.8 = 24.3 \text{ mm}$$

This is well within the limit of $\dfrac{\text{span}}{250} = 28$ mm

10.2 Cracking

Check by calculation that the longitudinal reinforcement in the reinforced concrete wall section shown in Figure 10.2 is sufficient to control cracking due to restraint of intrinsic deformation resulting in pure tension.

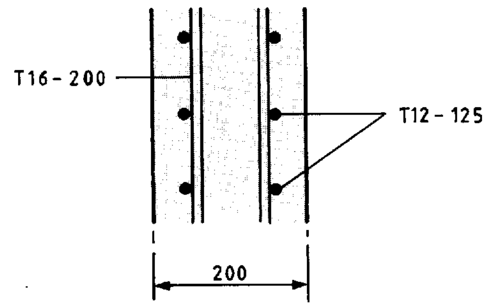


Figure 10.2 Wall section

10.2.1 Design data

Concrete strength class is C30/37.

Cover to reinforcement = 35 mm — NAD Table 6

High bond bars with f_{yk} = 460 N/mm^2

Exposure class 2(a)

10.2.2 Calculation method

Requirements for the control of cracking are given in EC2 Section 4.4.2. Crack control is normally achieved by the application of simple detailing rules.

The procedure for the calculation of crack widths is first to calculate the stress and hence the strain in the reinforcement, taking into account the bond properties of the bars and the duration of loading. Next, the average final crack spacing dependent on the type, size and disposition of the reinforcement and the form of strain distribution is established.

The design crack width may then be obtained and compared with the limiting design crack width. In the absence of specific requirements, a limiting crack width of 0.3 mm will generally be satisfactory for reinforced concrete members in buildings with respect to both appearance and durability. 4.4.2.1(6)

10.2.3 Check by calculation

10.2.3.1 Calculation of steel stress and strain

Steel stress:

$$\sigma_s = \frac{k_c k f_{ct,eff} A_{ct}}{A_s}$$ 4.4.2.2(3) Eqn 4.78

where

A_s = area of reinforcement within the tensile zone

= 905 × 2 = 1810 mm^2/m

A_{ct} = area of concrete within tensile zone

= $1000 \times 200 = 200 \times 10^3$ mm²

k_c = a coefficient taking account of stress distribution

= 1.0 for pure tension

k = a coefficient allowing for the effect of non-uniform self-equilibrating stresses

= 0.8 for tensile stresses due to restraint of intrinsic deformations ($h \leq 300$ mm)

$f_{ct,eff}$ = tensile strength of concrete effective at first cracking

= 3.8 N/mm² (taking $f_{ctk\,0.95}$ but see Section 10.2.3.4) 3.1.2.4(3) Table 3.1

Therefore

$$\sigma_s = \frac{1.0 \times 0.8 \times 3.8 \times 200 \times 10^3}{1810} = 336 \text{ N/mm}^2$$

Mean strain:

$$\epsilon_{sm} = \frac{\sigma_s}{E_s}\left[1 - \beta_1\beta_2\left(\frac{\sigma_{sr}}{\sigma_s}\right)^2\right]$$ 4.4.2.4(2) Eqn 4.81

where

E_s = modulus of elasticity of steel

= 200 kN/mm² 3.2.4.3(1)

β_1 = a coefficient taking account of bond properties of the bars

= 1.0 for high bond bars

β_2 = a coefficient taking account of load duration

= 0.5

σ_{sr} = the stress in the reinforcement based on a cracked section under the load causing first cracking

= σ_s for intrinsic imposed deformation

Therefore

$$\epsilon_{sm} = \frac{336}{200 \times 10^3}\left(1 - 0.5\right) = 8.4 \times 10^{-4}$$

10.2.3.2 Calculation of crack spacing

The average final crack spacing

$$s_{rm} = 50 + 0.25\, k_1 k_2 \left(\frac{\phi}{\rho_r}\right)$$ 4.4.2.4(3) Eqn 4.82

where

k_1 = a coefficient taking account of the bond properties of the bar

= 0.8 for high bond bars

In the case of imposed deformations, k_1 should be multiplied by k, with k being in accordance with EC2 Section 4.4.2.2.(3).

k_2 = a coefficient taking account of the form of strain distribution
= 1.0 for pure tension

ρ_r = the effective reinforcement ratio $= \dfrac{A_s}{A_{c,eff}}$

$A_{c,eff}$ = the effective tension area.

The effective tension area is generally the area of concrete surrounding the tension reinforcement to a depth of 2.5 times the distance from the tension face to the controid of the reinforcement or, for members in tension, half the actual member thickness, whichever is the lesser. This is calculated as:

$$2.5 \times (35 + 12/2) = 103 \not> h/2 = 100 \text{ mm}$$

Therefore

$$A_{c,eff} = 1000 \times 100 = 100 \times 10^3 \text{ mm}^2$$

$$\rho_r = \frac{1810}{2 \times 100 \times 10^3} = 0.009$$

$$s_{rm} = 50 + \frac{(0.25 \times 0.8 \times 0.8 \times 1.0 \times 12)}{0.009} = 263 \text{ mm}$$

10.2.3.3 Calculation of crack width

The design crack width

$$w_k = \beta s_{rm} \epsilon_{sm}$$ 4.4.2.4 Eqn 4.80

where

β = a coefficient relating the average crack width to the design value
= 1.3 for restraint cracking in members with a minimum dimension of 300 mm or less.

Therefore

$$w_k = 1.3 \times 263 \times 8.4 \times 10^{-4} = 0.29 < 0.3 \text{ mm (limit)}$$

10.2.3.4 Concluding remark

The Code suggests a minimum value of 3 N/mm^2 be taken when the time of cracking cannot be confidently predicted as being less than 28 days.

Whilst the values given for $f_{ct,eff}$ seem high, it is difficult at the design stage to assess accurately the as placed concrete strength because this often exceeds the class specified. Consequently, unless strict site control is exercised, it would be prudent to adopt the apparently conservative figures given in EC2 Table 3.1.

11 DEEP BEAMS

11.1 Introduction

The design of deep beams may be based on analyses applying:

(a) linear elastic analysis; 2.5.1.1(5) 2.5.3.7.3

(b) an equivalent truss consisting of concrete struts and arches with reinforcement, all preferably following the elastic field;

(c) non-linear analysis.

In EC2[1] details of the analysis model and, therefore, much of the design are not given and it is left for the Engineer to satisfy the principal Code requirements. This can be achieved using CIRIA Guide 2, *The design of deep beams in reinforced concrete*[18], which also provides recommendations on the detailed analysis and design. The Guide was written for use with the then current British Standard CP 110[19].

Here it has been assumed that a complete design to the CIRIA Guide would be carried out and then checks made to demonstrate compliance with the specific clauses for deep beams in EC2.

To highlight some of the differences between EC2 and design to the CIRIA Guide, the example in Appendix B of the Guide has been used.

A small number of EC2 clauses have been identified as relating specifically to deep beams:

(a) 2.5.2.1(2) – definition of deep beams

(b) 2.5.3.7.3 – analysis modelling

(c) 4.4.2.3(4) – skin reinforcement

(d) 5.4.5 – reinforcement detailing

11.2 Example

A proposed arrangement of walls and columns is shown in Figure 11.1. Loading details are presented in Figure 11.2. It is intended to justify a design using the Simple Rules of Section 2 of the CIRIA Guide.

The beam is a flat vertical plate and the thickness is small compared with other dimensions. **CIRIA Guide 2 Cl.2.1.1(1)**

There are two loads which may be defined as concentrated and no indirect loads or supports. **Cl.2.1.1(4) Cl.2.1.1(5)**

In EC2 a beam is classified as a deep beam if the span is less than twice the depth. **2.5.2.1(2)**

CIRIA Guide 2 classifies deep beams as 'Beams with span/depth ratios of less than 2 for single span beams or less than 2.5 for multi span beams', thus giving an extended range of elements to be designed as deep beams in comparison with EC2.

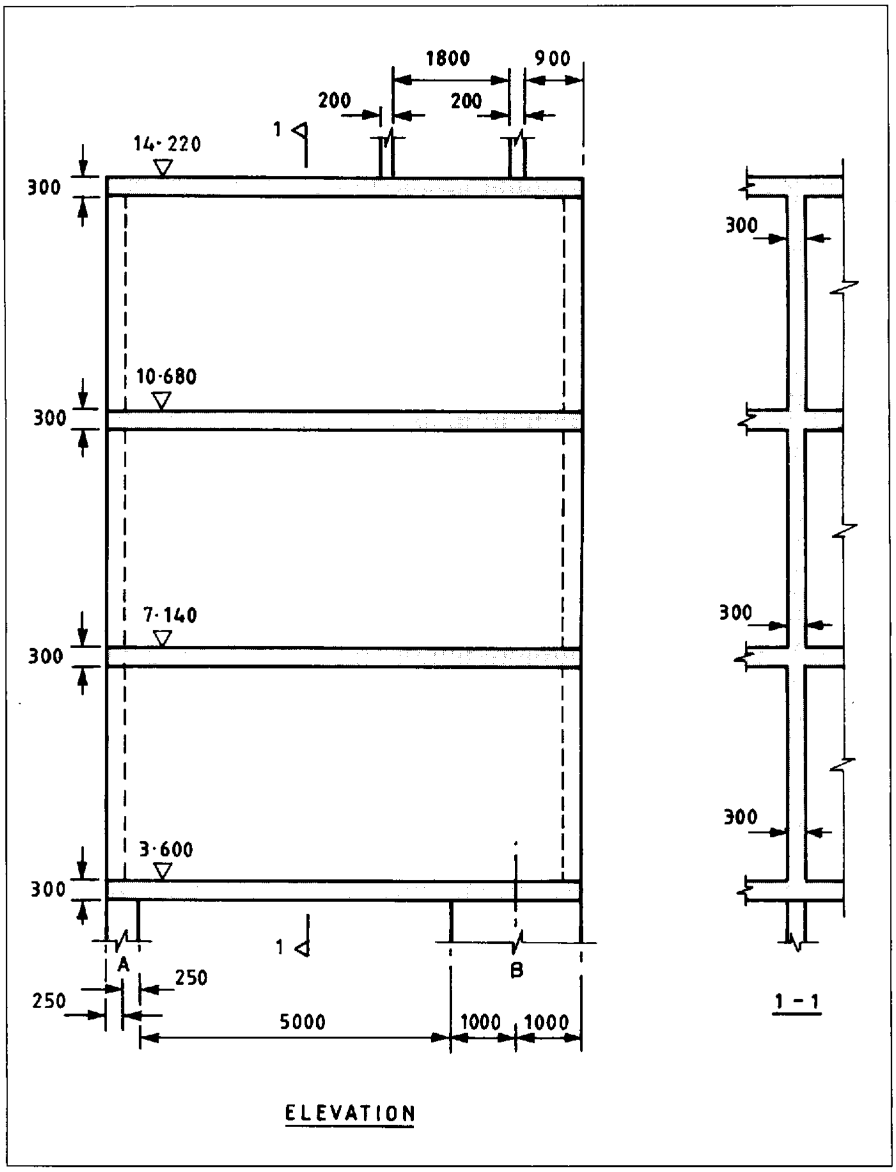


Figure 11.1 Structural arrangement

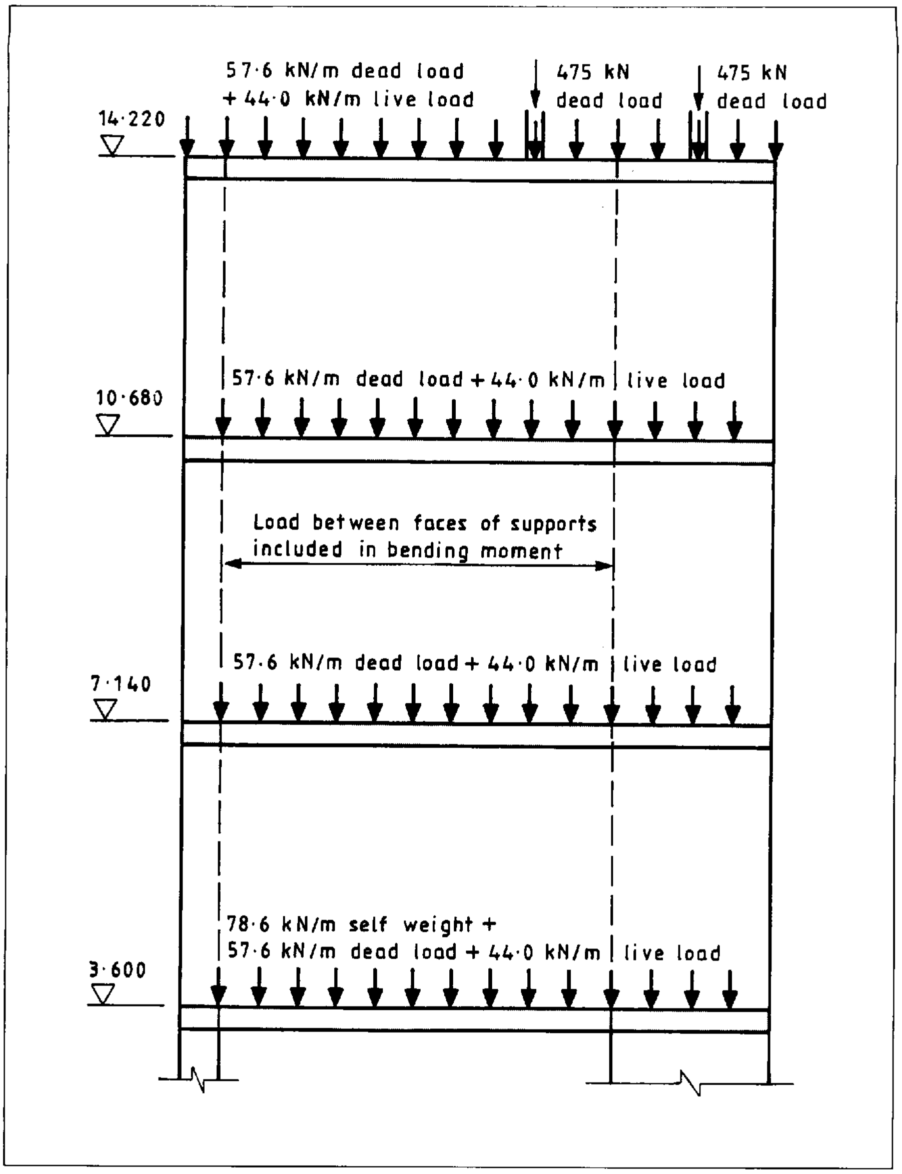


Figure 11.2 Loading details

11.2.1 Durability

For dry environment, exposure class is 1. Table 4.1

Minimum concrete strength class is C25/30. ENV 206 Table NA.1

The CIRIA Guide example uses f_{cu} = 30 N/mm^2

Use C25/30 for design to EC2 to keep examples broadly consistent.

Minimum cement content and water cement ratio ENV 206 Table 3

Minimum cover to reinforcement = 15 mm NAD Table 6

Assume nominal aggregate size = 20 mm

Assume maximum bar size = 20 mm

Nominal cover ≥ 20 mm NAD 6.4(a)

Use 25 mm nominal cover

Check requirements for fire resistance to BS 8110: Part 2[2]. NAD 6.1(a)

11.2.2 Materials

Type 2 deformed reinforcement with f_{yk} = 460 N/mm^2 NAD 6.3(a)

Concrete strength class C25/30, nominal aggregate size 20 mm

11.2.3 Effective dimensions of beam

CIRIA Guide 2 Cl.2.2.1

Effective span (l) $= l_o + (c_1/2 \le 0.1l_o) + (c_2/2 \le 0.1l_o)$

$$= 5000 + \frac{500}{2} + 0.1 \times 5000 = 5750 \text{ mm}$$

Note that EC2 effective spans typically come to the mid-point of the supports. Figure 2.4

Active height (h_a) = lesser of h and l

$h = 10920 > l = 5750$ mm

Therefore

$h_a = 5750$ mm

Thickness of beam = 300 mm

This thickness is used to be consistent with the CIRIA Guide 2 example. It will be necessary under EC2 to demonstrate that the required reinforcement can be accommodated within this width. The effective dimensions of the beam are shown in Figure 11.3

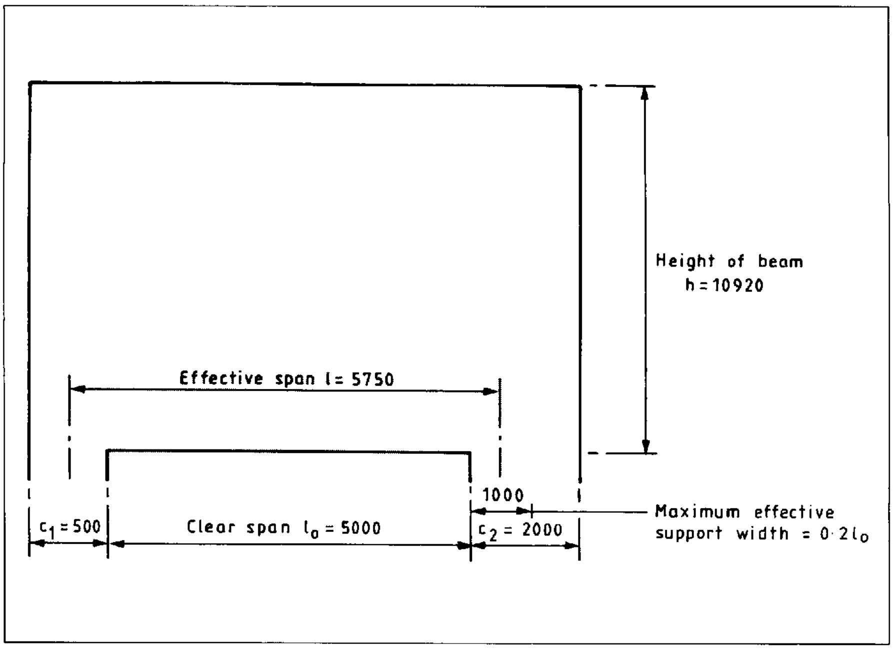


Figure 11.3 Effective dimensions of beam

11.2.4 Elastic stability – slenderness limits

The CIRIA Guide Simple Rules assume no reduction of capacity due to the slenderness of the section or to lack of adequate restraint. This is valid if every panel can be defined as braced and not slender.

In the examination of this condition, the CIRIA Guide states that the effective height of each panel is taken to be 1.2 × the shortest distance between centres of parallel lateral restraints (where there are effective lateral restraints at all four edges of the panel) or as 1.5 × the distance between the centres of the parallel lateral restraints (where one or two opposite edges of the panel are free). When both rotational and lateral movements are restrained the effective height may be taken as the clear distance between restraints.

In EC2 the demonstration is not quite as straightforward.

The floors are assumed to be held in position horizontally by an adequate bracing system and are 'braced' in accordance with EC2. **4.3.5.3.2**

The floor slabs are monolithic with the wall so the effective height, l_o, is calculated from the relevant clauses in EC2 referring to columns. However the design example does not give any information on the adjacent structure so k_A cannot be calculated but $\beta = l_o/l_{col}$ cannot exceed 1.0 and therefore **4.3.5.3.5 Eqn 4.60 Figure 4.27**

$$l_o \leq 1.0 \times 3540 = 3540 \text{ mm}$$ **4.3.5.3.5(1)**

The wall is considered slender if λ exceeds the greater of 25 or $15/\sqrt{\nu_u}$ **4.3.5.3.5(2)**

$$i = \text{radius of gyration} = 300/\sqrt{12} = 86 \text{ mm}$$

$$\lambda = l_o/i = 3540/86 = 41.2$$

$$\nu_u = N_{Sd}/A_c f_{cd}$$

N_{Sd} say in lower storey (bottom loads not considered)

$$= 57.6 \times 3 \times 1.35 + 44 \times 3 \times 1.5 + 2 \times 475 \times 1.35/5.75$$

$$= 654 \text{ kN/m}$$

$$\nu_u = \frac{654 \times 10^3}{1000 \times 300 \times 25/1.5} = 0.13$$

$$15/\sqrt{\nu_u} = 41.6 > \lambda = 41.2$$

The wall is not slender

11.2.5 Loading

Loading details are shown in Figure 11.2 and evaluated in Tables 11.1 and 11.2.

Table 11.1 Characteristic loads

Total loads	Q_k (kN/m)	G_k (kN/m)
Slab at level		
14.220	44.0*	57.6*
10.680	44.0*	57.6*
7.140	44.0	57.6
3.600	44.0	57.6
Self-weight = 0.3 × 10.92 × 24	–	78.6
	176.0	309.0
Point loads at level 14.220; 2 @ 475 kN*, which are considered as dead loads		
Vertical forces applied above a level of 3.30 + 0.75 × 5.75 = 7.620 are considered as top loading and loads below as hanging loads		

*considered as top loads

CIRIA Guide 2 Cl.2.3 Cl.2.3.1

CIRIA Guide 2 Cl.2.3.1(1)

In EC2 differing γ_F values produce slightly different design forces to those in CIRIA Guide 2

$$\gamma_G = 1.35, \quad \gamma_Q = 1.5$$

Ultimate distributed top load

$$\gamma_G G_k + \gamma_Q Q_k = 1.5 \times 88 + 1.35 \times 115.2 = 288 \text{ kN/m}$$

Table 11.2 Hanging loads

Loads applied within the depth of the beam	Q_k (kN/m)	G_k (kN/m)
Slab at level 7.140	44.0	57.6
Self-weight	–	78.6
	44.0	136.2
Loads applied to the bottom of the beam		
Slab at level 3.600	44.0	57.6
Total hanging load	88.0	193.8

The CIRIA Guide Simple Rules apply where the intensity of any load is less than $0.2f_{cu}$ and the load is applied over a length which exceeds $0.2l$. More intense loads and those applied over shorter lengths are considered to be 'concentrated', in which case reference should be made to the Supplementary Rules in the Guide.

CIRIA Guide 2 Cl.2.3.1(5)

To allow for design to EC2 where different γ_F values and concrete strength classes are used, the check for load intensity might reasonably be made against

$$0.2 \times \text{ratio of } \gamma_F \text{ values} \times (f_{cu}/f_{ck}) \times f_{ck}$$

$$= 0.2 \times \left(\frac{1.35}{1.4}\right)\left(\frac{30}{25}\right) f_{ck} = 0.23 f_{ck}$$

Ultimate concentrated top load = 1.35 × 475 = 641 kN

Allowing for 45° spread of load through thickness of slab

Load intensity = $641 \times 10^3/(800 \times 300) = 2.67 \text{ N/mm}^2$

This loading is well below $0.23f_{ck}$ but, because the length of the loaded area is less than $0.2l = 0.2 \times 5750 = 1150$ mm, some additional reinforcement may be required and must be calculated using the Supplementary Rules in the CIRIA Guide.

11.2.6 Moment and shears

CIRIA Guide 2 Cl.2.3.2

11.2.6.1 Reactions due to loads on clear span

The arrangement of loading and supports assumed for calculating the bending moments is shown in Figure 11.4. These loads are for the fully loaded system.

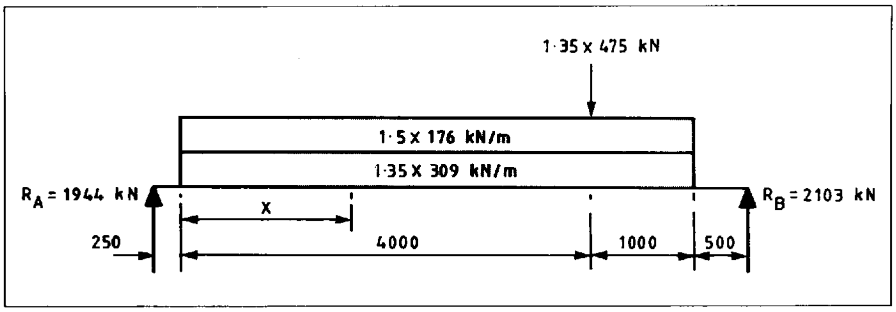


Figure 11.4 Total loads at ultimate limit state

Reactions from total loads are

$R_A = 1944$ kN, $R_B = 2103$ kN

Shear forces must be considered for top- and bottom-load cases separately. Consider the bottom-loaded case shown in Figure 11.5.

Total bottom load $= 1.35 \times 193.8 + 1.5 \times 88 = 393.6$ kN/m

Reactions from bottom applied loads are

$R_{Ab} = 1027$ kN, $R_{Bb} = 941$ kN

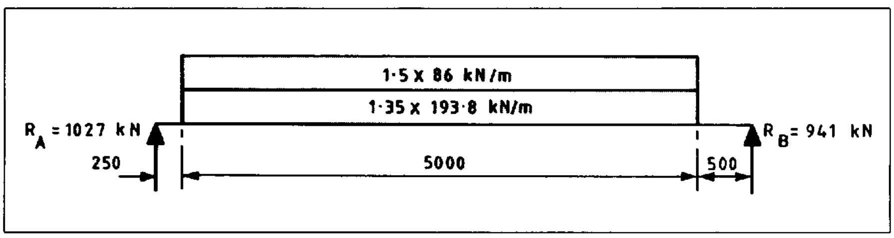


Figure 11.5 Hanging loads at ultimate limit state

Reactions from top loads are thus

$R_{At} = 1944 - 1027 = 917$ kN

$R_{Bt} = 2103 - 941 = 1162$ kN

11.2.6.2 Additional shear forces due to loads over supports

Loads acting over the effective support width apply an additional shear force to the critical section of the beam (i.e., at the support face). In this example, one of the point loads acts at the centre line of the actual support, B, as shown in Figure 11.6.

CIRIA Guide 2 Cl.2.3.2

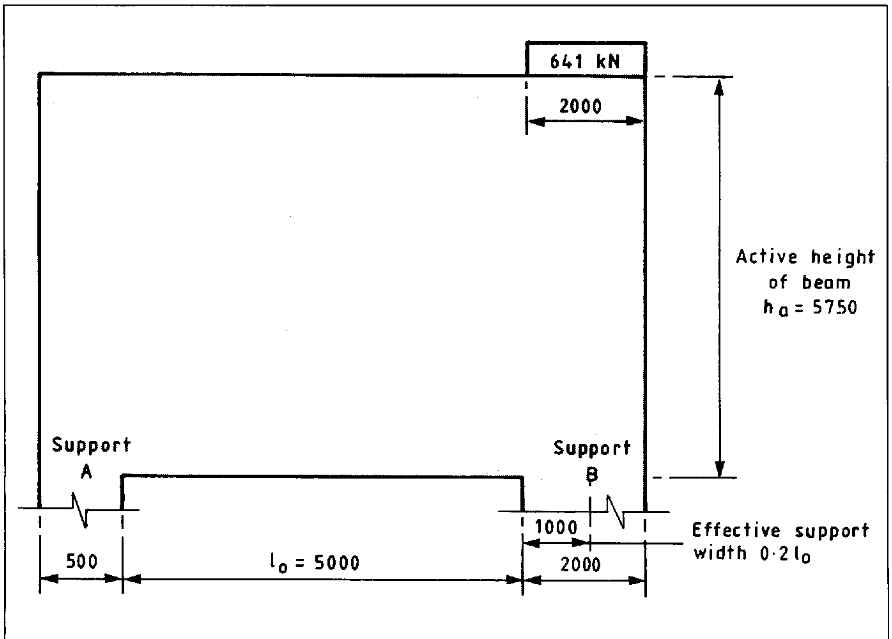


Figure 11.6 Additional load at support B

Since the effective support width is half the actual width, the additional shear force

$$= 0.5 \times 641 \times (h_a - 0.2l_o)/h_a$$

$$= 320.5(5.75 - 0.2 \times 5.0)/5.75 = 265 \text{ kN}$$

11.2.6.3 Total shear forces

At support A

Top loading	(V_{At})	=	917 kN
Bottom loading	(V_{Ab})	=	1027 kN
Total	(V_A)	=	1944 kN

At support B

Top loading	(V_{Bt})	=	1162 + 265 = 1427 kN
Bottom loading	(V_{Bb})	=	941 kN
Total	(V_B)	=	2368 kN

11.2.6.4 Maximum bending moment

Position of zero shear (where x is distance from face of support A) is given by

$$1944 - (1.5 \times 176 + 1.35 \times 309)x = 0$$

Therefore $x = 2.85$ m

$$M = 1944\,(2.85 + 0.25) - 681 \times \frac{2.85^2}{2} = 3261 \text{ kNm}$$

11.2.7 Strength design

Bending capacity check in accordance with CIRIA Guide 2 — CIRIA Guide 2 Cl.2.4 & 2.4.1

$l/h_a \quad = \quad 1 \; < \; 1.5$

Hence there is no need to check the compression in the concrete and the area of steel required may be calculated from a lever arm given as

$z \quad = \quad 0.2l + 0.4h_a \quad = \quad 3450 \text{ mm}$

For the reinforcement area there is no difference in using EC2 equations.

$$f_{yd} = \frac{f_{yk}}{\gamma_s} = \frac{460}{1.15} = 400 \text{ N/mm}^2$$ Table 2.3

$A_s = \dfrac{M}{f_{yd}z} = \dfrac{3261 \times 10^6}{400 \times 3450} = 2363 \text{ mm}^2$

This is 15% less than CIRIA Guide 2, predominantly due to a higher yield strength reinforcement used in this example, but also in part because of lower γ_F values used in EC2.

11.2.8 Detailing of principal bending moment reinforcement

CIRIA Guide 2 states 'Reinforcement is not to be curtailed in the span and may be distributed over a depth of $0.2h_a$. A minimum steel percentage in relation to the local area of concrete in which it is embedded is given in Table 1'. — CIRIA Guide 2 Cl.2.4.1 Cl.2.6.2

Minimum steel percentage = 0.71%

A maximum bar spacing for a maximum crack width of 0.3 mm is given in Table 2 of the CIRIA Guide.

Spacing ≤ 165 mm, for b = 300 mm

Reinforcement may be distributed over depth = 0.2 × 5750 = 1150 mm

i.e. minimum number of bars in face $= \dfrac{1150}{165} = 7$

Use 14T16 for main tension reinforcement

$A_s \quad = \quad 2814 \text{ mm}^2 \; > \; 2363 \text{ mm}^2$ required .... OK

EC2 requires for beams a maximum bar size or a maximum bar spacing to limit cracking under quasi-permanent loading. — 4.4.2.3

Quasi-permanent loading = $G_k + 0.3Q_k$ — 4.4.2.3(3)

Ratio to ultimate loading = 0.56

Estimate of steel stress $= 0.56f_{yd} \times \dfrac{A_{s,req}}{A_{s,prov}} = 188 \text{ N/mm}^2$

Maximum bar size is 25 > 16 mm .... OK — Table 4.11

Maximum bar spacing is 250 (pure flexure) > 165 mm .... OK — Table 4.12

Note that only one of these conditions needs to be met.

In CIRIA Guide 2, the bars must be anchored to develop 80% of the maximum ultimate force beyond the face of the support and 20% of the maximum ultimate force at or beyond a point $0.2l_o$ from the face of the support, or at or beyond the far face of the support, whichever is less. The main reinforcement must be anchored so that the concrete within the area of support relied upon for bearing is adequately confined. — CIRIA Guide 2 Cl.2.4.1

EC2 for deep beams requires that the reinforcement, corresponding to the ties considered in the design model, should be fully anchored beyond the nodes by bending up the bars, by using U-hoops or by anchorage devices, unless a sufficient length is available between the node and the end of the beam permitting an anchorage length of $l_{b,net}$. — 5.4.5(1)

The EC2 requirements are clearly more onerous.

Support A anchorage

$$l_{b,net} = \frac{\alpha_a l_b A_{s,req}}{A_{s,prov}} \geq l_{b,min}$$

5.2.3.4.1 Eqn 5.4

where

α_a = 0.7 for curved bars with side cover $\geq 3\phi$

f_{bd} = 2.7 N/mm^2 for good bond in bottom half of pour — Table 5.3

$$l_b = \frac{\phi}{4}\left(\frac{f_{yd}}{f_{bd}}\right) = \frac{\phi}{4}\left(\frac{400}{2.7}\right) = 37\phi$$

Eqn 5.3

$A_{s,req}$ = 2363 mm^2, $A_{s,prov}$ = 2814 mm^2

Therefore

$$l_{b,net} = 0.7 \times (37 \times 16) \times \frac{2363}{2814} = 348 \text{ mm}$$

There is insufficient distance to accommodate such an anchorage length beyond the centre-line of the column.

If U-bars or loops are provided, the minimum internal diameter of the bend needs to satisfy the requirement for curved bars. This is an indirect check on the crushing of the concrete inside the bend and the tabulated value could be multiplied by $A_{s,req}/A_{s,prov}$. — NAD Table 8

Minimum internal diameter of bend = 13ϕ = 208 mm

Note that it is necessary to check that sufficient space is available in the final detailing.

At support B a straight anchorage will be sufficient to meet both CIRIA Guide 2 and EC2 requirements.

11.2.9 Minimum longitudinal steel

CIRIA Guide 2 refers to the British Standard CP 110, and EC2 will be slightly more onerous.

For beams generally

$$A_s \nless 0.6 b_t d / f_{yk} \nless 0.0015 b_t d$$ 5.4.2.1.1

Basing the flexural steel on the active height assumed for the beam design

$$A_s = 0.0015 \times 300 \times 5750 = 2588 \text{ mm}^2$$

Deep beams should normally be provided with a distributed reinforcement near both sides, the effect of each being equivalent to that of an orthogonal mesh with a reinforcement ratio of at least 0.15% in both directions. 5.4.5(2)

$$A_s = 0.003 \times 300 \times 1000 = 900 \text{ mm}^2\text{/m}$$

This reinforcement should also satisfy the requirement that beams with a total depth of 1.0 m or more, where the main reinforcement is concentrated in only a small proportion of the depth, should be provided with additional skin reinforcement to control cracking on the side faces of the beam. This reinforcement should be evenly distributed between the level of the tension steel and the neutral axis, and should be located within the links. 4.4.2.3(4)

$$\frac{A_s}{A_{ct}} = k_c k f_{ct,eff} / \sigma_s$$ 4.4.2.2(3) Eqn 4.78

where

k_c = 0.4 assuming value for bending is to be used

k = 0.5 4.4.2.3(4)

$f_{ct,eff}$ = 3 N/mm² using suggested value

$\sigma_s = f_{yk} = 460$ N/mm² 4.4.2.3(4)

Hence

$$\frac{A_s}{A_{ct}} = 0.4 \times 0.5 \times 3/460 = 0.0013$$

The requirements of either Table 4.11 or Table 4.12 of EC2 should be met.

Assume steel stress = $(\frac{1}{2}) \times$ value for main bars = 94 N/mm² 4.4.2.3(4) Table 4.11

Maximum bar size

ϕ_{max} = 32 mm

Maximum bar spacing in 'pure tension' condition Table 4.12 4.4.2.3(4)

s_{max} = 200 mm

Use T10 @ 150 mm crs (EF) above level of main reinforcement

$A_s = 1048 > 900 \text{ mm}^2/\text{m}$ .... OK

11.2.10 Shear design

CIRIA Guide 2 separates top and bottom loads and deals with the design of these in different ways.

CIRIA Guide 2 Cl.2.4.2

In principle the bottom loads require vertical tension hangers to suspend the loads above the active beam height, h_a, with horizontal web reinforcement needed in the area of the supports.

The top-load shear calculations include taking into account any additional shear force induced by top loads over the supports.

Under the simple design rules the top-load shear capacity is not improved by web reinforcement.

A nominal, orthogonal pattern of web reinforcement not less than the minimum required for walls in BS 8110 is intended. This is augmented for bottom loads and in the area of the supports.

The detail of the CIRIA Guide calculation is not repeated here and reference should be made to the original document. The reinforcement details are shown in Figures 11.7 and 11.8.

CIRIA Guide 2 Figures 93 & 13

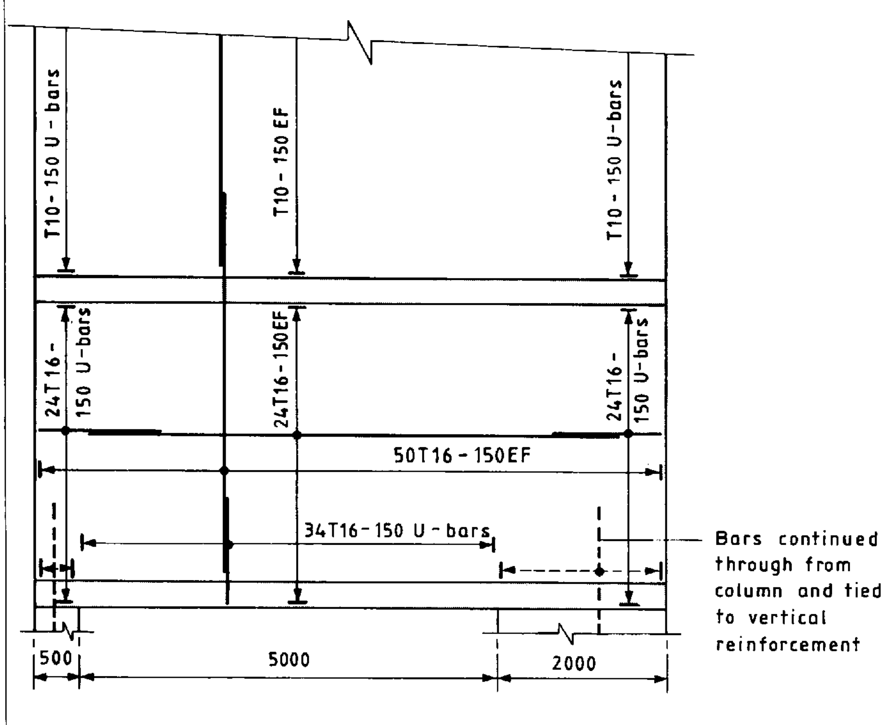


Figure 11.7 Arrangement of reinforcement

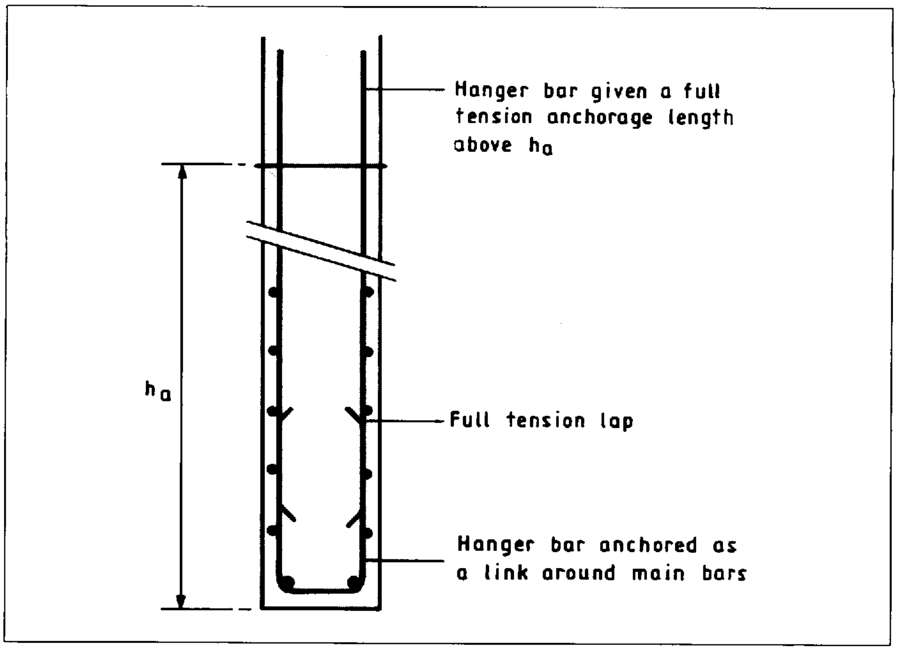


Figure 11.8 Detail at bottom of wall

Using the standard method in EC2 **4.3.2.2(7)**

V_{Sd} = 2368 kN maximum at support B

V_{Rd1} = $\tau_{Rd}k(1.2 + 40\rho_l)b_w d$ **Eqn 4.18**

τ_{Rd} = 0.30 N/mm^2 **Table 4.8**

A_{sl} = 0.15%A_c, therefore ρ_l = 0.0015

k = 1 as d > 0.6 m

d = $h - 0.2h_a$, say

= 10920 − 0.2 × 5750 = 9770 mm

b_w = 300 mm

Therefore

V_{Rd1} = 0.3 (1.2 + 40 × 0.0015)300 × 9770 × 10^{-3} = 1108 kN

V_{Sd} > V_{Rd1}, therefore shear reinforcement needed

V_{Rd3} = $V_{cd} + V_{wd}$, where V_{cd} = V_{Rd1} **Eqn 4.22**

Therefore

V_{wd} ≥ 2368 − 1108 = 1260 kN

$$\frac{A_{sw}}{s} \geq \frac{1260 \times 10^3}{0.9df_{ywd}} \text{ with } f_{ywd} = 400 \text{ N/mm}^2$$ **Eqn 4.23**

≥ 0.36 mm^2/mm

Where the load is not acting at the top of a beam, suspension reinforcement should be provided to transfer the load to the top. 4.3.2.4.1P(3)

The bottom load identified previously = 393.6 kN/m

Area of hanger steel needed with f_{yd} = 400 N/mm^2

$$\frac{A_{sh}}{s} = \frac{393.6}{400} = 0.99 \text{ mm}^2/\text{mm}$$

Use T16 @ 150 mm crs. (EF) when hangers are needed

$$\frac{A_s}{s} = \frac{2 \times 201}{150} = 2.68 > 0.36 + 0.99 = 1.35 \text{ mm}^2/\text{mm} \ldots \text{ OK}$$

Use T10 @ 150 mm^2(EF) elsewhere

$$\frac{A_s}{s} = \frac{2 \times 78}{150} = 1.04 > 0.36 \text{ mm}^2/\text{mm} \ldots\ldots\ldots\ldots\ldots \text{ OK}$$

Minimum shear reinforcement with f_{ywk} = 460 N/mm^2

ρ_w = 0.0012 by interpolation Table 5.5

$$\frac{A_{sw}}{s} = 0.0012 b_w = 0.36 \text{ mm}^2/\text{mm} \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{ OK}$$

For heavily loaded deep beams it may prove more complicated to justify the shear.

11.2.11 Further guidance

CIRIA Guide 2 has further guidance for reinforcement in support regions, under concentrated loads and around holes in beams. CIRIA Guide 2

Supplementary design rules are also provided for deep beams that include arrangements excluded by the simplified method.

It is because of the extent of this information that the initial suggestion was made, that a complete design to the CIRIA Guide is undertaken and then a parallel design to EC2 is performed as appropriate, to demonstrate compliance with the individual clauses.

12 LOAD COMBINATIONS

12.1 Introduction

EC2[1] considers all loads as variables in time and space and applies statistical principles to arrive at the loads for design. There is an underlying assumption that the basic loads themselves are described in statistical terms. Thus, when variable loads of different origins, for example superimposed loads on floors and wind loads on the faces of buildings, have to be considered acting together in a load case, the probability of both loads not being at their full characteristic values is allowed for by multipliers called ψ factors. These factors too are derived statistically and values are given in EC1[20] and the NAD to EC2[1].

Thus when a number of variable loads have to be considered simultaneously in any load case, each load is treated in turn as the primary load and others are considered secondary. The primary load is applied at its characteristic value multiplied by the partial safety factor. All secondary loads are applied at their characteristic values multiplied by the partial safety factor and further multiplied by a ψ factor. These ψ factors vary depending upon the limit state and the type of loading being considered.

Mathematically the design load for ultimate limit state may be represented as:

$$\Sigma\, \gamma_{G,j} G_{k,j} \;+\; \underbrace{\gamma_{Q,1} Q_{k,1}}_{\text{primary load}} \;+\; \underbrace{\sum_{i>1} \gamma_{Q,i} \psi_{o,i}\, Q_{k,i}}_{\text{secondary load}}$$

While the above procedure is the general approach, EC2 also provides simplified rules:

(a) where only one variable load occurs the design load
$= \Sigma\, \gamma_{G,j} G_{k,j} + 1.5 Q_k$

(b) when more than one variable load occurs the design load
$= \Sigma\, \gamma_{G,j} G_{k,j} + 1.35 \sum_{i>1} Q_{k,i}$

It is important to note that this Code permits the use of either approach although in some circumstances the general method may result in higher loading.

In practice the simplified procedure will be perfectly satisfactory for most situations and could be used.

The following examples are given to illustrate the thinking behind the general approach and indicate where the general approach may be required.

Usually, when dead loads produce a favourable effect, γ_G can be taken as unity. However, if the variation of the magnitude of the dead load is likely to prove sensitive then γ_G should be taken as 0.9.

For the particular case of continuous beams without cantilevers, the Code permits the use of $\gamma_G = 1.35$ for all the spans.

When calculating the loads on vertical elements of multi-storey structures the vertical loads may be based on either:

(a) loads from beams multiplied by suitable ψ and γ_f values; or

(b) loads on beams multiplied by γ_f values and a global reduction in loading applied using the procedure given in BS 6399[21]. This is the approach in the NAD.

12.2 Example 1 – frame

For the frame shown in Figure 12.1 identify the various load arrangements, to check the overall stability. Assume office use for this building.

Note that the load arrangements for the design of elements could be different.

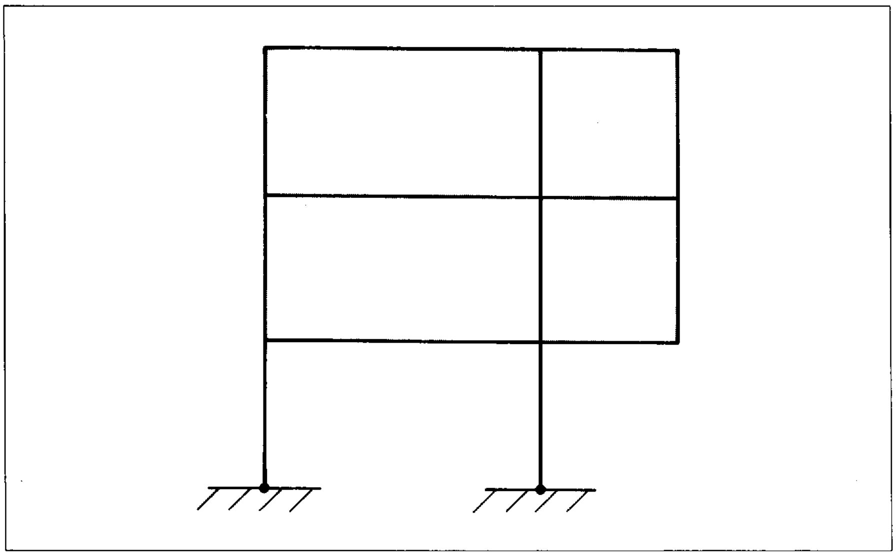


Figure 12.1 Frame configuration – example 1

12.2.1 Notation

Characteristic loads/m

G_{kr} = dead – roof

G_{kf} = dead – floor

Q_{kr} = imposed – roof

Q_{kf} = imposed – floor

Characteristic load/frame

W_k = wind – roof or floor

12.2.2 Load cases – example 1

Fundamental load combination to be used is 2.3.2.2P(2)

$$\Sigma\gamma_{G,j}G_{k,j} + \gamma_{Q,1}Q_{k,1} + \sum_{i>1}\gamma_{Q,i}\psi_{o,i}Q_{k,i}$$ Eqn 2.7(a)

As the stability will be sensitive to a possible variation of dead loads, it will be necessary to allow for this as given in EC2 Section 2.3.2.3(P3).

Take

$\gamma_{G,inf}$ = 0.9, $\gamma_{G,sup}$ = 1.35 Table 2.2

γ_Q = 1.5 NAD Table 1

ψ_o = 0.7 for imposed loads (offices)

12.2.2.1 Load case 1 – example 1

Treat the wind load as the primary load (see Figure 12.2).

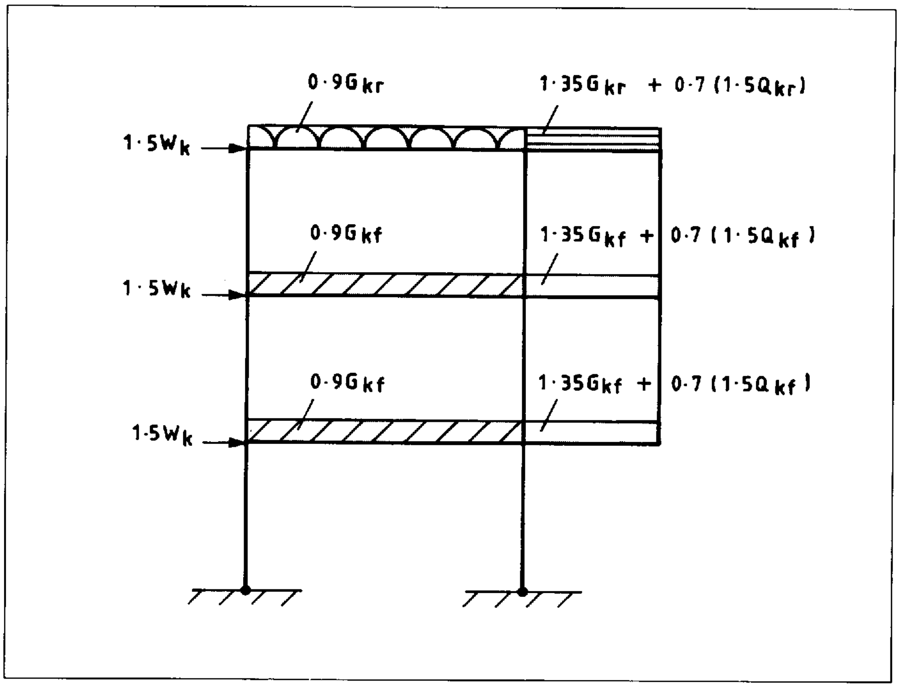


Figure 12.2 Load case 1 – example 1

12.2.2.2 Load case 2 – example 1

Treat the imposed load on the roof as the primary load (see Figure 12.3).

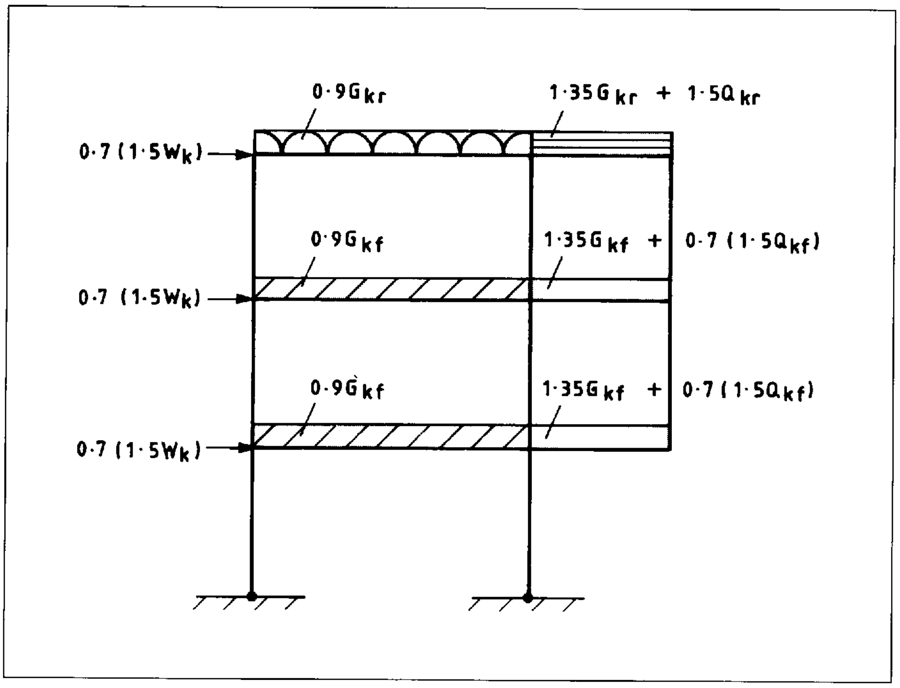


Figure 12.3 Load case 2 – example 1

12.2.2.3 Load case 3 – example 1

Treat the imposed load on the floors as the primary load (see Figure 12.4).

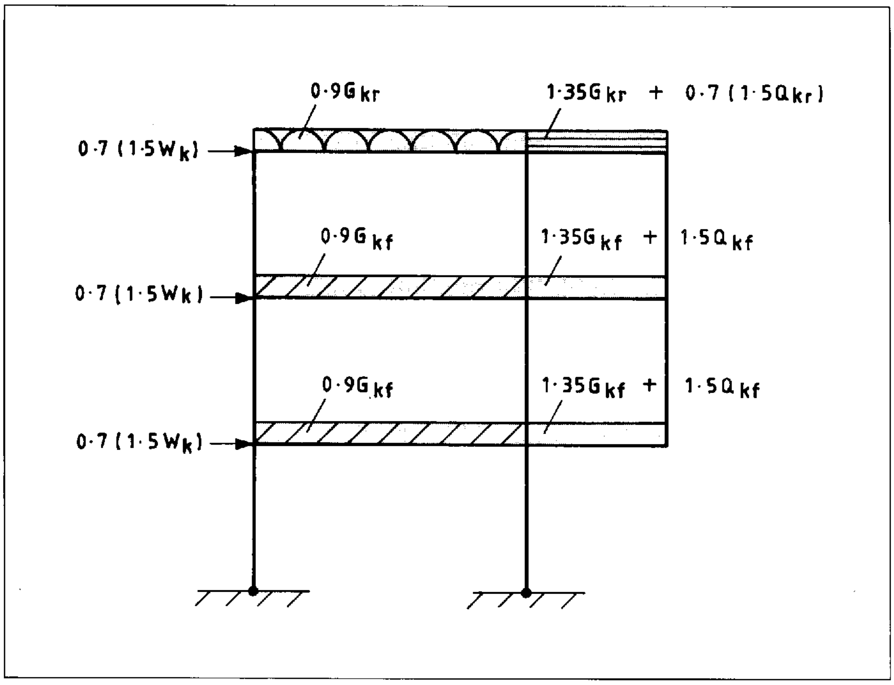


Figure 12.4 Load case 3 – example 1

12.2.2.4 Load case 4 – example 1

Consider the case without wind loading treating the imposed floor loads as the primary load (see Figure 12.5).

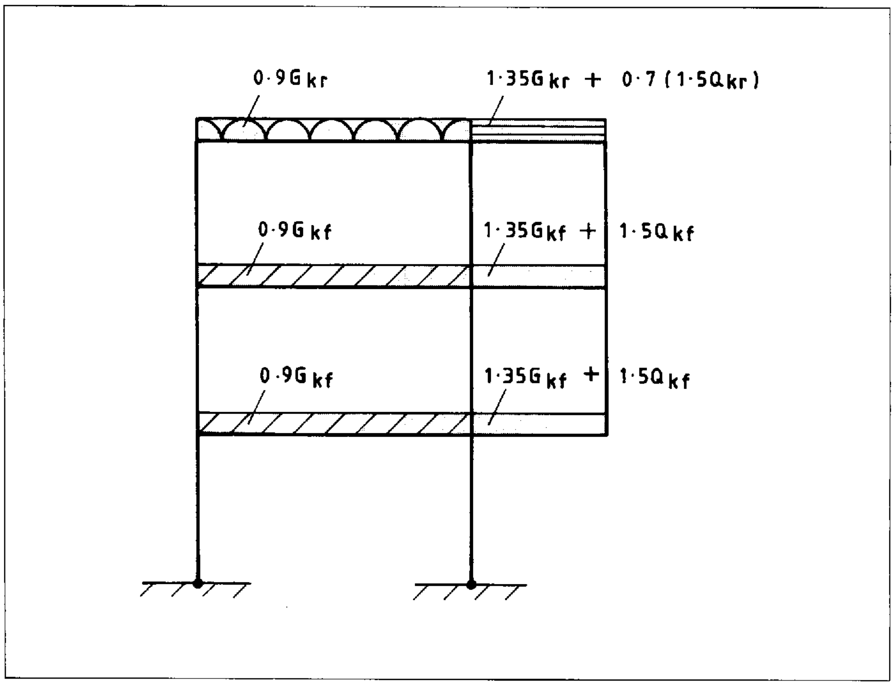


Figure 12.5 Load case 4 – example 1

12.2.2.5 Load case 5 – example 1

Consider the case without wind loading treating the imposed roof load as the primary load (see Figure 12.6).

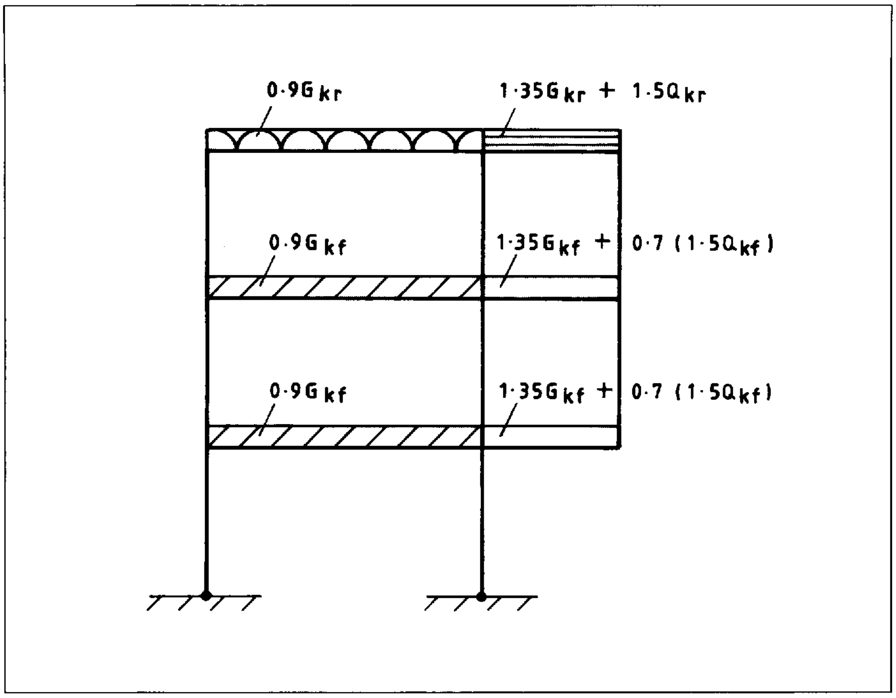


Figure 12.6 Load case 5 – example 1

Note:
When the wind loading is reversed, another set of arrangements will need to be considered. However, in problems of this type, the critical arrangements are likely to be found intuitively rather than by directly searching through all the theoretical possibilities.

12.3 Example 2 – continuous beam 1

Identify the various load arrangements for the ultimate limit state for the design of the four-span continuous beam shown in Figure 12.7.

Assume that spans 1–2 and 2–3 are subject to domestic use and spans 3–4 and 4–5 are subject to parking use.

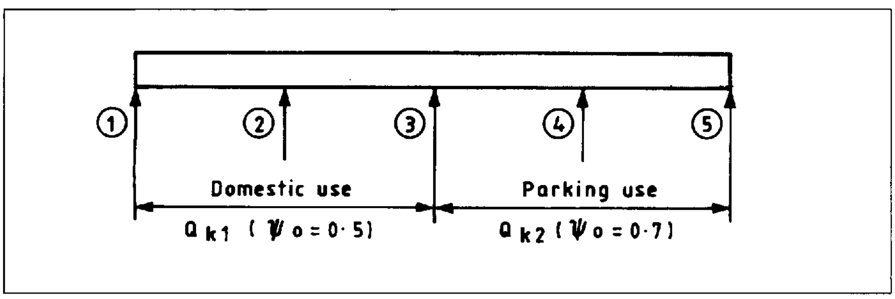


Figure 12.7 Beam configuration – example 2

12.3.1 Notation

G_k = characteristic dead load/m

Q_{k1} = characteristic imposed load/m (domestic use)

Q_{k2} = characteristic imposed load/m (parking use)

12.3.2 Load cases – example 2

Fundamental load combination to be used is 2.3.2.2P(2)

$$\Sigma \gamma_{G,j} G_{k,j} + \gamma_{Q,1} Q_{k,1} + \sum_{i>1} \gamma_{Q,i} \psi_{o,i} Q_{k,i}$$ Eqn 2.7(a)

For beams without cantilevers the same value of self-weight may be applied to all spans, i.e., $1.35G_k$. 2.3.2.3(4)

The load cases to be considered for the imposed loads are 2.5.1.2(4)

(a) alternate spans loaded; and

(b) adjacent spans loaded.

The various load arrangements are shown in Figure 12.8.

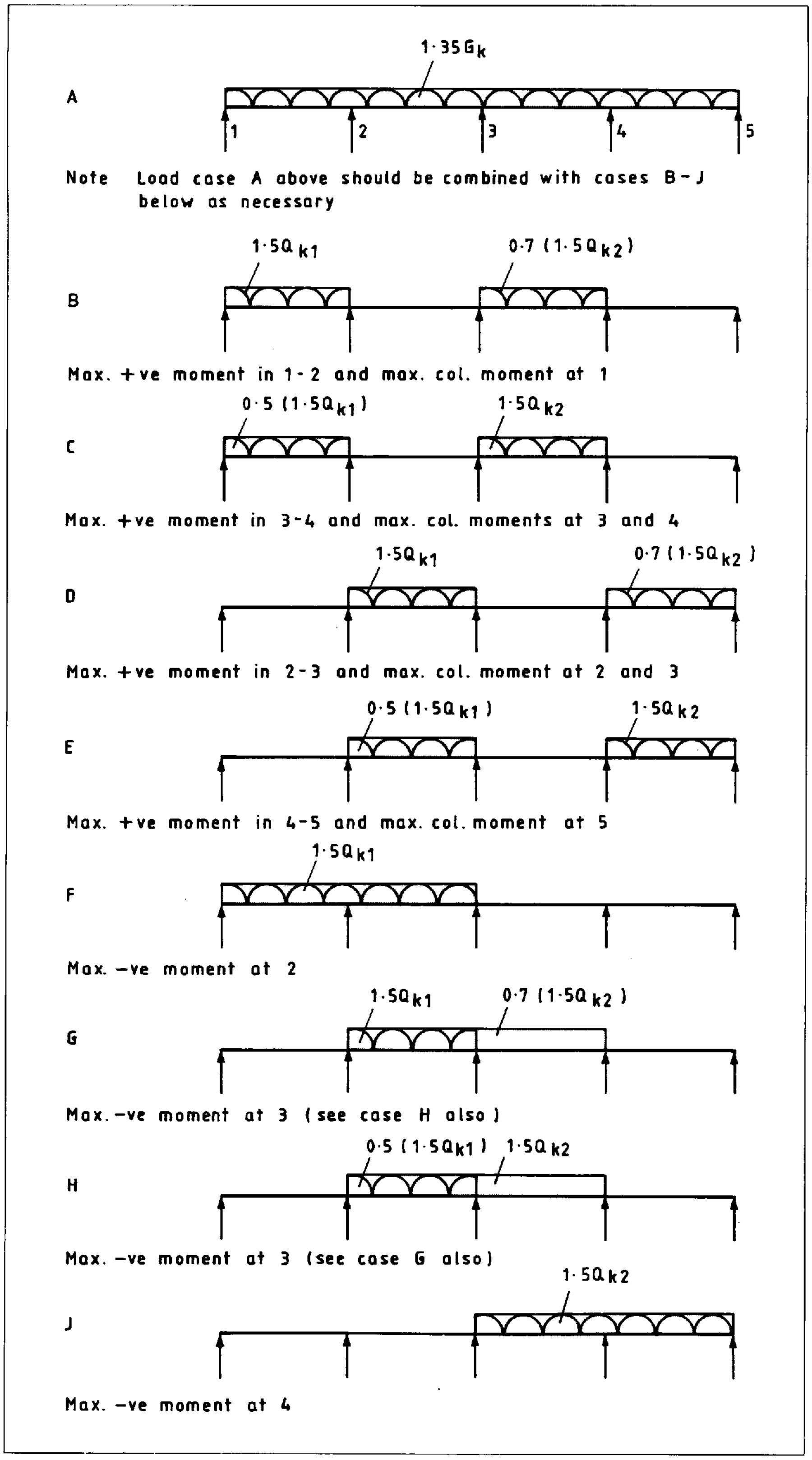


Figure 12.8 Load cases – example 2

12.4 Example 3 – continuous beam 2

For the continuous beam shown in Figure 12.9, identify the critical load arrangements for the ultimate limit state. Assume that the beam is subject to distributed dead and imposed loads, and a point load at the end of the cantilever arising from the dead load of the external wall.

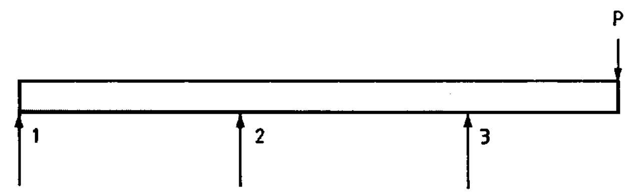


Figure 12.9 Beam configuration – example 3

12.4.1 Notation

G_k = characteristic dead load/m

Q_k = characteristic imposed load/m

P = characteristic point load (dead)

12.4.2 Load cases – example 3

The fundamental combinations given in EC2 Section 2.3.2.2 should be used. Note that the presence of the cantilever prohibits the use of the same design values of dead loads in all spans. 2.3.2.3(4)

The various load arrangements are shown in Figures 12.10 to 12.13.

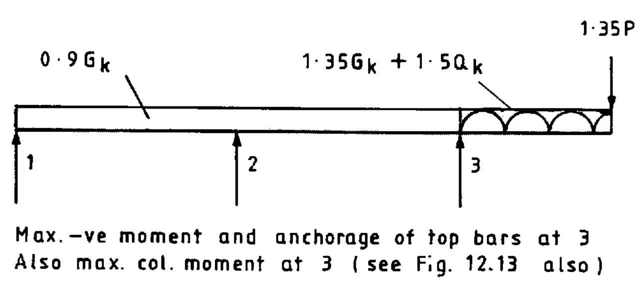


Figure 12.10 Load case 1 – example 3

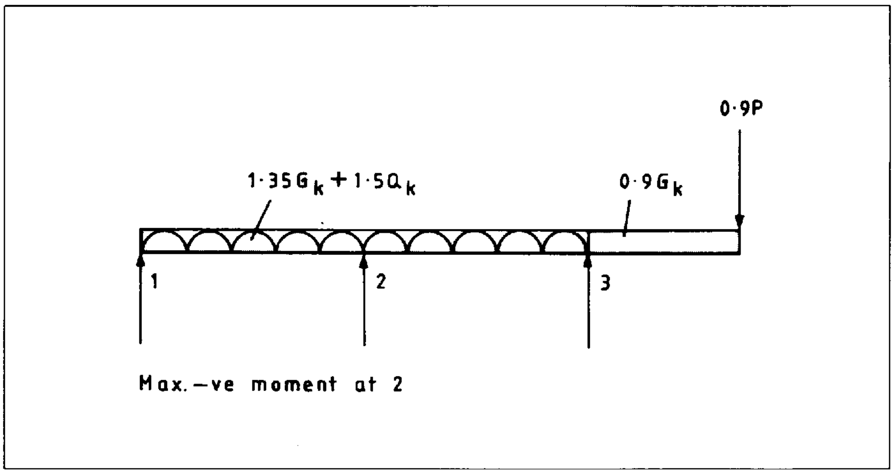


Figure 12.11 Load case 2 – example 3

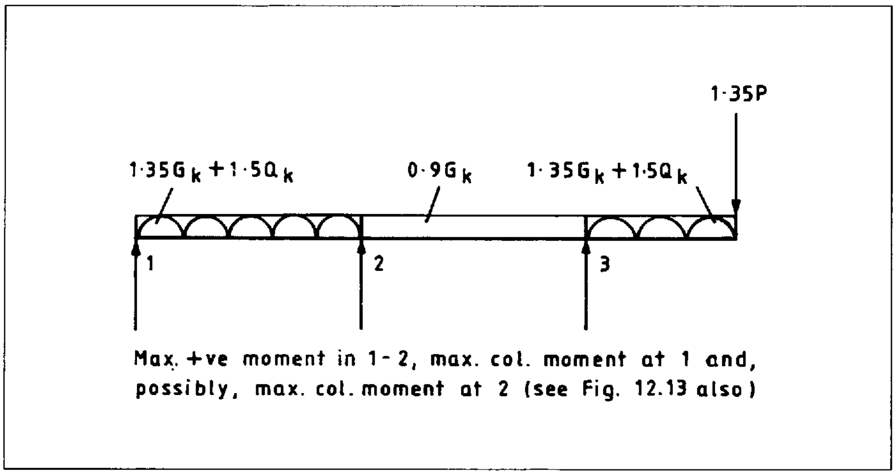


Figure 12.12 Load case 3 – example 3

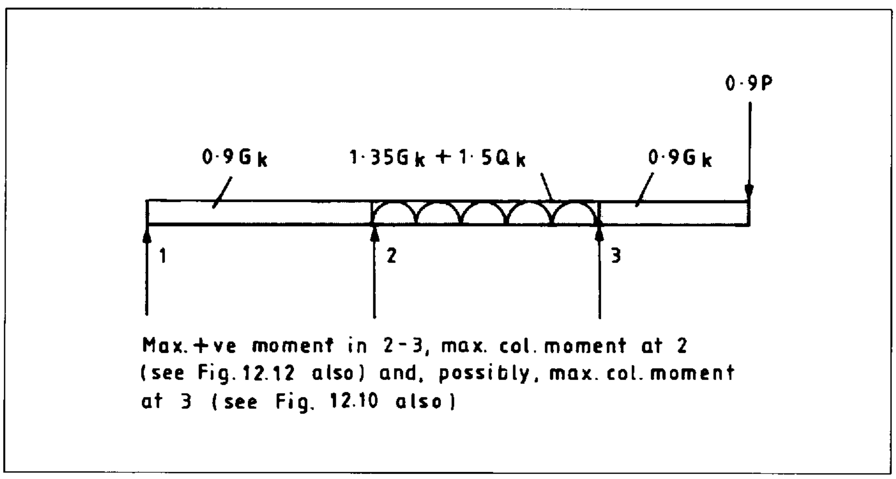


Figure 12.13 Load case 4 – example 3

12.5 Example 4 – tank

A water tank, as shown in Figure 12.14, of depth H (in metres) has an operating depth of water h (in metres). Calculate the design lateral loads for the ultimate limit state.

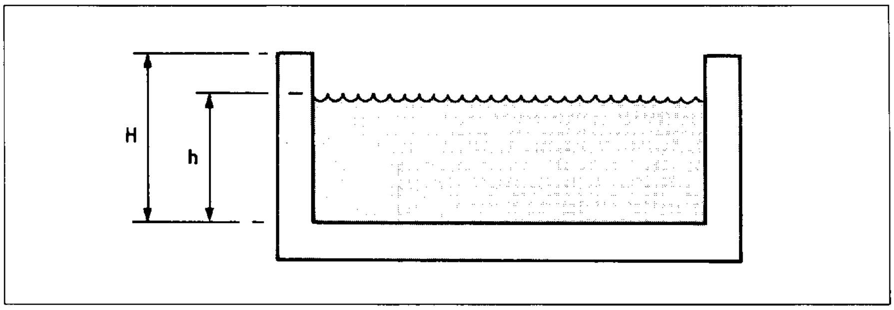


Figure 12.14 Tank configuration – example 4

According to the draft EC1, earth loads are permanent loads. The same reasoning can be applied to lateral pressures caused by water. The NAD for EC2 confirms this. NAD 6.2(c)

Design can therefore be based on the pressure diagram shown in Figure 12.15.

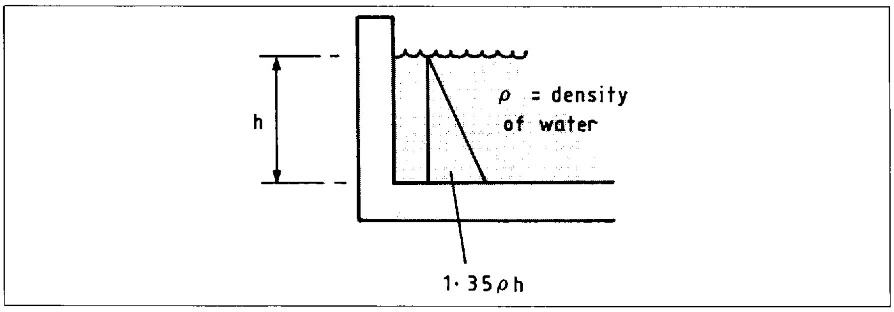


Figure 12.15 Design load based on operating water depth – example 4

Consideration should also be given to the worst credible water load, which in this case will correspond to a depth of H, i.e., water up to the top of the tank. EC2 permits the variation of the partial safety factor $\gamma_{G,j}$ depending on the knowledge of the load $G_{k,j}$.

However, the method of establishing $\gamma_{G,j}$ is not given. The basis adopted in BS 8110: Part 2[(2)] could be used and a factor of 1.15 applied instead of 1.35. In this case the alternative design loading will be as shown in Figure 12.16.

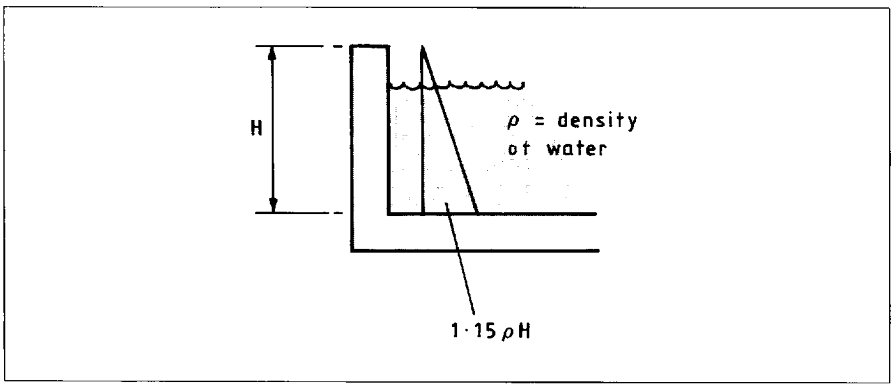


Figure 12.16 Design load based on worst credible water depth – example 4

13.1 Concrete grades

3.1.2.4

EC2[1] uses the cylinder strength, f_{ck}, to define the concrete strength in design equations, although the cube strength may be used for control purposes. The grade designations specify both cylinder and cube strengths in the form C cylinder strength/cube strength, for example C25/30.

It may occasionally be necessary to use cube strengths which do not exactly correspond to one of the specified grades. In such instances a relationship is required between cylinder and cube strength in order to obtain an appropriate value for f_{ck}. The relationship implicit in EC2 and ENV 206[6] is given in Figure 13.1.

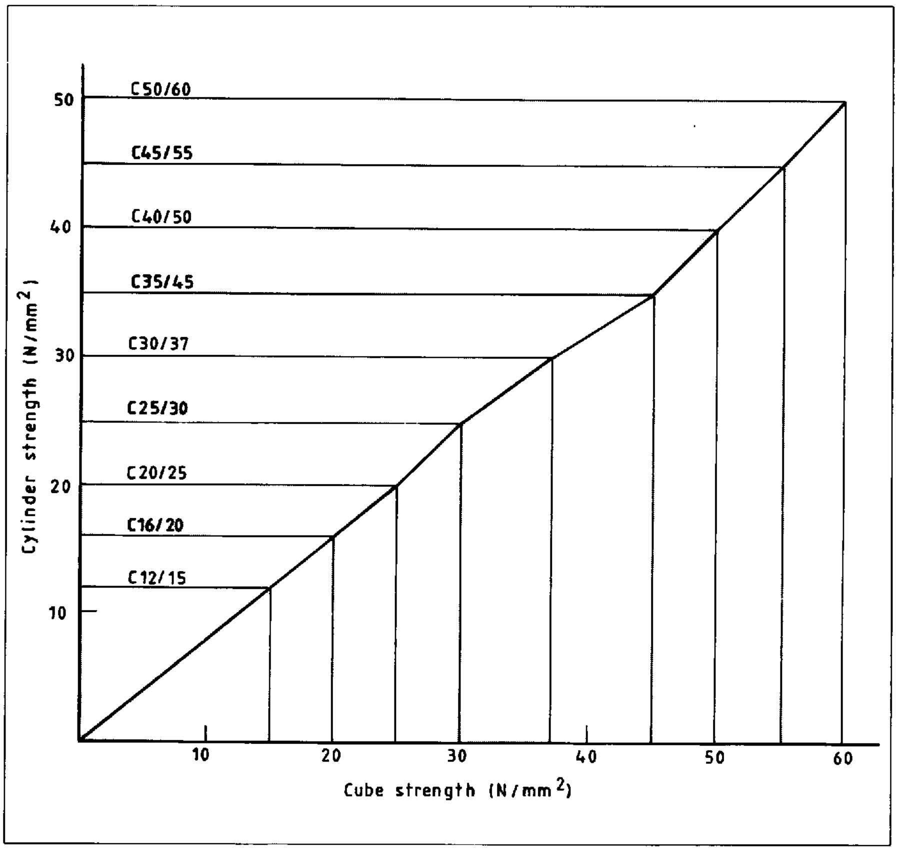


Figure 13.1 Relationship between cube and cylinder strength of concrete

13.2 Singly reinforced rectangular beam sections

The following equations and design tables have been derived from the assumptions given in 4.3.1 and 4.2.1.3.3(b) of the Code combined with the redistribution limits given in 2.5.3.4.2. They are entirely in accordance with EC2. 4.3.1 4.2.1.3.3 2.5.3.4.2

13.2.1 Equations for singly reinforced rectangular beam sections

$$\omega = \frac{A_s f_{yk}}{bdf_{ck}} = 0.652 - \sqrt{0.425 - 1.5\mu}$$

$$x/d = 1.918\,\omega$$

where

$$\mu = \frac{M}{bd^2 f_{ck}}$$

Table 13.1 gives ω and x/d as a function of μ.

Table 13.1 Flexural reinforcement in singly reinforced rectangular sections.

$\mu = \frac{M}{bd^2 f_{ck}}$	$\omega = \frac{A_s f_{yk}}{bdf_{ck}}$	x/d	z/d	$\mu = \frac{M}{bd^2 f_{ck}}$	$\omega = \frac{A_s f_{yk}}{bdf_{ck}}$	x/d	z/d
0.010	0.012	0.022	0.991	0.090	0.113	0.217	0.913
0.012	0.014	0.027	0.989	0.092	0.116	0.223	0.911
0.014	0.016	0.031	0.987	0.094	0.119	0.228	0.909
0.016	0.019	0.036	0.986	0.096	0.122	0.234	0.907
0.018	0.021	0.040	0.984	0.098	0.125	0.239	0.904
0.020	0.023	0.045	0.982	0.100	0.127	0.245	0.902
0.022	0.026	0.050	0.980	0.102	0.130	0.250	0.900
0.024	0.028	0.054	0.978	0.104	0.133	0.256	0.898
0.026	0.031	0.059	0.977	0.106	0.136	0.261	0.896
0.028	0.033	0.063	0.975	0.108	0.139	0.267	0.893
0.030	0.035	0.068	0.973	0.110	0.142	0.272	0.891
0.032	0.038	0.073	0.971	0.112	0.145	0.278	0.889
0.034	0.040	0.077	0.969	0.114	0.148	0.284	0.887
0.036	0.043	0.082	0.967	0.116	0.151	0.289	0.884
0.038	0.045	0.087	0.965	0.118	0.154	0.295	0.882
0.040	0.048	0.092	0.963	0.120	0.157	0.301	0.880
0.042	0.050	0.096	0.961	0.122	0.160	0.307	0.877
0.044	0.053	0.101	0.960	0.124	0.163	0.313	0.875
0.046	0.055	0.106	0.958	0.126	0.166	0.319	0.873
0.048	0.058	0.111	0.956	0.128	0.169	0.324	0.870
0.050	0.060	0.116	0.954	0.130	0.172	0.330	0.868
0.052	0.063	0.121	0.952	0.132	0.175	0.336	0.865
0.054	0.065	0.125	0.950	0.134	0.179	0.343	0.863
0.056	0.068	0.130	0.948	0.136	0.182	0.349	0.861
0.058	0.071	0.135	0.946	0.138	0.185	0.355	0.858
0.060	0.073	0.140	0.944	0.140	0.188	0.361	0.856
0.062	0.076	0.145	0.942	0.142	0.191	0.367	0.853
0.064	0.078	0.150	0.940	0.144	0.195	0.373	0.851
0.066	0.081	0.155	0.938	0.146	0.198	0.380	0.848
0.068	0.084	0.160	0.936	0.148	0.201	0.386	0.846
0.070	0.086	0.165	0.934	0.150	0.205	0.393	0.843
0.072	0.089	0.170	0.932	0.152	0.208	0.399	0.840
0.074	0.092	0.176	0.930	0.154	0.211	0.405	0.838
0.076	0.094	0.181	0.928	0.156	0.215	0.412	0.835
0.078	0.097	0.186	0.926	0.158	0.218	0.419	0.833
0.080	0.100	0.191	0.924	0.160	0.222	0.425	0.830
0.082	0.102	0.196	0.921	0.162	0.225	0.432	0.827
0.084	0.105	0.202	0.919	0.164	0.229	0.439	0.824
0.086	0.108	0.207	0.917	0.166	0.232	0.446	0.822
0.088	0.111	0.212	0.915				

13.2.2 Limits to use of singly reinforced beam sections

Limits to x/d as a function of the amount of re-distribution carried out are given in EC2. These can be re-written as 2.5.3.4.2

For concrete grades $\leq$ C35/45

$$(x/d)_{lim} = \frac{\delta - 0.44}{1.25}$$

For concrete grades > C35/45

$$(x/d)_{lim} = \frac{\delta - 0.56}{1.25}$$

Equations can be derived for ω_{lim} and μ_{lim} for rectangular sections as a function of $(x/d)_{lim}$. These are

$$\mu_{lim} = 0.4533(x/d)_{lim}[1 - 0.4(x/d)_{lim}]$$

$$\omega_{lim} = (x/d)_{lim}/1.918$$

Table 13.2 gives values of $(x/d)_{lim}$, μ_{lim} and ω_{lim} as a function of the amount of re-distribution carried out. EC2 states that plastic design, for example yield line analysis, can be used where $x/d \leq 0.25$. The limits corresponding to this value are also included in the table. 2.5.3.5.5

Table 13.2 Limiting values

% redistribution	δ	$(x/d)_{lim}$		μ_{lim}		ω_{lim}	
		$f_{ck} \leq 35$	$f_{ck} > 35$	$f_{ck} \leq 35$	$f_{ck} > 35$	$f_{ck} \leq 35$	$f_{ck} > 35$
0	1.00	0.448	0.352	0.1667	0.1371	0.2336	0.1835
5	0.95	0.408	0.312	0.1548	0.1238	0.2127	0.1627
10	0.90	0.368	0.272	0.1423	0.1099	0.1919	0.1418
15	0.85	0.328	0.232	0.1292	0.0954	0.1710	0.1210
20	0.80	0.288	0.192	0.1155	0.0803	0.1502	0.1001
25	0.75	0.248	0.152	0.1013	0.0647	0.1293	0.0792
30	0.70	0.208	0.112	0.0864	0.0485	0.1084	0.0584
Plastic design		0.25		0.1020		0.1303	

13.3 Compression reinforcement

Compression reinforcement is required in any section where $\mu > \mu_{lim}$. The amount can be calculated from

$$\omega' = \frac{\mu - \mu_{lim}}{0.87(1 - d'/d)}$$

where

ω' = mechanical ratio of compression steel

$$= \frac{A'_s}{bd} \times \frac{f_{yk}}{f_{ck}}$$

d' = depth from compression face to centroid of compression reinforcement

A'_s = area of compression reinforcement

The area of tension reinforcement can now be obtained from

$$\omega = \omega_{lim} + \omega'$$

Equations above for ω' and ω are valid for $d'/x \leq 1 - f_y/805$.

13.4 Flanged beams

For beams with flanges on the compression side of the section, the formulae for rectangular sections may be applied provided

$$x/d \leq h_f/d$$

where

h_f = thickness of the flange

For beams where the neutral axis lies below the flange, it will normally be sufficiently accurate to assume that the centre of compression is located at mid-depth of the flange. Thus, for singly reinforced beams, approximately

$$M = 0.87\, A_s f_{yk}(d - h_f/2)$$

The neutral axis depth is given approximately by

$$x/d = 1.918\,(b/b_r)\omega - 1.25\,(b/b_r - 1)h_f/d$$

where b_r is the rib width and the definition of ω is identical to that for a rectangular section.

13.5 Symmetrically reinforced rectangular columns

Figures 13.2(a) to (e) give non-dimensional design charts for symmetrically reinforced columns where the reinforcement can be assumed to be concentrated in the corners. The broken lines give values of K_2 in Eqn 4.73 of EC2. Eqn 4.73

Where the reinforcement is not concentrated in the corners, a conservative approach is to calculate an effective value of d' as illustrated in Figure 13.3.

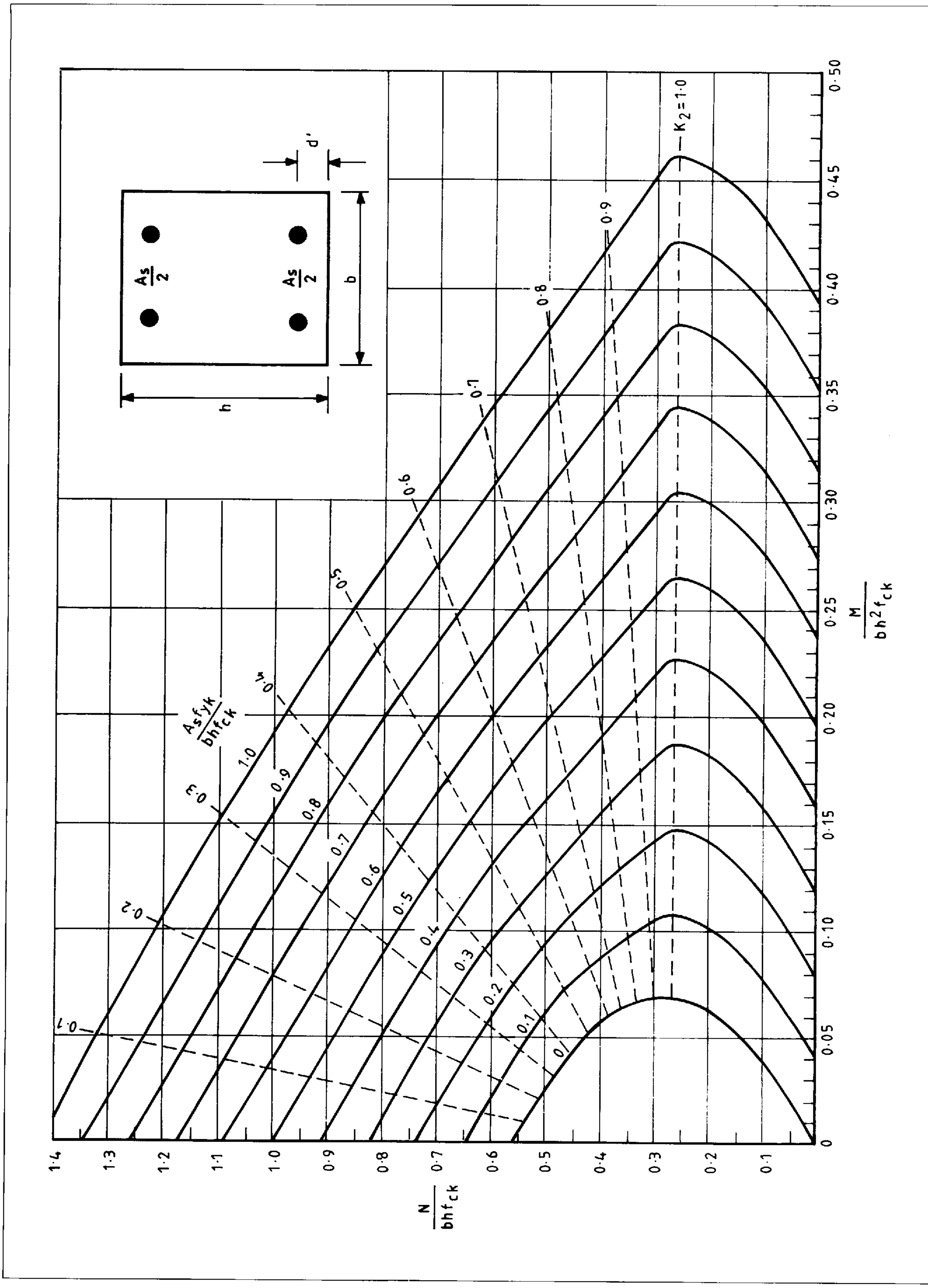


Figure 13.2(a) Rectangular columns (d'/h = 0.05)

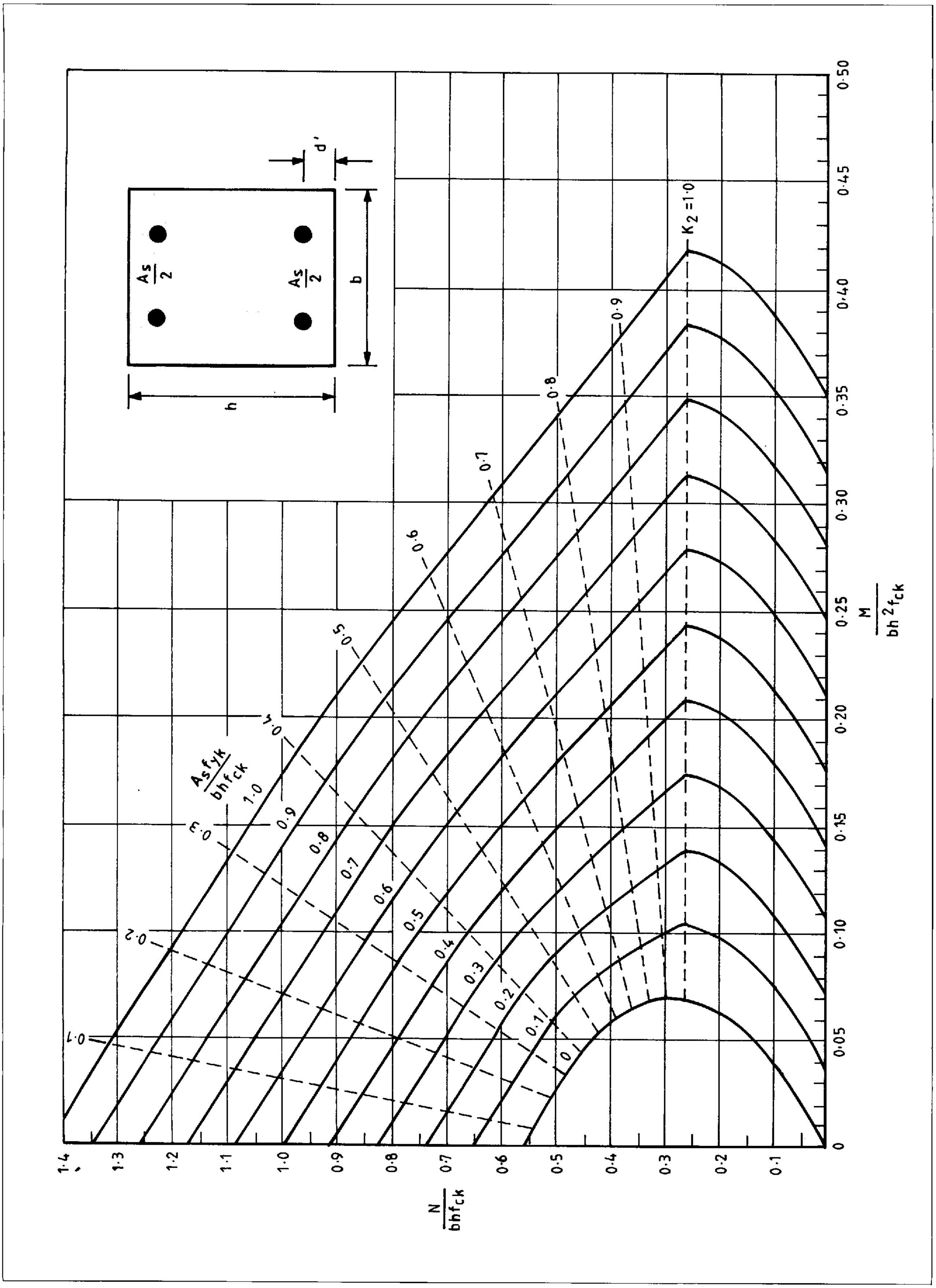


Figure 13.2(b) Rectangular columns (d'/h = 0.10)

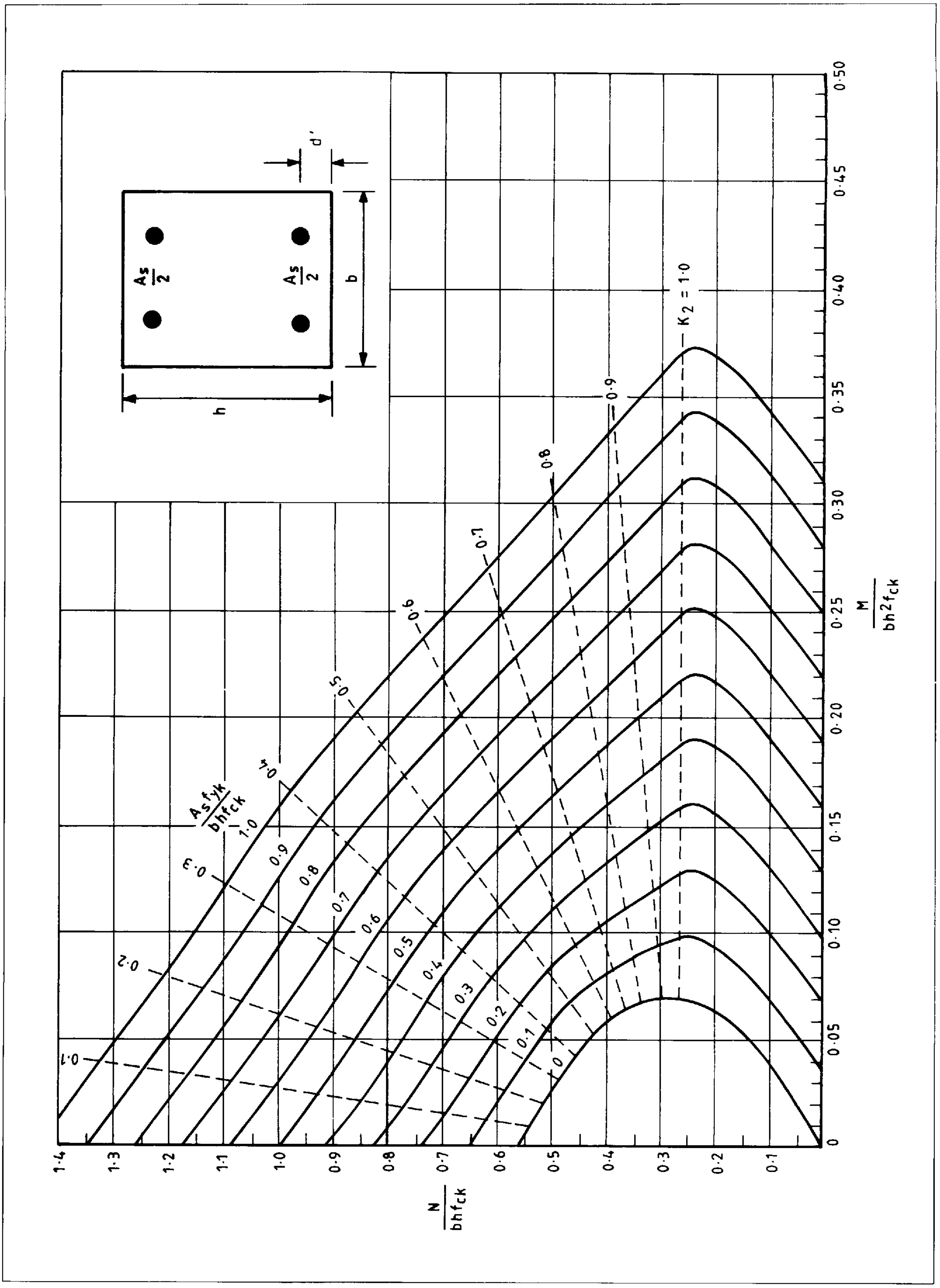


Figure 13.2(c) Rectangular columns (d'/h = 0.15)

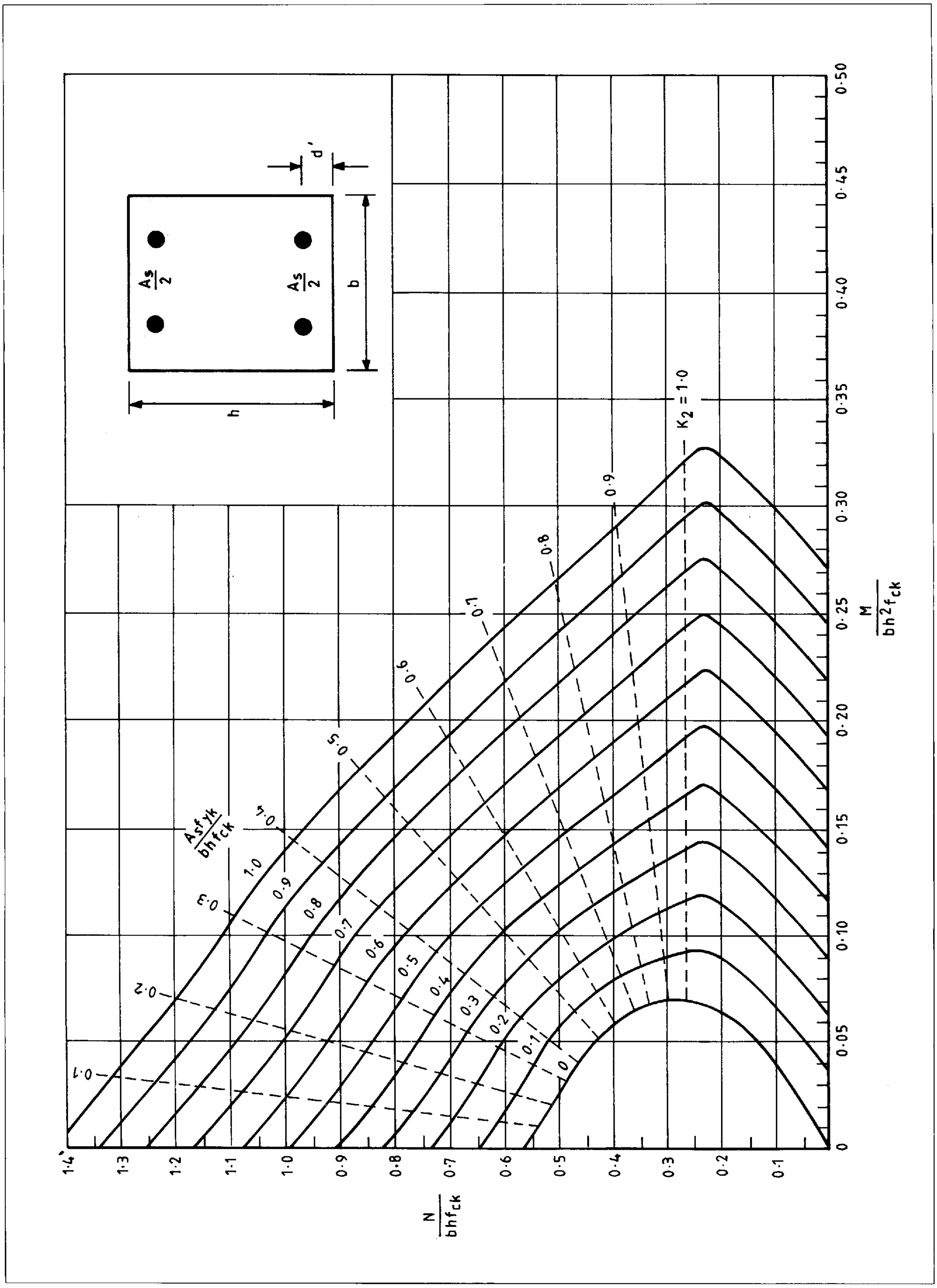


Figure 13.2(d) Rectangular columns (d'/h = 0.20)

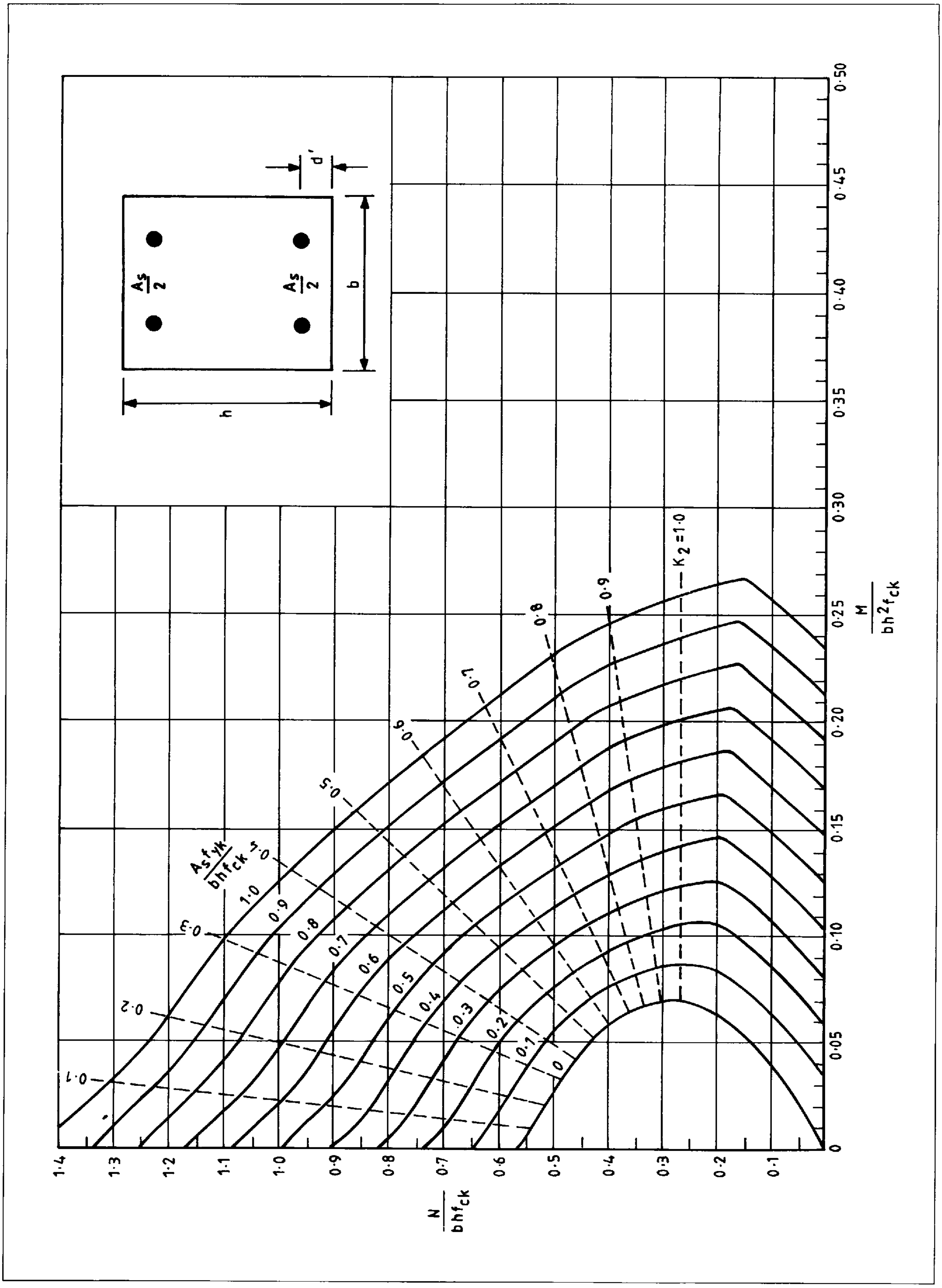


Figure 13.2(e) Rectangular columns (d'/h = 0.25)

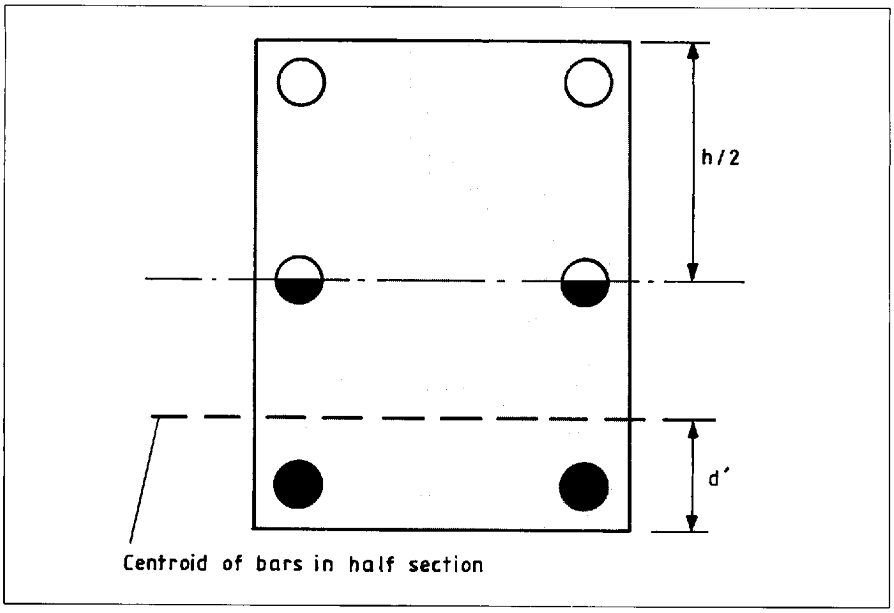


Figure 13.3 Method of assessing an effective value for d'

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